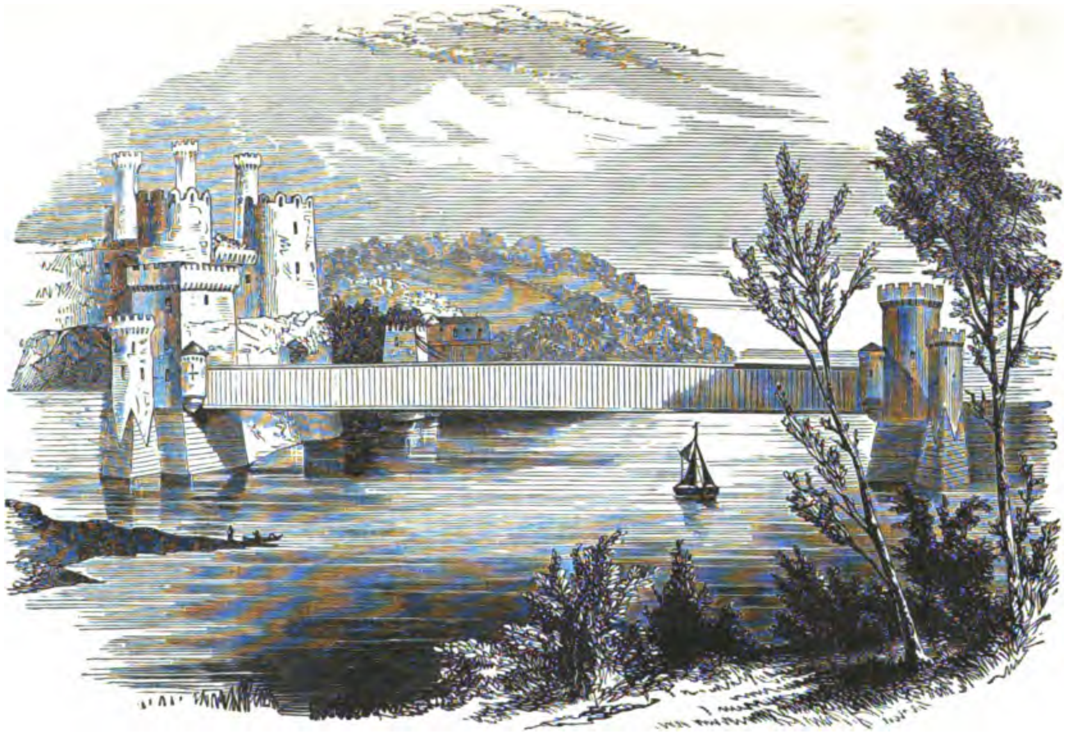

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PRINCIPLES

OF

THE MECHANICS

OF

MACHINERY AND ENGINEERING.

BY JULIUS WEISBACH,

PROFESSOR OF MECHANICS AND APPLIED-MATHEMATICS IN THE ROYAL MINING ACADEMY OF FREIBERG.

FIRST AMERICAN EDITION.

EDITED

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IN TWO VOLUMES.

ILLUSTRATED WITH EIGHT HUNDRED AND THIRTEEN ENGRAVINGS ON WOOD.

VOL. II.

APPLIED MECHANICS.

PHILADELPHIA:

LEA AND BLANCHARD.

1849.

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PREFACE BY THE AMERICAN EDITOR.

In submitting to the American reader the second volume of Weisbach's Mechanics of Machinery and Engineering, we cannot, perhaps, better express our own appreciation of the value of this part of his labors, than by citing a passage from the advertisement of the English translator, Prof. L. Gordon.

"The usefulness of this second volume will be manifest from the practical interest and importance of the subjects treated. The first part of the volume, though far from giving a complete theory of engineering and architectural construction, brings many important questions of practice before the student in a simple form, and in a light by which he will more readily recognize the bearings of the mathematical calculations on this subject, than has been usually the case in English works. The second part of the volume contains the only Theoretical Treatise on Water Power of the least practical value hitherto printed in the English language. The real *importance* of such a treatise will be variously estimated; but as it is the first publication in which a systematic attempt is made to familiarize English Machinists with the application of exact reasoning in developing the theory of the machines treated of, it is believed that it must be interesting to them, and if so, it cannot fail to be useful likewise."

The most available treatise on the numerous forms of reaction wheels, and other turbines to which the American student has access, is believed to be embraced in this volume. The author, it may be observed, has not contented himself with giving a general theory on that subject, but by skillfully analyzing the several effects produced, and computing separately the prejudicial and the useful resistances

to the action of the water, has presented conclusions challenging the highest confidence, especially as they stand confirmed, in most cases, by the results of numerous direct experiments.

In reference to the water-pressure engine, also, it may be said that the present volume will afford to the American student the most direct and positive information as to the useful application of water in that species of motor.

In the original work of Prof. Weisbach, the second volume embraced the science applicable to the steam engine, but as that subject has now assumed so distinct an importance, and as its numerous topics and improvements could scarcely be presented with sufficient clearness, in a less space than an entire volume, it has been deemed expedient, in imitation of the English translator, to reserve that branch of the mechanics of engineering for a separate treatise.

In assigning to their appropriate chapters the additions of the translator, which had in the English edition been thrown into the form of an appendix, we have been guided by a desire of rendering the work more serviceable to the student, by placing before him the whole matter pertaining to each branch under its appropriate head.

We have added a few articles particularly relating to the strength of materials, which, we hope, may not be found uninteresting to the student. Indeed, when we take into view the lamentable, and often wilful and obstinate disregard of the truths which science has elicited relative to this department of our subject; when we see machines and engines intended to perform the most powerful operations, and edifices, or monuments, designed to endure for ages, constructed of materials, either utterly worthless, or, at best, of very inferior character and durability, or containing in their composition the elements of weakness and decay, we may estimate, with some justness, the importance of those researches and computations, which prove what may be expected from the employment of good or bad materials respectively, for any of the purposes of the architect and engineer.

The fact that the public has often been basely imposed upon by reason of employing as architects and engineers those who would pander to the cupidity of contractors for materials and labor, and

erect public works wholly discreditable to the nation, is an additional reason why works, written for the purpose of imparting correct information on the physical properties and the relative values of materials, ought to be diligently studied by those who desire correct and reliable knowledge.

The list of illustrations which we have added will much facilitate reference to the several topics to which they relate, and the execution of the cuts, with the creditable manner in which they have been used by the printer, will be sufficiently apparent to the most casual observer.

WASHINGTON, *August*, 1849.

AUTHOR'S PREFACE.

In writing this, the second volume, I have adhered as closely as possible to my views of what the work should be, as explained in the preface to the first volume.

I am aware that these views are not adopted by all who are capable of judging in the matter, and that a more general and mathematical treatment of the subject would have been preferred by many. But I have now long experience in teaching to fall back upon, and am thereby convinced, that the comparatively elementary style adopted as it can be followed by those who have not made extensive mathematical acquirements, will more surely lead to the introduction of applications of Mechanical Science in the routine practice of engineers, than the more general methods of treating these subjects have done.

A basis on true principles and established facts, and simplicity in the method of analysis, are the main requisites in a work intended for the instruction and guidance of practical men. And it is chiefly the want of these, in technical literature, that has retarded the introduction of science amongst those engaged in the execution of works, and the erection of machinery. If in evolving rules of art, imperfect facts be assumed, or unwarranted hypotheses be adopted—if the essential be not distinguished from that which is merely collateral, and if important considerations be neglected, it cannot be expected that the rules deduced, however correct the process of deduction, will be available for any useful application. But this is no uncommon fault. Authors forget that the mathematics can only *guide* our ideas, and not *give* us any: and thus, in admiration of their analytical processes, they often overlook the worthlessness of the premises. Hence it arises that practicians not unfrequently reproach *theory* as valueless, whilst it is, in reality, the facts of the case that have been erroneously stated or applied. Besides, it is not an easy matter to deduce rules of art by the principles of

science; for this requires not only an intimate acquaintance with the subject investigated, but generally requires special observations or experiments to be made, in order to *create the facts*, so to speak, that are to be reasoned upon and reduced to a theory which shall interpret them.

In this second volume of his work, the Author has done his utmost to develop theories that will be found applicable in practice—to furnish the guide above alluded to—well aware, however, that his endeavors have only imperfectly succeeded.

This volume is divided into two parts; the first, the application of Mechanics in Construction, and the other to the theory of Machines recipients of Water and Wind Power. The Author regrets now his not having entered more at large into a discussion of the theory of the construction of wooden and stone bridges, and more particularly not to have been able to avail himself of the information contained in Ardant's *Etudes sur l'établissement des charpentes à grande portée*, as this subject is, in these times of railway extension, of especial importance (in Germany).

The second part of the volume is as concisely written as was consistent with the object I had in view. I now regret having been so brief on the important subject of Dynamometers. The chapter on Turbines may appear to some to err in excess, from my having given the details of the theory and construction of the old impact and pressure turbines; but I consider that it is important to be aware of the faults or imperfections of one construction of a machine, in order fully to appreciate the improvements introduced in a more perfect one. Again, the application of Water-pressure Engines, being almost entirely confined to the Mining Engineer's province, the fullness with which I have treated this engine may appear to exceed its relative importance. The circumstance, however, that there is no work in any language, that I am aware of, treating of these engines, must be my apology for attempting to fill that gap in technical literature.

I hope soon to preface a volume, containing a Treatise on Mechanism, and on the principle *Operators*, or machines performing various mechanical operations.

JULIUS WEISBACH.

FREIBERG, *December*, 1847.

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PRINCIPLES

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MACHINERY AND ENGINEERING.

SECTION I.

THE APPLICATION OF MECHANICS IN BUILDING.

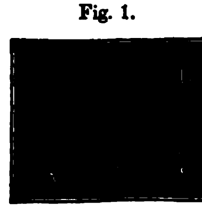
CHAPTER I.

OF THE EQUILIBRIUM AND PRESSURE OF SEMI-FLUIDS.

§ 1. *Sand, earth, corn seeds, shot, &c. &c.*, may be considered as *semi-fluids.*—They resemble fluids in so far as, like these, they require external support that they may preserve a particular form. The mutual adhesion of the parts of semi-fluids is of course greater than in the case of water. Water always requires external support, while this is only the more frequent case with so called semi-fluids; and whilst water is in equilibrium only when its surface is horizontal, the *disintegrated masses* or semi-fluids in question, may be in stable equilibrium, though their surface be inclined.

If the parts of a disintegrated mass be connected by their mutual friction alone, the mass will be in equilibrium when its surface is not inclined to the horizon at a greater angle than the *angle of repose* ρ (Vol. I. § 159). The *natural slope* of disintegrated masses is determined by the angle of repose. If by the *slope* of a declivity AB, Fig. 1, we understand the ratio $\frac{b}{a}$ of the base $AC=b$ to

Fig. 1.



the height $CB = a$, it is evidently $= cotang.\ \rho$, or, as $tang.\ \rho$ is equal to the *co-efficient* of friction f, $\frac{b}{a} = \frac{1}{f}$.

According to ***Martony de Köszegh***, the natural slope of perfectly dry soil, for example, $= 1,243$, and for moist soil $= 1,083$. Hence, the angle of slope in the first case is $= 39°$, and in the second $43°$.

For very fine sand, the *slope* $= \frac{5}{3}$, therefore, the angle of slope $= 31°$. For rye seeds, the author found $\rho = 30°$, for fine shot $\rho = 25°$, and for the finest shot $\rho = 22\frac{1}{2}°$.

Remark. Experiments on the slope of disintegrated masses are made by heaping them up, and dressing them off from below upwards.

§ 2. *Pressure of Earth.*—If a disintegrated mass, such as earth, be supported by a retaining wall, it exerts a pressure (*poussée*) against it, a knowledge of which is of importance in practice. Suppose a body of earth M, Fig. 2, supported by a retaining wall AC, the *back* of which is vertical. Take as a first case that the earth and wall are the same height, and the earth's surface in no way extraneously loaded. Suppose that a wedge-shaped piece ADE separates from the general mass, and thus rests on the retaining wall on the one side, and on the earth on the other; put the height AD of the earth and wall $= h$, the density of the earth $M = \gamma$, and the angle AED which the surface of separation AE makes with the *horizontal* $= \phi$. Let us consider a length of the mass (at right angles to the plane of the figure) equal unity, then the weight of the wedge ADE:

Fig. 2.

$$G = \frac{AD \,.\, DE}{2} \,.\, 1 \,.\, \gamma = \tfrac{1}{2}\, h \,.\, h\ cotg.\ \phi \,.\, \gamma = \tfrac{1}{2}\, h^2\ cotg.\ \phi \,.\, \gamma.$$

The vertical back AD is acted upon by the pressure $SP = P$ at right angles to it; and, therefore, it may be assumed that an equal opposite horizontal pressure maintains the prism ADE on the inclined plane. We know also (Vol. I. § 159) that a force will be taken up by a body if its direction does not deviate from the normal to the plane of contact by more than the angle of repose, and we may, therefore, assume that the second component force R of G is taken up by the mass below AE, even supposing its direction to deviate from the normal SN by angle $RSN = \rho$. As $NSG = AED = \phi$, we have $RSG = \phi - \rho$, and, therefore, the horizontal pressure on the retaining wall $P = G\ tang.\ (\phi - \rho)$, (compare Vol. I. § 162), or $P = \frac{1}{2}\, h^2\, \gamma\ cotg.\ \phi\ tang.\ (\phi - \rho)$.

This force depends upon an unknown angle ϕ, or upon the dimensions of the prism of pressure, and is thus different for different values of ϕ, and a maximum for a certain value. If, now, ADE be the prism of greatest pressure, and ADO a prism exerting a less pressure, we have in AEO a prism which requires no force to maintain it on its basis, but which would rather require some force

to pull it downwards. And so for other wedges AOH, &c., into which we might divide AEF, because these rest on still less inclinations; we may therefore assume, that by an opposite force equal to the maximum pressure P, not only the prism ADE, but also the prism below AE and AEF, is perfectly sustained, and that therefore this maximum pressure is that which the retaining wall is subjected to from the whole mass.

§ 3. *Prism of greatest Pressure.*—We must now determine the prism of maximum pressure. We have manifestly only to determine that value of ϕ for which *cotang.* ϕ *tang.* $(\phi - \rho)$ is a maximum.

Now *cotang.* ϕ *tang.* $(\phi-\rho) = \frac{\sin. (2\phi - \rho) - \sin. \rho}{\sin. (2\phi - \rho) + \sin. \rho}$, and as this fraction is greater, the greater *sin.* $(2\phi - \rho)$ is, we shall have *cotang.* ϕ *tang.* $(\phi - \rho)$ a maximum when *sin.* $(2\phi - \rho)$ is a maximum, that is $= 1$, or $2\phi - \rho = 90°$, *i. e.* $\phi = 45° + \frac{\rho}{2}$. Hence we name the pressure of the earth against the retaining wall:

$$P = \tfrac{1}{2} h^2 \gamma \; cotang. \left(45° + \frac{\rho}{2}\right) tang. \left(45° - \frac{\rho}{2}\right),$$

or since $cotang. \left(45° + \frac{\rho}{2}\right) = tang. \left(45° - \frac{\rho}{2}\right)$,

$$P = \tfrac{1}{2} h^2 \gamma \left[tang. \left(45° - \frac{\rho}{2}\right)\right]^2.$$

The complement of $\phi = 45° + \frac{\rho}{2}$, is $DAE = 45° - \frac{\rho}{2}, = \frac{90° - \rho}{2}$ = one half of DAF the complement to 90° of the angle of friction ρ. Therefore the surface AE of the prism of pressure bisects the angle DAF which the natural slope AF makes with the vertical AD. We can now very easily compare the pressure of a disintegrated or semi-fluid mass with that of water. In the latter the pressure is $\frac{1}{2} h^2 \gamma_1$ (Vol. I. § 276), when h = height, 1 = breadth of the pressed surface. In the case of earth, on the other hand, we have the pressure

$$P = \tfrac{1}{2} h^2 \varepsilon \gamma_1 \left[tang. \left(45° - \frac{\rho}{2}\right)\right]^2,$$

where γ_1 = the density of water, and ε the specific gravity of the semi-fluid. Hence the pressure of earth is always

$\varepsilon \left[tang. \left(45° - \frac{\rho}{2}\right)\right]^2$ times as great as the pressure of water, or the pressure of a semi-fluid may be set as equal to the pressure of perfect fluid of specific gravity $\varepsilon \left[tang. \left(45° - \frac{\rho}{2}\right)\right]^2$.

Thus we see that the pressure of earth increases gradually from the surface downwards, or is proportional to the pressure height.

It follows, likewise, that the *centre of pressure* of earth-works, &c. &c., coincides with the centre of pressure of water, and that, therefore, in the case in question, where the surface is a rectangle, it is at one-third of the height h from the base (Vol. I. § 278).

Example. If the specific gravity of a mass of corn seeds, heaped 6 feet high, be 0,776 (Vol. I. § 291, remark 1), it exerts a pressure against each foot in length of a vertical wall: $P = \frac{1}{2} . 6^2 . 0{,}776 . 63 . \left[\left(tang.\ 45^\circ - 15^\circ\right)\right]^2 = 18 . 63 . 0{,}776\ (tang.\ 30^\circ)^2 =$ $880 \times 0{,}5773{,}5^2 = 293\frac{1}{4}$ lbs. (English.)

§ 4. *Cohesion of Semi-fluids.*—In the above investigations we have omitted to consider the *cohesion*, or that mutual union of the parts of the mass, increasing with the surface of contact. As this cohesion, however, in the case of the less disintegrated masses, as, for instance, in well compacted earth, is not unimportant, we shall now introduce it into the formula. Let us put the *modulus of cohesion*, or the force of union for the unit of surface of contact $= x$, we have for the case shown in Fig. 2, the force required to separate the prism ADE on the surface

$$AE, = 1 . AE . x = \frac{x h}{sin.\ \phi}.$$

The vertical component $\frac{xh}{sin.\ \phi}\ sin.\ \phi = x\ h$ counteracts gravity, and the horizontal component $\frac{x h}{sin.\ \phi} . cos.\ \phi = x\ h\ cotg.\ \phi$, counteracts the pressure. If, therefore, we introduce into the formula $P = G\ tang.\ (\phi - \rho)$, instead of P, $P + x\ h\ cotang.\ \phi$, and instead of G, $G - x\ h$, we then obtain the equation:

$$P = (G - x\ h)\ tang.\ (\phi - \rho) - x\ h\ cotang.\ \phi.$$

If again we substitute $G = \frac{1}{2}\ h^2\ \gamma\ cotang.\ \phi$, we have.:

$$P = (\tfrac{1}{2}\ h^2\ \gamma\ cotang.\ \phi - x\ h)\ tang.\ (\phi - \rho) - x\ h\ cotang.\ \phi.$$

It is, however, convenient to make the following transformations in this formula.

$$P = h\ [(\tfrac{1}{2}\ h\ \gamma + x\ cotang.\ \rho)\ cotg.\ \phi\ tang.\ (\phi - \rho) - x\ cotang.\ \phi - x\ (1 + cotg.\ \phi\ cotg.\ \rho)\ tang.\ (\phi - \rho)],$$

or, as $tang.\ (\phi - \rho) = \frac{tang.\ \phi - tang.\ \rho}{1 + tang.\ \phi\ tang.\ \rho}$

$$= \frac{tang.\ \phi - tang.\ \rho}{1 + cotang.\ \phi\ cotang.\ \rho} . cotang.\ \phi . cotang.\ \rho,$$

we have $P = h\ [(\frac{1}{2}\ h\ \gamma + x\ cotang.\ \rho)\ cotg.\ \phi\ tang.\ (\phi - \rho) - x\ (cotg.\ \phi + cotg.\ \rho - cotg.\ \phi)]$, hence

$$P = h[(\tfrac{1}{2}\ h\ \gamma + x\ cotg.\ \rho)\ cotg.\ \phi\ tang.\ (\phi - \rho) - x\ cotang.\ \rho].$$

This force becomes a maximum when the product $cotang.\ \phi\ tang.\ (\phi - \rho)$ is a maximum, and as we have seen this latter is so, when $\phi = 45^\circ + \frac{\rho}{2}$; therefore, the entire horizontal pressure of the earth against the wall:

$$P = h\left[(\tfrac{1}{2}\ h\ \gamma + x\ cotg.\ \rho)\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 - x\ cotang.\ \rho\right]$$

$$= \tfrac{1}{2}\ h^2\ \gamma\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2$$

$$- x\ h\ cotang.\ \rho\left[1 - \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2\right]$$

or as $cotang.\, \rho = \dfrac{2}{tang.\left(45^\circ + \frac{\rho}{2}\right) - tang.\left(45^\circ - \frac{\rho}{2}\right)}$, and

$$1 - \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2$$

$$= \left[tang.\left(45^\circ + \frac{\rho}{2}\right) - tang.\left(45^\circ - \frac{\rho}{2}\right)\right] tang.\left(45^\circ - \frac{\rho}{2}\right),$$

$$P = \tfrac{1}{2} h^2 \gamma \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 - 2\, h\, \varkappa\, tang.\left(45^\circ - \frac{\rho}{2}\right)$$

$$= h\, tang.\left(45^\circ - \frac{\rho}{2}\right)\left[\frac{h\gamma}{2}\, tang.\left(45^\circ - \frac{\rho}{2}\right) - 2\,\varkappa\right].$$

This force is 0 for $\tfrac{1}{2}\, h_1\, \gamma\, tang.\left(45^\circ - \frac{\rho}{2}\right) = 2\,\varkappa$, that is,

$$\text{for } h_1 = \frac{4\,\varkappa}{\gamma\, tang.\left(45^\circ - \frac{\rho}{2}\right)}.$$

For this height, therefore, a coherent mass may be cut vertically, and should continue so to stand. Inversely, from the height h_1 of the vertical face of any soil, we may deduce the modulus of cohesion, for

$$\varkappa = \tfrac{1}{4}\, h_1\, \gamma\, tang.\left(45^\circ - \frac{\rho}{2}\right).$$

Therefore, the cohesion of a mass is greater or less, according to the height h_1 for which it maintains a vertical face.

If we introduce h_1 into the expression for P, we obtain:

$$P = \frac{h\gamma}{2}\,(h - h_1)\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2.$$

For sand, seeds, shot, and for newly turned soils, h_1 is very small; for compressed compact soils, it is sometimes considerable; for disintegrated, moist earth, Martony found $h_1 = 0{,}9$ feet, whilst, for the same material soaked with water, $h_1 = 0$. According to circumstances, a vertical face of from 3 to 12 feet maintains itself in different soils.

In most cases of practical application, it is advisable to omit the effects of cohesion.

§ 5. *Surcharged Masses of Earth.*—If the earth-work M, Fig. 3, be loaded on the surface, with buildings or otherwise, as DEH, the retaining wall undergoes an increased pressure. To determine this increased pressure, let us put the pressure on each square foot of the horizontal surface $= q$, then the pressure on the surface for $ADE = q\,.\,DE = q\,h\, cotang.\, \phi$, and, therefore, the horizontal pressure, without reference to cohesion:

Fig. 3.

$$P = (G + q\, h\, cotang.\, \phi)\, tang.\, (\phi - \rho)$$

$= (\frac{1}{2}\, h^2\, \gamma + q\, h)\ .\ cotg.\ \phi\ tang.\ (\phi - \rho)$, or as $\phi = 45^\circ + \frac{\rho}{2}$

$$P = (\tfrac{1}{2}\, h^2\, \gamma + qh) \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2.$$

To find the point of application of this force, we must decompose it into its two parts $\frac{1}{2}\, h^2\, \gamma \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2$ and $q\, h \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2$. The first part has its point of application at $\frac{1}{3}$ of the height h above the base A, and, therefore, its statical moment referred to this point:

$$= \frac{h}{3} \cdot \tfrac{1}{2}\, h^2\, \gamma \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2 = \frac{h^3\, \gamma}{6} \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2;$$

By the second part, however, equal portions of the vertical wall are equally pressed, and therefore the resultant pressure of this part passes through the centre of gravity of the wall, and acts at half the height $\frac{h}{2}$ from the base. Hence the statical moment of the second force

$$= \frac{h}{2} \cdot qh \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2 = \frac{q\, h^2}{2} \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2.$$

The moment of the entire pressure is thus:

$(\frac{1}{6}\, h^3\, \gamma + \frac{1}{2}\, q\, h^2) \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2$, and, therefore, the leverage of the force, or the distance $AO = a$ of its point of application O from the base:

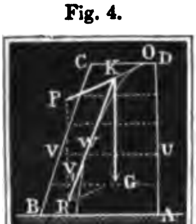
$$a = \frac{(\frac{1}{6}\, h^3\, \gamma + \frac{1}{2}\, h^2\, q) \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2}{(\frac{1}{2}\, h^2\, \gamma + h\, q) \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2} = \frac{\frac{1}{6}\, h^2\, \gamma + \frac{1}{2}\, h\, q}{\frac{1}{2}\, h\, \gamma + q}$$

$$= \left(\frac{h\, \gamma + 3\, q}{3\, h\, \gamma + 6\, q}\right) . h.$$

Remark.—If the earth be carried above the cope of the wall, and form from it a natural slope, the formula of § 3 is still applicable, if h be put equal to the height of the earth, and not that of the wall.

§ 6. ***Retaining Walls.***—The pressure of earth has often, in engineering, to be withheld by retaining walls (Fr. ***revêtements***), or by ***walling-timbers***, and ***sheet-piling***. Retaining walls of masonry are most usual, and we shall, therefore, here treat of these in greater detail.

Fig. 4.



A wall AC, Fig. 4, may be either pushed forward, or turned over by a force $KP = P$. If we suppose this wall composed of horizontal courses of stone bedded on each other, we may assume that, should the wall give way, a horizontal crack

will form, upon which the upper part CU slides forward or turns about. For security we shall neglect the effects of mortar, and take only the friction between the beds into consideration. From the force P, and the weight G of the part CU of the wall, there results a force $KR = R$ upon the magnitude and direction of which the possibility of an overturn or sliding forward of this part of the wall depends. If the angle RKG, by which this resultant deviates from the normal to the plane of separation UV, be less than the angle of friction ρ, the wall cannot slide forward (Vol. I. § 159); and if the direction of the resultant pass within the *joint* or plane of separation, then rotation about the axis V is not possible (Vol. I. § 130).

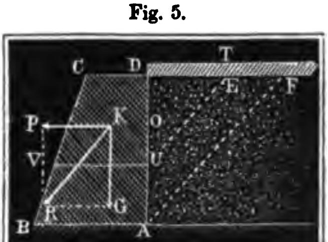
In most cases of application it will be found that rotation more readily takes place than sliding, and therefore, in building retaining walls, provision against the former has to be made. Rotation, or *heeling* is the more apt to occur, in as much as it not unfrequently takes place, not about the axis V, but about a point V_1 nearer the resultant R; because the pressure concentrated in V, compresses or breaks the stone near the point V.

If the points of intersection W, for a series of resultants R passing through the joints, be found, and a line drawn through these, we have what is termed the *line of resistance*, and it is easy to perceive that an overturn of the wall cannot take place, so long as this line does not pass beyond the joints of the wall.

If the force P, which the wall has to withstand, deviates in direction from the vertical more than the angle ρ, there can be no question of its sliding, because the resultant of P and G always makes a smaller angle with the vertical than P alone.

§ 7. *Slipping of Walls.*—If we substitute for P the pressure of earth found above, we can determine the thickness, having which, a wall will be sufficient to withstand this pressure. Let us consider, in the first place, the case of slipping for a wall AC, Fig. 5. Suppose that the earth-work pushes forward the part UC, on the joint UV. If we put the thickness at top of wall $CD = b$, the relative batter $= n$, and the height $DU = x$, we have the thickness:

Fig. 5.



$UV = b + n\,x$, and the contents of UC for 1 foot in length $= b\,x + \frac{n\,x^2}{2}$, and, therefore, the weight,

$G = \left(b + \frac{nx}{2}\right) x\,\gamma_1$; γ_1 being the density of the masonry. For the pressure of the earth on DU, we have generally:

$$P = (\tfrac{1}{2}\,x^2\,\gamma + qx)\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2;$$

and hence, for the angle $RKG = \phi$ made by the resultant R with the vertical:

$$tang.\,\phi = \frac{P}{G} = \frac{\frac{1}{2}x^2\gamma + q\,x}{\left(b + \frac{nx}{2}\right)x\,\gamma_1} \cdot \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2;$$ or as ϕ must be less than ρ, therefore, $tang.\,\phi < f$,

$$\frac{\frac{1}{2}x\gamma + q}{\left(b + \frac{nx}{2}\right)\gamma_1} \cdot \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 < f,$$ from which we have the thickness of wall:

$$b > \frac{\frac{1}{2}x\gamma + q}{f\gamma_1}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 - \frac{nx}{2}.$$

For $x = 0$ we have the thickness at the top:

$b > \frac{q}{f\gamma_1}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2$, therefore, for $q = 0$, we have $b = 0$; for $x = h$, the whole height of wall, the thickness is:

$$b > \frac{\frac{1}{2}h\gamma + q}{f\gamma_1}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 - \frac{nh}{2}.$$

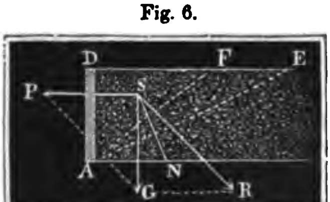
To apply this formula to a dyke or dam, we must put $\rho = 0$, and $q = 0$; then we get $b > \left(\frac{\gamma}{f\gamma_1} - n\right)\frac{x}{2}$, (Vol. I. § 280).

The formulas give for $q = 0$, (that is, when the surface of the fluid or semi-fluid reaches to the top of the wall), the breadth at top $= 0$; but experience has proved that the thickness here should rarely be less than 2 feet, and in positions liable to wear and tear, always above this dimension.

Remark. The co-efficient of friction for stones and bricks in contact with each other (Vol. I. § 161), is from 0,67 to 0,75. And when a bed of fresh mortar is interposed, only 0,60 to 0,70. Mortar once *set*, acts by cohesion or adhesion, and, according to Boistard, the cohesion of mortar is from 800 to 1500 lbs. per square foot. According to Morin, this amounts to from 2000 to 5000 lbs.

§ 8. *Abutting Resistance of Earth.*—We must distinguish between the *active* and *passive* pressure of the earth. In the cases hitherto considered, the pressure is active, pressing against a passive resistance. The pressure of earth, however, becomes passive when it opposes an active force as resistance, as when it resists the thrust of an arch, &c. &c. Poncelet has termed this effect of earth-works *butée des terres* (German, *Hebekraft der Erde*), and Moseley has termed it *resistance of earth.* The resistance which a body opposes to being pushed up an inclined plane, is greater than the force necessary to prevent the sliding of the body down the inclined plane, and just so, in the case of disintegrated masses, the resistance which they oppose to a vertical surface, moved horizontally, is

Fig. 6.



greater than the force with which they press against a vertical plane at rest. Whilst we have above (Vol. I. § 162) put the latter force, $P = G\ tang.\ (\phi - \rho)$, the resistance of the latter must be set $P = G\ tang.\ (\phi + \rho)$, or, as G is the weight $\frac{1}{2} h^2 \gamma\ cotang.\ \phi$ of the prism of pressure ADE, Fig. 6, $P = \frac{1}{2} h^2 \gamma\ cotang.\ \phi\ tang.\ (\phi + \rho)$. This resistance P depends on the angle $AED = \phi$ at which the assumed plane of separation intersects the horizontal, and is a minimum for a certain value of ϕ. But in order to find this value, let us put:

$$cotang.\ \phi\ tang.\ (\phi + \rho) = \frac{sin.\ (2\phi + \rho) + sin.\ \rho}{sin.\ (2\phi + \rho) - sin.\ \rho},$$

and we see at once that this is a minimum, when $sin.\ (2\phi + \rho)$ is a maximum, that is when

$$2\phi + \rho = 90^\circ, \text{ therefore, } \phi = 45^\circ - \frac{\rho}{2}.$$

If we now introduce this value into the formula for P, we obtain the least resistance of the earth-work.

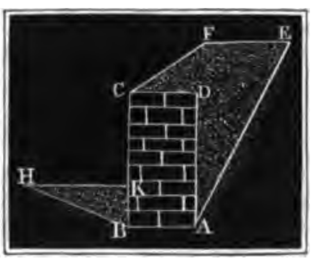
$$P = \frac{1}{2} h^2 \gamma\ cotang.\ \left(45^\circ - \frac{\rho}{2}\right) tang.\ \left(45^\circ + \frac{\rho}{2}\right)$$

$$= \frac{1}{2} h^2 \gamma \left[tang.\ \left(45^\circ + \frac{\rho}{2}\right)\right]^2.$$

This is, generally, the resistance with which earth or any other disintegrated mass withstands a moving force; for as soon as this force is equal to that resistance, a yielding of the mass takes place.

§ 9. *Depth of Foundations.*—An important application of the passive resistance of earth, arises in the founding of retaining and other walls. If the ground on which the retaining wall is to stand be clayey, or wet, the co-efficient of friction between the wall and the ground may fall as low as 0,3, and then a slipping of the wall may very easily occur. It is, therefore, necessary in such cases to dig the foundation to such a depth that the passive resistance on the outside, combined with the friction on the bottom, may counterbalance the active pressure on the inside. If G be the weight of a supporting wall AC, Fig. 7, therefore fG its friction on the bottom, AB, if h be the height of the earth at the back, and h_1 the height in front; if further, ρ and γ be the angle of friction, and the density for the one, and ρ_1 and γ_1 those for the other earthy mass, we have:

Fig. 7.



$$fG + \frac{1}{2} h_1^2 \gamma_1 \left[tang.\ \left(45^\circ + \frac{\rho_1}{2}\right)\right]^2 = \frac{1}{2} h^2 \gamma \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2$$

and therefore the depth BK of the foundation for such a wall:

$$h_1 = \sqrt{\frac{h^2 \gamma \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 - 2fG}{\gamma_1}} .\ tang.\ \left(45^\circ - \frac{\rho_1}{2}\right).$$

For security, a *co-efficient* of stability 1,4 has been introduced (by French engineers for the *revêtement* walls of fortifications), and therefore the depth:

$$h_1 = 1{,}4\ tang.\left(45^\circ - \frac{\rho_1}{2}\right)\sqrt{\frac{h^2\gamma\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 - 2fG}{\gamma_1}}$$

is given to such walls.

Example. To what depth must a parallel wall 8 feet thick, and 13 feet clear height, have its foundation sunk, that it may withstand the pressure of water standing level with the top of the wall? In this case $\rho = 0$, $\gamma = 62{,}25$ lbs. (for which we take 63) $h = 13$ feet; also $f = 0{,}3$, $\rho_1 = 30^\circ$, $\gamma = 1{,}6 \times 63 = 100{,}8$ lbs., and G (the density of the masonry) being $2 \times 63 = 126$ lbs., must be $= 8 \times 13 \times 126 = 13104$ lbs., therefore,

$$h_1 = 1{,}4\ tang.\,(45^\circ - 15^\circ)\sqrt{\frac{13^2 \times 63 - 2 \times 0{,}3 \times 13104}{100{,}8}} = 4{,}25 \text{ feet very nearly.}$$

§ 10. *Heeling of Retaining Walls.*—In order to appreciate a retaining wall in reference to stability, it is necessary to determine its *line of resistance.* For simplicity, we shall first take a parallel wall AC, Fig. 8. If we had only a horizontal force $KP = P$ to deal with, the point of application of which is at a distance $DO = a$ from the cope of the wall, the line of resistance would be a hyperbole, as the following simple view of the subject shows. Of the force P (whose point of application we assume in the line passing through the centre of gravity of the wall) and the weight G of $UVCD$, the resultant is R which intersects UV in W, a point in the line of resistance sought. If we now put the thickness of the wall $AB = CD = b$, its density $= \gamma_1$, the abscissa $KN = x$, and the ordinate $NW = y$, we have $G = (a + x)\, b\, \gamma_1$, and from similarity of triangles:

Fig. 8.

$$KWN \text{ and } KRG\text{: } \frac{WN}{KN} = \frac{RG}{KG}, \text{ that is } \frac{y}{x} = \frac{P}{(a+x)\, b\, \gamma_1},$$

and hence the equation of the line of resistance $y = \dfrac{P\,x}{(a+x)\, b\, \gamma_1}$.

From this we see that when $x = 0$, $y = 0$, and for $x = \infty$, $y = \dfrac{P}{b\,\gamma_1}$, and for $x = -a$, $y = -\infty$. The curved line of resistance, therefore, passes through K, and has not only the horizontal CD, but likewise a vertical EF for asymptote, distant $ST = \dfrac{P}{b\,\gamma_1}$ from the centre of gravity S of the wall.

Fig. 9.

It is otherwise, of course, for a wall to withstand pressure of earth or water as AC, Fig. 9, for here a is variable, because P is applied at a point U at $\frac{1}{3}$ of the height

DU from the base. If we draw the end of the vertical line through *S* as origin of the co-ordinates, that is, if we put $HN = x$, we have:

$$\frac{y}{\frac{1}{3}x} = \frac{P}{b\,x\,\gamma_1}, \text{ or as } P = \tfrac{1}{2}\,x^2\gamma\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2,$$

$y = \frac{\gamma}{6\,b\,\gamma_1}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 . x^2$. This equation corresponds to the common parabola with absciss y and ordinate x.

If, however, we suppose the earth-work carried a height h_1 above the cope of the wall, we must adopt the proportion:

$\frac{y}{\frac{1}{3}(x + h_1)} = \frac{\frac{1}{2}\gamma}{b\,x\,\gamma_1}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 (x + h_1)^2$, whence we have the equation $y = \frac{\gamma}{6\,b\,\gamma_1}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 \frac{(x + h_1)^3}{x}$.

§ 11. The stability of a retaining wall requires not only that the line of resistance be within the wall, but also that it shall not come too near the outside of it. The famous Marshal Vauban gives the practical rule: that the line of resistance should intersect the basis of the wall in a point whose distance from the vertical passing through the centre of gravity of the wall is at most $\frac{4}{9}$ of the distance of the outer axis of the wall from this line. If, as Poncelet does, we call the reciprocal of this number, or the ratio $\frac{FB}{FL}$ between the distance of the outer axis from the vertical passing through the centre of gravity, and the distance of the point of intersection *L* of the line of resistance from this gravity line, the *co-efficient of stability*, and represent it generally by δ, we have for the stability of a parallel wall, withstanding the pressure of earth, (by introducing into the last formula instead of x, the height h of the wall, and instead of y $\frac{\frac{1}{2}b}{\delta}$).

$$\frac{b}{2\,\delta} = \frac{\gamma}{6\,b\,\gamma_1}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2 \frac{(h + h_1)^3}{h},$$

and, therefore, the requisite thickness of the wall:

$$b = (h + h_1)\; tang.\left(45^\circ - \frac{\rho}{2}\right)\sqrt{\frac{\delta\,\gamma}{3\,\gamma_1}\cdot\frac{h + h_1}{h}}.$$

If for δ we substitute $\frac{9}{4} = 2{,}25$, and for $\frac{\gamma}{\gamma_1}$, $\frac{2}{3}$ a mean value, we get:

$$b = 0{,}707\;(h + h_1)\sqrt{\frac{h + h_1}{h}}\,.\; tang.\left(45^\circ - \frac{\rho}{2}\right).$$

If we take $\rho = 30^\circ$, we obtain $b = 0{,}4\;(h + h_1)\sqrt{\frac{h + h_1}{h}}$.

Poncelet gives:

$b = 0{,}865\;(h + h_1)\; tang.\left(45^\circ - \frac{\rho}{2}\right)\sqrt{\frac{\gamma}{\gamma_1}}$, or approximately:

$b = 0{,}285\;(h + h_1)$, for cases in which h_1 varies from 0 to $2\,h$.

Example. What must be the thickness of a parallel wall of 28 feet in height to retain broken stones, mine rubbish for a height of 35 feet? Assuming that the density of the wall $= 2{,}4 \times 63 = 151{,}2$ lbs. The density of the rubbish $1{,}3 \times 63 = 81{,}3$ lbs., and $\rho = 50°$. According to Poncelet's formula:

$$b = 0{,}865 \times 35 \; tang. \; (45° - 25°) \sqrt{\frac{13}{24}} = 30{,}3 \sqrt{\frac{13}{24}} \,.\, tang. \; 20° = 8{,}11 \text{ feet.}$$

§ 12. *Poncelet's Tables.*—To facilitate applications of the formula, Poncelet has calculated the following table, which contains values of $\frac{b}{h}$ corresponding to given values of $\frac{h_1}{h}$, $\frac{\gamma}{\gamma_1}$, and ρ. There are two cases distinguished in the table, namely, the case when the earth-work is heaped, as is shown in Fig. 7, the coping being covered, and the case shown in Fig. 10, where a *berme* of the breadth $0{,}2\ h$, from the outer edge of the cope of the wall, is left before the natural slope of the embankment begins: so that, in short, a promenade is left of the width $CL = 0{,}2\ h$.

The headings of the table explain themselves.

Values of $\frac{h_1}{h}$	Values of $\frac{b}{h}$ for										
	$\frac{\gamma_1}{\gamma} = 1$; $f = 0{,}6$. Berme.		$\frac{\gamma_1}{\gamma} = 1$; $f = 1{,}4$. Berme.		$\frac{\gamma_1}{\gamma} = 1{,}5$; $f = 1$. Berme.			$\frac{\gamma_1}{\gamma} = \frac{2}{3}$; $f = 0{,}6$. Berme.		$\frac{\gamma_1}{\gamma} = \frac{2}{3}$; $f = 1{,}4$. Berme.	
	$= 0$	$= 0{,}2\,h$	$= 0$	$= 0{,}2\,h$	$= 0$	$= 0{,}2\,h$	$= b$	$= 0$	$= 0{,}2\,h$	$= 0$	$= 0{,}2\,h$
0,0	0,452	0,452	0,258	0,258	0,270	0,270	0,270	0,350	0,350	0,198	0,198
0,1	0,498	0,507	0,282	0,290	0,303	0,306	0,303	0,393	0,398	0,222	0,229
0,2	0,548	0,563	0,309	0,326	0,336	0,342	0,326	0,439	0,445	0,249	0,262
0,3	0,604	0,618	0,338	0,361	0,368	0,375	0,343	0,485	0,489	0,274	0,283
0,4	0,665	0,670	0,369	0,394	0,399	0,405	0,357	0,532	0,522	0,303	0,299
0,5	0,726	0,717	0,402	0,423	0,436	0,431	0,368	0,579	0,549	0,332	0,314
0,6	0,778	0,754	0,436	0,450	0,477	0,457	0,377	0,617	0,572	0,360	0,328
0,7	0,824	0,790	0,472	0,476	0,512	0,481	0,385	0,645	0,593	0,387	0,343
0,8	0,847	0,820	0,510	0,501	0,544	0,504	0,391	0,668	0,610	0,413	0,357
0,9	0,903	0,848	0,541	0,524	0,575	0,523	0,398	0,690	0,624	0,437	0,371
1,0	0,930	0,873	0,571	0,546	0,605	0,540	0,405	0,707	0,636	0,457	0,384
1,4	1,023	0,945	0,684	0,624	0,696	0,602	0,416	0,762	0,672	0,537	0,428
2,0	1,107	1,004	0,812	0,714	0,795	0,655	0,425	0,811	0,705	0,622	0,475
3,0	1,180	1,060	0,981	0,835	0,892	0,717	0,435	0,852	0,731	0,726	0,531
5,0	1,247	1,101	1,206	0,994	1,002	0,779	0,445	0,883	0,751	0,862	0,596
1,00	1,283	1,137	1,508	1,182	1,109	0,839	0,452	0,909	0,771	1,013	0,667
2,00	1,309	1,156	1,757	1,327	1,171	0,878	0,456	0,922	0,780	1,129	0,712
3,00	1,316	1,162	1,866	1,389	1,194	0,894	0,458	0,926	0,783	1,174	0,730
∞	1,337	1,175	2,144	1,541	1,243	0,927	0,461	0,934	0,789	1,279	0,769

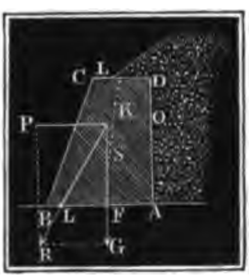
In this table the ***limiting values*** have been principally held in view. Thus $\frac{\gamma_1}{\gamma} = 1$ corresponds pretty nearly to one limit of the ratio of the densities of masonry and earth, while $\frac{\gamma_1}{\gamma} = \frac{5}{8}$ corresponds to the other. Again, f or *tang.* $\rho = 0{,}6$, is the value for the least coherent earths, and $f = 1{,}4$ the value for stiff compact earthwork. In many practical cases, the required proportion has to be deduced by interpolation.

Remark. The formula $b = .865\,(h + h_1)$ *tang.* $\left(45^\circ - \frac{\rho}{2}\right)\sqrt{\frac{\gamma_1}{\gamma}}$ gives results corresponding with those in the table to within $\frac{1}{14}$.

The values in the table refer to parallel walls, built with mortar. If the external batter of the wall does not exceed 0,2, that is 2,4 inches per foot, the breadth b found above, will be that of the wall at $\frac{1}{3}$ of its height from the base, and through this point the line of batter is to be drawn.

Remark. The dimensions resulting from Poncelet's rules or tables are applied in France for walls of fortifications, but give dimensions *nearly one-fourth greater* than the average practice of civil engineers in Britain for the same relative circumstances.—Tr.

Fig. 10.



§ 13. ***Retaining Walls with Batter.***—If the wall has a *batter*, or if its profile be a trapezium *AC*, Fig. 10, the thickness necessary to insure resistance to rotation can only be determined by aid of a complicated expression. If we assume the face *AB* as the plane of separation, and put $KF = OA = x$, and $FL = y$, we then have again: $\frac{y}{x} = \frac{P}{G}$, and

$FB = \delta\, y = \frac{\delta\, P x}{G}$. But as $x = \frac{1}{3}(h + h_1)$, and

$$P = \tfrac{1}{2}\,(h + h_1)^2\,\gamma\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2,$$

and if b the thickness at top, and n the relative batter, therefore, $n\,h$ the absolute batter, $G = (bh + \frac{1}{2}\,nh^2)\,\gamma_1$, and, therefore,

$$FB = \frac{\delta\gamma}{\gamma_1} \cdot \frac{(h + h_1)^3 \cdot \left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2}{6\,h\,(b + \frac{1}{2}\,nh)}$$

The distance BF of the outer edge of the wall from the vertical passing through the centre of gravity, is:

$$= \frac{b + nh}{2} + \frac{3\,b + nh}{2\,b + nh} \cdot \frac{nh}{6} = \frac{3\,b^2 + 6\,n\,h\,b + 2\,n^2\,h^2}{3\,(2\,b + n\,h)},$$

and hence we may put:

$$3\,b^2 + 6\,nhb + 2\,n^2\,h^2 = \frac{\delta\,\gamma}{\gamma_1} \cdot \frac{(h + h_1)^3}{h}\left[tang.\left(45^\circ - \frac{\rho}{2}\right)\right]^2,$$

and hence the thickness of wall at top:

$$b = -nh + \sqrt{\frac{\delta \gamma}{3 \gamma_1} \cdot \frac{(h + h_1)^3}{h} \left[tang. \left(45^\circ - \frac{\rho}{2}\right)\right]^2 + \frac{1}{3} n^2 h^2}.$$

Remark 1. If the back of the wall have a batter likewise, we have a different prism of greatest pressure to deal with, because the force applied to the wall is no longer horizontal. The investigation becomes complicate, and we forbear to enter upon it, but shall refer to works treating of the subject.

Remark 2. Coulomb was the first to propound a good theory of the pressure of earth. *See* "Théorie des machines simples." Prony, in his "Leçons sur la poussée des terres, (1802,)" extended Coulomb's theory. Navier pursues the same notions, with much elegance and precision, in his "Leçons sur l'application de la mécanique, tome I." Mayniel, in 1808, published a special treatise on the pressure of earth, in which the observations and theories of his predecessors are reviewed, "Traité expérimental, &c., de la poussée des terres." C. Martony de Köszegh made experiments on a large scale for the Austrian government, which were published in 1828, under the following title: "Versuche über den Seitendruck der Erde, ausgeführt auf höchsten Befehl, &c., und verbunden mit den theoretischen Abhandlungen von Coulomb und Français, Wien, 1828." The most complete work on the pressure of earth is that of Poncelet in the "Mémorial de l'officier du génie, 1838," and which has been translated into German by Lahmeyer, Braunschweig, 1844. In Moseley's "Engineering and Architecture," this subject is handled with great elegance and success. Hagen has a chapter on this subject in the second part of his admirable "Wasserbaukunst," in which he takes a peculiar view of it.

CHAPTER II.

THEORY OF ARCHES.

§ 14. *Arches.*—An arch (Fr. *voûte*, Ger. *Gewölbe*), is a system of bodies resting upon each other, and supported by two fixed points, in such manner that they are in equilibrium not only among themselves, but with certain external forces. The material of these bodies is usually stone, and hence are termed *arch-stones* (Fr. *voussoirs*, Gr. *Gewölbesteine*). The planes of contact of the stones are the *beds* or *joints*. The fixed points upon which the arch rests are termed *abutments* (Fr. *Pieds-droits*, Ger. *Widerlager*), and in certain cases *piers* (Fr. *culées*, *piliers*, Ger. *Pfeiler*). Of the arch-stones, the highest is termed the *key-stone* (Fr. *clef*, Ger. *Schlussstein*), and those which rest on the abutments or piers, are termed *imposts* or *springers*, (Fr. *coussinets*, Ger. *Kämpfer*.) An arch is included between two more or less curved surfaces, the *intrados* and *extrados*, which are sometimes termed *the soffit*, and *the back of the arch*.

As regards the intrados and extrados, arches are very various. Cylindrical surfaces are most usual, but conical surfaces occur, and we have domes, and variously proportioned groinings. We shall treat of cylindrical arches only, and limit ourselves still further, to the consideration of those having a horizontal axis. Such arches are bounded by two vertical parallel planes, the *faces* of the arch,

(Fr. *parements*, Ger. *Stirnflächen.*) According as the faces are perpendicular or inclined to the geometrical axis of the arch, the arch is *direct*, or *oblique*, or *skewed* (Fr. *droites* or *biaises*); groined arches or vaults (Fr. *voûtes d'arête*, Ger. *Kreuz*, or *Klostergewölbe*), are merely intersecting cylindrical arches. Domes or cupolas (Fr. *voûtes en dome*, Ger. *Kuppel* or *Kesselgewölbe*), are arches generated by the revolution of a curve about a vertical axis.

As regards the curvature of arches, it is very various. The section is sometimes circular, sometimes elliptical, catenarian, or formed of several circular arcs, and plate bands, or straight arches are sometimes built.

Remark.—As experience has abundantly proved that arches fail or give way by a rotation of determinate parts round the edges where certain joints meet the extrados or intrados, and not by sliding dislocation, we need here only consider the conditions of equilibrium in reference to the former circumstance, omitting our author's investigation of the latter, which show, as is usually done in elementary treatises of mechanics, that for the case of equilibrium without friction, *the weight of the arch-stones must be to each other as the differences of the cotangents of the angles of inclination of the joints to the horizon.*—Tr.

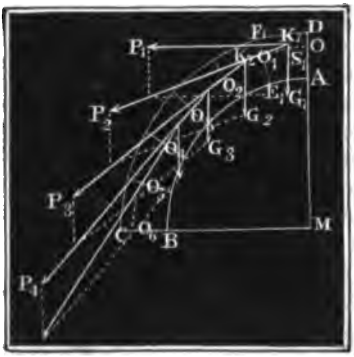
Remark.—The dislocation of an arch by slipping of voussoirs might occur in two ways: according as the joint of maximum pressure lies above or below the joint of minimum pressure. In the former case, Fig. 11, the hauches of the arch slide out, and

Fig. 11.

Fig. 12.

the crown slips down. In the other case, the reverse happens, Fig. 12. This second case scarcely ever occurs, so that we shall not farther recur to it.

Fig. 13.



§ 15. *Line of Pressure and Resistance.*—An arch is so much more likely to fall in by rotation round the outer or inner edge of a joint, than by slipping, that the former may be considered as the usual accident. The stability of an arch in reference to rotation may be considered exactly in the same manner as the stability of a pier or wall (Vol. II. § 6). From the horizontal force P_1 applied at any point O, Fig. 13, in the crown of the arch and the weight of the first arch-stone acting in its centre of gravity S_1, there results the force P_2 acting on the first joint, and the intersection O_1 of the direction of this force with the joints $E_1 F_1$.

Again, from the pressure P_2, and the weight G_2 of the second arch-stone, acting in its centre of gravity S_2, there results the pressure P_3 in the second joint, and the intersection O_2 of the direction of this force with the second joint. Proceeding in this manner, we obtain the remaining normal pressures, and the intersections O_3, O_4, &c., in the other joints. But the lines O, O_1, O_2, O_3 . . . , which unite the intersections or points of application of the pressures P_1, P_2, P_3 . . . , is the ***line of resistance*** (Fr. *ligne de Pression*, Ger. *Widerstandslinie*), (Vol. II. § 6). So long as at least one line of resistance can be found in the face of an arch, which neither passes beyond the intrados nor the extrados at any point, so long dislocation of the arch by rotation cannot occur. If, on the other hand, the line of resistance intersects the intrados, the arch will fall inwards, and if it goes beyond the extrados, the arch will rise upwards, and so fall to pieces. Fig. 14 represents the former case, and Fig. 15 the latter.

Fig. 14.

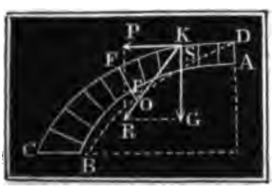
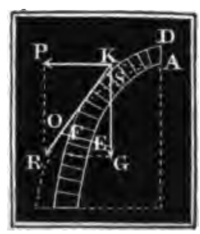


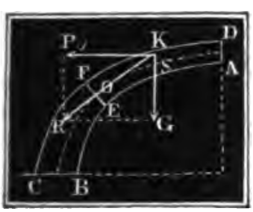
Fig. 15.



The dislocation becomes inevitable, however, from the circumstance that the incompressibility of the stones opposes resistance to the forces RR, acting with the leverages EO, FO. The cohesion of the mortar alone resists this force; but as a very slight concussion is sufficient to destroy this cohesion, its effects should not be relied upon as available.

It is easy to perceive that arches are so much the more stable (in reference to rotation) the greater the number of lines of resistance that can be drawn within them; the less, therefore, the number of lines of resistance that intersect the intrados or extrados. The arch of greatest stability, Fig. 16, is necessarily that in which a line of resistance may be drawn, which passes through the centre of all the arch-stones, or ***bisects their depth.***

Fig. 16.

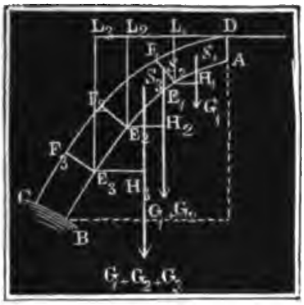


For the usual construction of arches, that is, for circular arches, a rotation or rising upwards, that is, an intersection of all lines of resistance with the extrados, cannot possibly occur; we may, therefore, limit ourselves in the investigation of stability to the rotation from which the arch falls inwards. That we may be certain that at least one line of resistance passes beyond neither intrados nor ex-

trados, we may start to draw it from the point D in the crown, and try whether it intersects the intrados.

§ 16. *Equilibrium in Reference to Rotation.*—The conditions of stability in reference to rotation may be considered in another point of view, and one more adapted for calculation. We may eliminate the forces P_1, P_2, $P_3 \ldots$, acting in the crown D, Fig. 17, which are necessary to hinder a rotation of the arch-stones round the inner edges E_1, E_2, E_3, &c., and then investigate which is the greatest of these forces. If we designate the leverages E_1L_1, E_2L_2, E_3L_3, ... of the force P referred to the points E_1, E_2, E_3, &c., as axis of rotation by a_1, a_2, a_3, &c., and the leverages E_1H_1, E_2H_2, E_3H_3, &c., of the weights G_1, $G_1 + G_2$, $G_1 + G_2 + G_3$, &c., in reference to these same axis by b_1, b_2, b_3, &c., we have for the force P acting at the crown:

Fig. 17.



$$P_1 = \frac{b_1}{a_1} G_1,\ P_2 = \frac{b_2}{a_2}(G_1 + G_2),\ P_3 = \frac{b_3}{a_3}(G_1 + G_2 + G_3), \text{ \&c.}$$

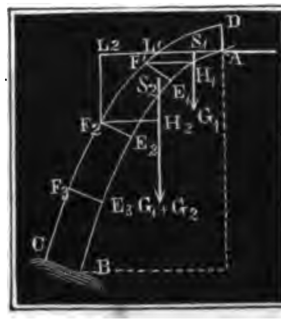
But not only the factors b_n, and $G_1 + G_2 + \ldots + G_n$ of the numerator increase from the crown towards abutments, but the denominator A_n increases also; hence one of the values of P_1, P_2, P_3, &c., is a maximum; and it is necessary for equilibrium, that the effective force P_m, acting in the crown, should be equal to it. That joint which corresponds to the maximum pressure, or the pressure on the crown, is termed the *joint of rupture* (Fr. *joint de rupture*, Ger. *Bruchfuge*), because dislocation by rotation first begins round its lower edge, if the force P_m at the crown diminishes. It is determined by the *angle of rupture*, which its plane makes with the horizon (or with the vertical). It is also easy to perceive, that the angle of rupture gives that point in the arches, in which the line of resistance, starting in D in the crown of the arch, touches the intrados.

If we compare the maximum effort required to hinder rotation inwards with the maximum effort required to resist slipping, we find that in most cases the force required to resist rotation is greater than that to resist slipping, and, therefore, *the pressure in the crown of an arch is equal to the greatest of all the forces P_1, P_2, P_3, &c., which oppose the rotation of the parts of the arch G_1, $G_1 + G_2$, $G_1 + G_2 + G_3$, &c., round the inner edges.* If, therefore, we have once determined this pressure at the crown of the arch, it is easy to find the pressure in any other part of the arch.

Arches falling by rotation *outwards* are exceptional cases. To discriminate by calculation as to the possibility of such an accident occurring, the point of application of the force P is taken at the

lower edge A, Fig. 18, of the joint of the key-stone, because the leverage, in reference to rotation about F_1, F_2, F_3, &c., is here the least. If now we again designate the leverages: F_1L_1, F_2L_2, F_3L_3, &c., by a_1, a_2, a_3, &c., and the leverages F_1H_1, F_2H_2, F_3H_3, &c., of the weights G_1, $G_1 + G_2$, $G_1 + G_2 + G_3$, &c., we have the values of P:

Fig. 18.



$$P_1 = \frac{b_1}{a_1} G, \; P_2 = \frac{b_2}{a_2}(G_1 + G_2),$$

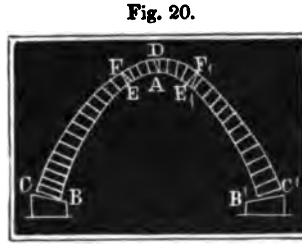
$$P_3 = \frac{b_3}{a_3}(G_1 + G_2 + G_3),$$

and if the *least* of these values be greater than the pressure in the crown, or the greatest of the forces which prevent a falling inwards, the arch is stable; unless this be the case, dislocation takes place.

Remark. The falling to pieces of an arch by rotation may likewise happen in two ways: according as the joint of rupture of the maximum value is above or below the joint of rupture of the minimum value. Fig. 19 represents the first, and Fig. 20 the second case.

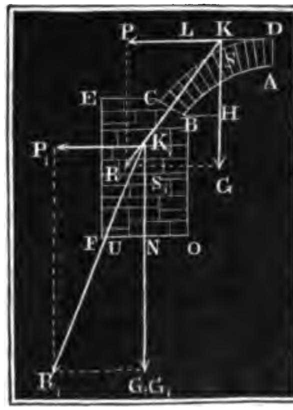
Fig. 19.

Fig. 20.



§ 17. *Stability of Abutments.*—If we have satisfied ourselves by the calculations indicated in the foregoing paragraphs, that an arch is stable, and have also determined the pressure in the key-stone, we have still to investigate the stability of the abutment walls; that is, to determine the thickness of abutment wall necessary to resist a detrusion or an overturn. This investigation is the more important, as it is not unfrequently in consequence of insufficient resistance of these, that arches, in themselves stable, fall in.

Fig. 21.



It is evident that a retaining wall FB, Fig. 21, is stable when the direction of the resultant force $K_1 R_1 = R_1$ of the weight of the one semi-arch acting at its centre of gravity S, the hori-

zontal thrust P acting at the crown, and S_1, the weight of the retaining wall, passing through its centre of gravity, passes through the base FO of the retaining or *abutment* wall, and deviates from the vertical K_1N by an angle less than the angle of repose ρ.

For the angle ϕ, which the resultant R_1 of the forces $P = \overline{KP_1}$ and $G + G_1 = \overline{K_1 G_1}$, makes with the vertical, we have $tang.\ \phi = \frac{P}{G + G_1}$; but $tang.\ \rho =$ the co-efficient of friction f, and hence to insure stability in reference to sliding, we should have $\frac{P}{G + G_1} < f$.

In order, further, that the resultant may pass through the outer edge F of the abutment, let us put the moment of P, referred to this edge, equal to the sum of the moments of the weights G and G_1. If a be the rise of the arch BL, and h the height of the abutment, then the moment of the force P referred to the edge F as an axis $= P(a + h)$; if, again, b be the horizontal distance BH of the vertical passing through the centre of gravity of the semi-arch AC from the inner edge B of the springing point, c the thickness of the abutment wall, and e the distance FN of the vertical gravity-line of the abutment wall from the edge F, we have the moment of the weights G and $G_1 = G(b + c) + G_1 e$, and thus we get the equation: $P(a + h) = G(b + c) + G_1 e$.

In order to insure permanence, experience dictates, according to Audoy's deductions, the employment of 1,9 P instead of P, so that the equation for determining the thickness of the abutment becomes: $1{,}9\, P(a + h) = G(b + c) + G_1 e$. If h_1 be the mean height of the abutment or pier, and γ the density of its masonry, we have for each foot in length of the pier the weight $G_1 = h_1 c \gamma$, and if we put $e = \frac{1}{2} c$, the moment $G_1 e = \frac{1}{2} h_1 c^2 \gamma$, and hence:

$$\tfrac{1}{2} h_1 c^2 \gamma + G c = 1{,}9\, P(a + h) - G b, \text{ or,}$$

$$c^2 + \frac{2 G c}{h_1 \gamma} = \frac{1{,}9\, P(a + h) - G b}{\frac{1}{2} h_1 \gamma},$$

and hence the thickness of the abutment in question:

$$c = -\frac{G}{h_1 \gamma} + \sqrt{\frac{1{,}9\, P(a + h) - G b}{\frac{1}{2} h_1 \gamma} + \left(\frac{G}{h_1 \gamma}\right)^2}.$$

In order to secure this wall against sliding, we must have:

$$G_1 > \frac{P}{f} - G, \text{ i. e. } c > \frac{P - f G}{f h_1 \gamma}.$$

It will usually be found that the former value of c is greater than the latter; and that, therefore, the thickness of the abutment must be regulated by the former condition of stability.

For very high piers, as $G c$, $1{,}9\, P a$ and $G b$, are very small compared with $1{,}9\, Ph$ and $\frac{1}{2} h_1 c^2 \gamma$ (which may be put $\frac{1}{2} h c^2 \gamma$), we have: $\frac{1}{2} h c^2 \gamma = 1{,}9\, P h$, i.e. $\frac{1}{2} c^2 \gamma = 1{,}9\, P$, and hence the greatest strength:

$$c = \sqrt{\frac{3{,}8\, P}{\gamma}}.$$

§ 18. *Loaded Arches.*—We have hitherto neglected to consider the influence of the *backing* on the arch; which, however, it is essential to examine. That the stability of an arch, such as a bridge, may not be altered by the passage of heavy weights upon it, it is necessary that the arch should in itself possess such weight, or be permanently loaded with backing, that any weight arising from traffic, such as heavy wagons, locomotives and the like, can only occasion a slight change in the entire load, or forces in action.

The backing consists usually of a system of walling (*spandril walls*), supporting the road-way, and carried up either to form a horizontal line EF, Fig. 22, or an inclined line, Fig. 23. In very

Fig. 22.

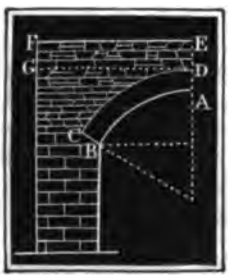


Fig. 23.

many cases, the spandril walls or backing of arches consist of the same materials as the arches; and if it be uniformly built, we may assume a common density for the whole, and thus considerably abbreviate calculation. If, according to Vol. I. § 58, we take the specific gravity of masonry at from 1,6 to 2,4, we have for the density of the masonry 100 to 150 lbs. per cubic foot, the former answering to brick-work, the latter to ashlaring. The loading of arches generally increases their thrust, and also their stability. That the voussoirs may resist crushing, they must have a certain depth proportioned to the pressure of the arch; and as this increases from the crown towards the springing, the depth of the voussoirs should likewise increase from the crown to the springing. Perronet has given as a rule for the depth at the crown, the formula: $d = 0,0694r + 0,325$ metres, or, in English measure $d = 0,0694r + 1$ foot, in which formula r is the greatest radius of curvature of the intrados.

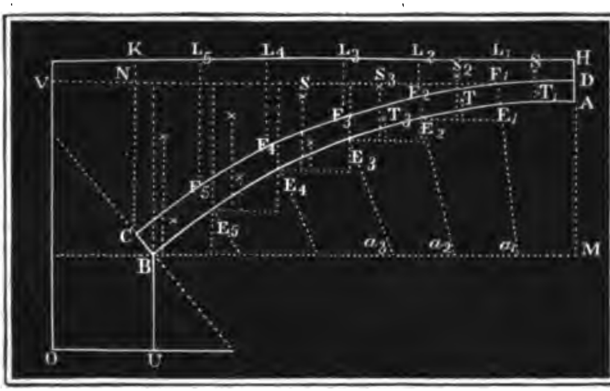
For arches whose radius is above 48 feet, or 15 metres, this formula gives greater dimensions than is given in ordinary practice. The depth of voussoirs must be regulated by the strength of the materials, and the position of the line of resistance in the arch. The joints being kept very thin, so that the mortar serves rather to distribute the pressure uniformly over the bed of the stone, it will be found that a thickness which reduces the strain to 225 lbs. per square inch of surface, allows of ample security for the average of materials. One-half this thickness must, however, exist on *each side* of the line of resistance.

Remark 1. 225 lbs. per square inch is only $\frac{1}{30}$ of the absolute strength of sound sandstone and limestone. In the celebrated bridge at Neuilly, near Paris, built in 1768 to 1780, by Perronet, the estimated pressure per square inch is 280 lbs.

Remark 2. When, as in the sequel we always do, we take the thrust or pressure at the top of the crown of the arch, and in like manner, only consider a rotation round the lowest point of the angle of rupture, it is the more necessary to assume this high degree of security, and to give the arch corresponding depth of voussoirs, as in these assumptions we only get the least value of the pressure. Besides, it is chiefly the upper edges of the voussoirs at the crown of the arch, and the lower edges of those at the joints of rupture that have to withstand the pressure, and, therefore, soonest give way; the depth we have indicated on each side the line of pressure is, therefore, necessary to insure stability.

§ 19. *Test of the Equilibration of Arches.*—The investigation of the stability of an arch may be gone through as follows: let $ABCD$, Fig. 24, be the one-half of the arch to be examined, and $CDHK$ the

Fig. 24.



spandril wall, which for simplicity's sake we shall assume to be of the same density as the arch. First, divide the arch in any convenient number, in this case six, equal or unequal parts, by lines E_1F_1, E_2F_2, E_3F_3, &c., in the direction of the joints, and determine, not only the area and the centres of gravity T_1, T_2, T_3 . . . of these parts, but also the areas and centres of gravity S_1, S_2, S_3 . . . of the superincumbent parts F_1H, F_2L_1, F_3L_2 . . . This done, take the statical moment of the first part AF_1 and F_1H referred to the first point of division E_1, and divide their sum by the vertical distance of this point of division from the horizontal DN drawn through the crown. In like manner take the moments of AF_1, E_1F_2, F_1H and F_2L_1, referred to the second point of division E_2, and divide the sum of these moments by the vertical distance of this second point from the horizontal DN. Again, determine the moments of the parts of the arch AF_1, E_1F_2, E_2F_3, and the parts of the spandril F_1H, F_2L_1, F_3L_2, referred to the edge E_3, and divide that sum by the vertical distance of the point E_3, from the horizontal DN, &c. By going through this process for all the parts, from A to B, we arrive at the forces that must be applied at D to prevent rotation round the points

E_1, E_2, E_3, &c., and the greatest of these forces is that which has to be taken as acting at the crown.

Having done this, multiply the sum of the areas $AF_1 + F_1H$ by the *tang.* $(a_1 - \rho)$, and again $AF_1 + E_1F_2 + F_1H + F_2L_1$ by *tang.* $(a_2 - \rho)$, &c., (where $a_1, a_2 \ldots$ are the several angles of inclination of the joints with the horizon), and find the greatest value of these products. If the greatest of these values be *less* than that necessary to prevent rotation round $E_1, E_2, E_3 \ldots$, there need be no further consideration of these forces; but if it be *greater*, then must this value be introduced as the pressure in the crown, and not that first found.

Lastly, it has to be determined whether the horizontal force so found is not sufficient to dislocate the arch by *pushing* or *turning* out a part of it.

With the horizontal thrust, determined as above, we can examine, as shown in § 16, the conditions of stability of the abutment.

Example. The relative stability of the arch in Fig. 24, may be calculated as follows: area of the part $AF_1 = 6{,}89$ square feet; area of the piece F_1H above $d = 8{,}48$ square feet, the lever of the former referred to $E_1 = 2{,}50$, and of the latter $= 2{,}45$; *i. e*, the moment of both $= 6{,}89 \,.\, 2{,}5 + 8{,}48 \,.\, 2{,}45 = 38{,}001$. The distance of E_1 from DN, or leverage of horizontal force in $D = 1{,}50$; and, therefore, the first value of this force $= \dfrac{38 \,.\, \gamma}{1{,}50} = 25{,}33 \,.\, \gamma$ lbs. Area of second part $E_1F_2 = 7{,}15$, and the part of spandril above it $F_2L_1 = 11{,}02$ square feet; the moment of both referred to $E_2 = 17{,}52 + 23{,}69 = 41{,}21$, adding to this the moment of $AL_1 = 38 + 15{,}37 \,.\, 5{,}10 = 38 + 78{,}39 = 116{,}39$, and hence the moment of the whole piece $AL_2 = 157{,}60$; the distance of E_2 from $DN = 2{,}35$, and hence the second value of the horizontal force in $D = \dfrac{157{,}60 \,.\, \gamma}{2{,}35} = 67{,}05 \,.\, \gamma$ lbs. Again, the area of the third piece $E_1F_3 = 7{,}68$, and of the part of spandril above it $F_3L_2 = 16{,}51$ square feet; the moment of both $= 46{,}61$, adding to this the moment of the piece $E_2H = 157{,}60 + 166{,}02 = 323{,}62$; we find the moment of the whole $= 370{,}23$; and as the distance of the point E_3 from $HN = 3{,}90$, the value of the force in $D = \dfrac{370{,}23 \,.\, \gamma}{3{,}90} = 94{,}93 \,.\, \gamma$ lbs. Proceeding in this manner, a value of the force that has to counteract the tendency to rotation round $E_4 = \dfrac{701{,}92 \,.\, \gamma}{5{,}9} = 118{,}97 \,.\, \gamma$ lbs.; and a fifth force in reference to rotation round $E_5 = \dfrac{1163{,}43 \,.\, \gamma}{8{,}45} = 137{,}68 \,.\, \gamma$ lbs.; and, lastly, in reference to rotation round B, a force $= \dfrac{1760{,}21 \,.\, \gamma}{11{,}6} = 157{,}74 \,.\, \gamma$ lbs. As this is the greatest value found, we put the pressure or thrust at the crown, $P = 151{,}74 \,.\, \gamma$, or, taking the weight of masonry as 150 lbs. per cubic foot, $P = 22761$ lbs. The depth of arch at crown is 1,3 feet; and, therefore, the area for each foot of length of the arch $= 144 \,.\, 1{,}3 = 187{,}2$ square inches; and hence the pressure on each square inch $\dfrac{22761}{187{,}2} = 122$ lbs., supposing the line of resistance to bisect the voussoirs.

If, with M. Petit, we take the angle of repose $= 30°$, we obtain for the force to prevent dislocation of the arch by sliding, the following values. The joints E_1F_1, E_2F_2, $E_3F_3 \ldots$ are inclined to the horizon at the angles 83° 40′, 77° 20′, 71°, 64° 40′, 58° 20′, 52°, respectively, therefore,

$P_1 = (6{,}89 + 8{,}48)$ *tang.* $(83°\,40' - 30°) \,.\, \gamma = 15{,}37 \,.$ *tang.* $53°\,40' \,.\, \gamma = 20{,}9 \,.\, \gamma$ lbs.;
$P_2 = (15{,}37 + 18{,}17)$ *tang.* $(77°\,20' - 30°) \,.\, \gamma = 33{,}54 \,.$ *tang.* $47°\,20' \,.\, \gamma = 36{,}4 \,.\, \gamma$ lbs.;
$P_3 = 57{,}73 \,.$ *tang.* $41° \,.\, \gamma = 50{,}1 \,.\, \gamma$ lbs.
$P_4 = 90{,}56$ *tang.* $34°\,40' \,.\, \gamma = 62{,}6 \,.\, \gamma$ lbs.
$P_5 = 134{,}13$ *tang.* $28°\,20' \,.\, \gamma = 72{,}3 \,.\, \gamma$ lbs.
$P_6 = 188{,}53$ *tang.* $22° \,.\, \gamma = 76{,}2 \,.\, \gamma$ lbs.

and, therefore, the greatest horizontal pressure counteracting sliding $= 76{,}2 \cdot \gamma$ lbs. As, however, this pressure, in its tendency to cause rotation round an inner joint, amounts to $151{,}7 \cdot \gamma$, it is evident that a sliding cannot take place. And in like manner it is easy to convince ourselves that neither rotation nor sliding outwards is possible. As to the stability of the abutment *OUK*, the moment of the force *P* referred to *O* as an axis of rotation $= 151{,}74 \cdot \gamma \cdot \overline{OV} = 151{,}74 \cdot 18 \cdot \gamma = 2731 \cdot \gamma$ lbs.; the moment of the loaded arch *ABKH*, is:

$$1760{,}2 \cdot \gamma + 188{,}53 \cdot \overline{OU} \cdot \gamma = (1760{,}2 + 188{,}53 \cdot 6{,}8)\,\gamma = 3042 \cdot \gamma \text{ lbs.},$$

and that of the pier $= 343 \cdot \gamma$ lbs.; hence the moment resisting rotation round $O = (3042 + 343) \cdot \gamma = 3385 \cdot \gamma$ lbs., and, therefore, *heeling* cannot possibly take place. If, however, more ample security be desired, we must substitute for *P*, $1{,}9\,P$, as above explained, and, therefore, take the moment of the force to produce heeling $= 5189 \cdot \gamma$, and thus we see that our abutment would be too thin; instead of 5,45 feet thickness, it would require from 11 to 12 feet. For a thickness of 11 feet, the moment of stability $= 1760{,}2 \cdot \gamma + 188{,}53 \cdot 11\,\gamma + 1281 \cdot \gamma = 5115 \cdot \gamma$, which would prove a *sufficient* stability.

§ 20. *Tables for Arches.*—In order to facilitate investigations on the stability of arches of the more usual forms, M. Petit calculated a series of tables of which we shall here give a short abstract. The first of these tables refers to semi-circular vaults, as in Fig. 25, the second refers to semi-circular arches with spandril walls at an angle of 45° as shown by the dotted line in Fig. 23. The third table refers to semi-circular arches with horizontal spandrils, as shown by the dotted line in Fig. 22, and the fourth table refers to segmental arched vaults.

Fig. 25.

In the first three tables, the two first vertical columns contain the proportions of the arches; the third column contains the angle of rupture; the fourth and fifth co-efficients of *horizontal thrust*, in terms of the radius or half span, and the weight of the materials (see example 1 following); and in the sixth, the co-efficient of the maximum thickness of abutment in terms of the half span.

To apply these tables, we have to look in column 1 for the ratio $k = \frac{r_2}{r_1}$ of the radius of the extrados to that of the intrados, and pass along horizontally to the fourth and fifth columns, and the greater of the numbers found in these two columns is to be taken as a co-efficient by which to multiply the square of r_1, the radius of intrados, and the weight per cubic foot γ of the masonry, the product of which gives the horizontal thrust in question. The sixth column gives the thickness of abutment, supposing the height infinite, by multiplying the co-efficients there found by the radius r_1. For low abutments, the thickness is less, and should be calculated according to § 17.

The fourth table contains in its first column the ratio $k = \frac{r_1}{r_2}$, and in the other columns the thrust of the arch for various proportions of the span s to the versed sine or height h. This latter table is only applicable when the angle of rupture, given in the first table, is *less* than the half of the central angle a, and of the arc of the vault.

TABLE I.

SEMICIRCULAR ARCH WITH PARALLEL VAULTED SURFACES.

Ratio of the radii. $k = \frac{r_2}{r_1}$.	Ratio of radius of intrados to depth of voussoir.	Angle of rupture.	Co-efficient p of the thrust of arch;		Co-efficient for greatest thickness of abutments.
			for rotation.	for sliding.	
2,732	1,154	0° 00′	0,00000	0,98923	
2,70	1,176	13 42	0,00211	0,96262	
2,50	1,333	35 52	0,02283	0,80846	
2,20	1,666	51 4	0,08648	0,58767	
2,00	2,000	57 17	0,13017	0,45912	1,3223
1,80	2,500	61 24	0,16373	0,34281	1,1414
1,60	3,333	63 49	0,17517	0,23874	0,9525
1,55	3,636	64 3	0,17478	0,21464	0,9031
1,50	4,000	64 9	0,17254	0,19130	0,8527
1,45	4,444	64 5	0,16798	0,16872	0,8007
1,40	5,000	63 48	0,16167	0,14691	0,7838
1,35	5,714	63 19	0,15287	0,12587	0,7622
1,30	6,666	62 14	0,14330	0,10559	0,7370
1,25	8,000	61 15	0,12847	0,08608	0,6987
1,20	10,000	59 41	0,11140	0,06733	0,6504
1,15	13,333	57 1	0,09176	0,04935	0,5905
1,10	20,000	53 15	0,06754	0,03213	0,5066
1,05	40,000	46 22	0,03813	0,01568	
1,02	100,000	38 12	0,01691	0,00618	
1,00	∞	0 00	0,00000	0,00000	

TABLE II.

SEMICIRCULAR ARCHES, MASONRY AT THE BACK, OF 45° INCLINATION.

Ratio of the radii. $k = \frac{r_2}{r_1}$.	Ratio of radius of intrados to depth of voussoir.	Angle of rupture.	Co-efficient p of the thrust of arch;		Co-efficient for greatest thickness of abutments.
			for rotation.	for sliding.	
2,00	2,000	60°	0,26424	0,74361	1,7264
1,80	2,500	60	0,29907	0,57383	1,5147
1,60	3,333	60	0,31245	0,42191	1,2990
1,55	3,636	61	0,31222	0,38673	1,2437
1,50	4,000	61	0,30996	0,35266	1,1877
1,45	4,444	60	0,30587	0,31971	1,1308
1,40	5,000	59	0,30001	0,28787	1,0954
1,35	5,714	58	0,29285		1,0823
1,30	6,666	57	0,28281	0,22756	1,0626
1,25	8,000	54	0,27102		1,0412
1,20	10,000	50	0,25806	0,17171	1,0160
1,15	13,333	47	0,24477		0,9894
1,10	20,000	42	0,23292	0,12032	0,9652
1,05	40,000	36	0,22902		0,9571

TABLE III.

SEMICIRCULAR ARCHES, WITH HORIZONTAL MASONRY ABOVE.

Ratio of the radii $k = \frac{r_2}{r_1}$.	Ratio of radius of intrados to depth of voussoir.	Angle of rupture.	Co-efficient p of the thrust of arch; for rotation.	for sliding.	Co-efficient for greatest thickness of abutments.
2,00	2,000	36°	0,05486	0,50358	1,3834
1,80	2,500	44	0,08508	0,37901	1,2001
1,60	3,333	52	0,12300	0,26755	1,0082
1,55	3,636	54	0,13027	0,24173	0,9584
1,50	4,000	56	0,13648	0,21673	0,9075
1,45	4,444	57	0,14122	0,19256	0,8554
1,40	5,000	59	0,14421	0,16920	0,8018
1,35	5,714	60	0,14504	0,14666	0,7465
1,30	6,666	61	0,14332	0,12495	0,7379
1,25	8,000	62	0,13872	0,10405	0,7260
1,20	10,000	63	0,13073	0,08397	0,7048
1,15	13,333	64	0,11895	0,06471	0,6723
1,10	20,000	65	0,10279	0,04627	0,6249
1,05	40,000	69	0,081755	0,02865	0,5573
1,00	∞	75	0,055472	0,01185	

TABLE IV.

VAULTED ARCHES, WITH PARALLEL ARCHED SURFACES.

Ratio of the radii $k = \frac{r_2}{r_1}$.	Co-efficient p of the thrust of the arch. $s = 4\,h$	$s = 5\,h$	$s = 6\,h$	$s = 7\,h$	$s = 8\,h$	$s = 10\,h$	$s = 16\,h$
1,40	0,15445	0,14691	0,14691	0,14691	0,14691	0,14478	
1,35	0,14771	0,13030	0,12587	0,12587	0,12587	0,12405	
1,30	0,13764	0,12331	0,10682	0,10559	0,10559	0,10406	
1,25	0,12547	0,11402	0,10009	0,08668	0,08608	0,08483	0,07180
1,20	0,11023	0,10196	0,09102	0,07999	0,06981	0,06636	0,05616
1,15	0,09123	0,08634	0,07866	0,07050	0,06259	0,04904	0,04116
1,10	0,06737	0,06563	0,06158	0,05666	0,05160	0,02414	0,02681
1,05	0,03776	0,03804	0,03709	0,03550	0,03357	0,02944	0,01882
1,01	0,00834	0,00871	0,00886	0,00889	0,00885	0,00862	0,00747

The following table contains a synopsis of the relative dimensions of segmental arches.

Ratio of the span to height $\frac{s}{h}$	Half central angle α.	*sin.* α.	Ratio of radius of intrados r_1 to height h $\frac{r_1}{h}$.
4	53° 7′ 30″	0,8000	2,500
5	43 36 10	0,6897	3,625
6	36 52 10	0,6000	5,000
7	31 53 26	0,5283	6,625
8	28 4 20	0,4706	8,500
10	22 37 10	0,3846	13,000
16	14 15 0	0,2462	32,500

Example 1. A semicircular arch with horizontal road-way over it, having radius of intrados $r_1 = 10$ feet. What should be the dimensions? What will be the thrust? According to Perronet's formula, $d = 0,0694 \,.\, 10 + 1 = 1,694$ feet, for which take 1,7 feet. We have now $r_2 = 11,7$ and $k = \frac{r_2}{r_1} = 1,17$. From Table 3, the angle of rupture is $63\frac{1}{3}°$, the co-efficient of horizontal thrust $= 0,1190 + \frac{2}{3} \,.\, 0,0118 = 0,1237$ (0,0118 being the difference between .119, and the number next above it). Taking 150 lbs. per cubic foot as weight of masonry, the thrust at crown $= 0,1237 \,.\, 150 \,.\, 10^2 = 1855$ lbs. For the extreme thickness of abutment, we have from the same table the co-efficient $0,6723 + \frac{2}{3} \,.\, 0,0325 = 0,6855$, and, therefore, the thickness $= 0,6855 \,.\, 10 = 6,85$ feet. For low abutments, the formula of § 17 gives smaller dimensions.

Example 2. What dimensions and forces correspond to a vault of 10 feet span, and 2 feet rise? Here we have $\frac{h}{s} = \frac{1}{5}$, therefore, the half central angle $\alpha = 43° \; 36' \; 10''$, and $\sin. \alpha = 0,6897$, and the radius $r = 3,625 \,.\, 2 = 7,25$ feet. Table 4 gives the co-efficient of horizontal thrust, (as $s = 5\,h$, and according to Perronet's formula: $d = 1,5$, so that $k = 1,2) = 0,10196$, and hence the thrust $= 0,102 \,.\, 150 \,.\, 7,25^2 = 804$ lbs.

Remark 1. That the part of the abutment on which the arch rests may not be thrust away, it is necessary that the horizontal thrust $P = pr^2 \gamma$ should be *less* than $\frac{1}{2} f \alpha \, (r_2^2 - r_1^2) \, \gamma$ the friction on the bed. If this be not the case, as, for example, in very flat arches, this sliding out of the upper part of the abutment must be prevented by artifices, such as iron tie rods. The co-efficient of friction $f = 0,76$, therefore, $\frac{1}{2} f = 0,38$, and, therefore, the strength of the ties must be such as to resist a force $= p - 0,38 \, \alpha \, (k^2 - 1) \, r_1^2 \, \gamma$. This is the state of the case when $s = 4\,h$ and k is less than 1,06; when $s = 5\,h$ to $10\,h$, and k less than 1,15, and when $s = 16\,h$, this sliding is sure to take place.

Remark 2. The literature on the subject of arches is very extensive; but the theories treated therein are not always admissible, because the assumptions are inconsistent with experience. We shall here only mention the authors whose theories and investigations are generally accepted as the best approximations. We refer, therefore, to Coulomb, "Théorie des machines simples," who first gave a rational theory of the arch, and such as is in substance given in the foregoing paragraphs. This theory is given with greater completeness by Navier, "Résumé des Leçons sur l'application de la Mécanique," t. I. There are papers by Audoy, Garidel, Poncelet and Petit, in the "Mémorial de l'officier du génie." The substance of the papers of Garidel and Petit, and their tables, are given by Mr. Hann in his Treatise on Bridges, published by Weale, 1839. Moseley's paper on the "Theory of the Arch," is, perhaps, the most elegant exposition of this interesting and important subject. The works of Robison, Whewell, Eytelwein, Gerstner, and others, contain particular expositions of Coulomb's theory. Hagen has published an interesting essay, entitled "Über Form und Stärke gewölbter Bogen," Berlin, 1844.

CHAPTER III.

THEORY OF FRAMINGS OF WOOD AND IRON.

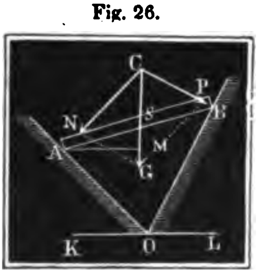
§ 21. *Wooden Structures.*—Structures of wood and of iron differ essentially from those in stone, in that these materials are subjected to what have been termed tensile and transverse, as well as compressive strains, to which latter alone masonry is exposed. Hence, in carpentry and iron-work, the *pieces* of which the framings are composed are not only laid one upon the other, but are *morticed, tenoned, fished, bolted, strapped*, &c., to unite them together. The principal axis of the pieces of any framing may be *horizontal, inclined*, or *vertical*. In the first case, they are termed *beams* or *joists;* in the second, *rafters, braces*, or *spears*, &c.; in the other, *posts, pillars, uprights*, &c. According to the function they fulfil, some pieces are termed *struts* or *spears* (viz: those resisting compression), and others, *ties* or *braces* (*i. e.* those resisting tension).

To investigate the *stability* or *equilibrium* of a framing, it is essential, in the first place, to know the forces and weights which the framing has to counteract. From these we determine, not only the forces which individual pieces have to withstand, but the forces acting at the points of connection, and the strains or pressures upon the points of support. Each part should have such form, position and dimensions, as to completely withstand every force acting on it.

As to the connection of the pieces of a framing with each other, we have principally to distinguish *bolts* and *pins, tenons* and *mortices, scarfs* and *shoulders*. Bolts and pins counteract, or take up all forces passing through their axes. Tenons and mortices counteract only forces acting in certain directions, and shoulders or scarfs counteract such forces as are directed at right angles to the plane of the shoulder.

§ 21*. A beam AB, Fig. 26, lying on inclined planes, is in an instable condition, unless friction or some artificial fastening, as *bolts* or *mortices* retain it. To establish equilibrium, it is a necessary condition that the vertical SG passing through the centre of gravity of the beam, should pass through the point C, in which the normals to the ends A and B of the planes intersect each other, for only then are the two components N and P, into which the weight G of the beam may be decomposed, taken up or

Fig. 26.



counteracted by the planes. If α and β be the angle AOK and BOL of the planes to the horizon, these forces are:

$$N = \frac{G \sin. \beta}{(\sin. \alpha + \beta)}, \text{ and } P = \frac{G \sin. \alpha}{\sin. (\alpha + \beta)}.$$

If, again, l be the length AB of the beam, s the distance AS of its centre of gravity S from the end A, and δ the angle of inclination BAM of the beam to the horizon, then the horizontal projection of $AS = s$ is $AM = s \cos. \delta$, or $= AC \sin. \alpha$, but as

$$AC = \frac{AB \sin. ABC}{\sin. ACB} = \frac{l \sin. (90^\circ - \beta + \delta)}{\sin. (\alpha + \beta)} = \frac{l \cos. (\beta - \delta)}{\sin. (\alpha + \beta)},$$

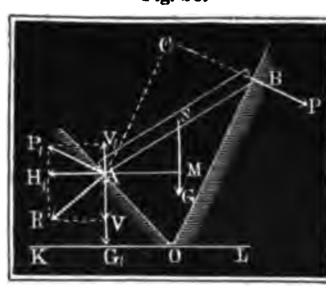
we have $AM = \dfrac{l \sin. \alpha \cos. (\beta - \delta)}{\sin. (\alpha + \beta)}$, and, therefore, we have the equation of condition:

$$s \sin. (\alpha + \beta) \cos. \delta = l \sin. \alpha \cos. (\beta - \delta).$$

If one of the planes be horizontal as AO in Fig. 27, then as $\alpha = 0$, we have $s \sin. \beta \cos. \delta = 0$, *i. e.* $\beta = 0$, or the other plane must likewise be horizontal. In order to prevent slipping of the beam in every other position, we must, Fig. 28, mortice one end of the beam

Fig. 27.

Fig. 28.



as A, or fasten it in some way. The pressure which the end of the beam there exerts on the inclined plane OB may be deduced from the theory of the bent lever MAC, whose arm $AM = AS \cos. SAM = s \cos. \delta$, and $AC = AB \cos. BAC = l \cos. (\beta - \delta)$, and hence P the pressure required

$$= \frac{G s \cos. \delta}{l \cos. (\beta - \delta)}$$

As the pressure on the point of support A is equal to the mean of all the forces acting on AB, we may assume that the vertical pressure $G_1 = G$, and its counter pressure $P_1 = P$, acts at this point. If, therefore, we decompose this latter into the horizontal force $H_1 = P_1 \sin. \beta$, and the vertical force $V_1 = P_1 \cos. \beta$, we obtain for the total pressure in A the horizontal component or *thrust* $H_1 = \dfrac{G s \sin. \beta \cos. \delta}{l \cos. (\beta - \delta)}$, and the vertical component, or vertical pressure:

$$V = G - V_1 = G\left(1 - \frac{s \cos. \beta \cos. \delta}{l \cos. (\beta - \delta)}\right),$$

from which we can easily calculate the magnitude and direction of the total pressure or strain.

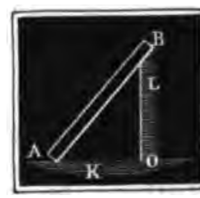
For the case of a beam leaning on a wall, Fig. 29, $\beta = 90°$, hence:

$$H = \frac{G\, s \cos. \delta}{l \sin. \delta} = G \frac{s}{l} \operatorname{cotg.} \delta = P, \text{ and } V = G =$$

the weight of the beam.

Fig. 29.

Fig. 30.



For the case of a beam leaning on a wall inclined at the same angles as the beam, as in Fig. 30 at B, $\beta = \delta$, hence:

$$P = G \frac{s}{l} \cos. \delta,$$

$$H = G \frac{s}{l} \sin. \delta \cos. \delta, \text{ and } V = G\left(1 - \frac{s}{l} \cos. \delta^2\right).$$

§ 22. *Thrust of Roofs.*—The formulas found in the preceding paragraphs are immediately applicable to calculating the thrust of rafters or "couples" for roofs (Fr. *fermes*). According to these, we have in the case of simple *lean-to* and coupled roofs, as in Figs. 31 and 32,

Fig. 31.

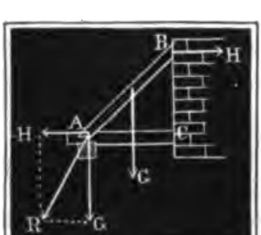
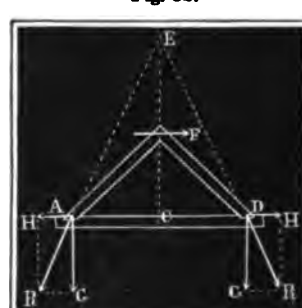


Fig. 32.



for the horizontal thrust acting at the lower and upper end: $H = \frac{Gs}{l} \operatorname{cotg.} \delta$, or, as in this case $s = \frac{1}{2} l$, $H = \frac{1}{2} G \operatorname{cotg.} \delta$; again the vertical pressure at the upper end $= 0$, and $= G$ the weight

of the couple and its load at the lower end. If we put the height of roof $BC = h$, and the span or width $AC = DC = b$, then $cotg.\ \delta = \frac{b}{h}$, and hence the thrust of the couple $= \frac{1}{2}\ G\ \frac{b}{h}$; *and thus we see that the horizontal thrust increases directly as the span, and inversely as the height or pitch of the roof.* The usual limits of h are between $2\ b$ and $\frac{1}{2}\ b$. The former ratio is that of church roofs of the Saxon and Norman period, and the latter that of the flat Italian roofs of modern houses. In the former, $\delta = 26° 34'$, and in the latter $63° 26'$. The thrust of the couples is very great in flat roofs; in the Italian roof, for instance, as above specified, the thrust equals the whole weight of the couple and load; in the Saxon roof the thrust is not above one-fourth of this. The feet of the couple must be mortised, or otherwise fastened into the beam (tie-beam) to prevent sliding. The entire pressure of a rafter at its foot A is:

$$R = \sqrt{H^2 + V^2} = \sqrt{1 + \tfrac{1}{4}\,(cotg.\ \delta)^2}\ .\ G = \sqrt{1 + \left(\frac{b}{2h}\right)^2}\ .\ G,$$

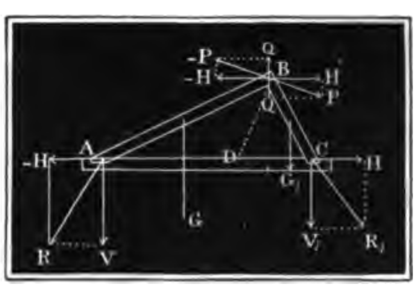
and for $RAH = \phi$, the angle made by the line of pressure with the horizon, we have

$$tang.\ \phi = \frac{G}{H} = \frac{G}{\frac{1}{2}\ G\ \frac{b}{h}} = \frac{2h}{b} = 2\ tang.\ \delta.$$

Thus we may find the direction of the total thrust at the foot, by doubling the height of the couple; or, by making $CE = 2\ .\ CB$, and drawing a line through the foot A, from the point E, and producing it to R.

For the pair of rafters, Fig. 32, in which the rafters are of equal length, these exert on each other only a horizontal thrust; but if the rafters be of unequal length, as in Fig. 33, the force P with which one rafter presses upon the other, deviates by a certain angle from the horizontal. If G be the weight of one rafter AB, and G_1 that of the other CB, and if δ and δ_1 be the angle of inclination of these rafters to the horizon, and if β be the angle of inclination BDC of the plane in which we may conceive the rafters to abut on each other, and against which the force P acts at right angles, we have:

Fig. 33.



$$P = \frac{1}{2}\ \frac{G\ cos.\ \delta}{cos.\ (\beta - \delta)},\ \text{and} = \frac{1}{2}\ \frac{G_1\ cos.\ \delta_1}{cos.\ (180° - \beta - \delta_1)},\ \text{hence}$$

$$-\ G\ cos.\ \delta\ cos.\ (\beta + \delta_1) = G_1\ cos.\ \delta_1\ cos.\ (\beta - \delta),\ \text{or}$$

$$\frac{G\,(sin.\ \beta\ sin.\ \delta_1 - cos.\ \beta\ cos.\ \delta_1)}{sin.\ \beta\ cos.\ \delta_1} = \frac{G_1\,(sin.\ \beta\ sin.\ \delta + cos.\ \beta\ cos.\ \delta)}{sin.\ \beta\ cos.\ \delta};$$

dividing, we have:

$$G\,(tang.\ \delta_1 - cotg.\ \beta) = G_1\,tang.\ \delta + cotg.\ \beta), \text{ thus}$$

$$cotang.\ \beta = \frac{G\,tang.\ \delta_1 - G_1\,tang.\ \delta}{G + G_1}.$$

And from this we have the horizontal thrust of both rafters:

$$H = P\,sin.\ \beta = \tfrac{1}{2}\,\frac{G\,sin.\ \beta\ cos.\ \delta}{cos.\ (\beta - \delta)} = \frac{\tfrac{1}{2}\,G}{cotg.\ \beta + tang.\ \delta}$$

$$= \frac{\tfrac{1}{2}\,(G + G_1)}{tang.\ \delta + tang.\ \delta_1}.$$

As to the vertical pressures V and V_1 at the rafter feet, the one is equal to the weight G, minus the vertical component $Q = P\ cos.\ \beta$, and the other is equal to the weight G_1 plus this component; or,

$$V = G - H\,cotg.\ \beta = G - \tfrac{1}{2}\,\frac{(G\,tang.\ \delta_1 - G_1\,tang.\ \delta)}{tang.\ \delta + tang.\ \delta_1},$$

$$\text{and } V_1 = G_1 + \tfrac{1}{2}\left(\frac{G\,tang.\ \delta_1 - G_1\,tang.\ \delta}{tang.\ \delta + tang.\ \delta_1}\right).$$

Example. The roof *ABD* (Fig. 32), is 40 feet span, and 30 feet height, and consists of couples 4 feet from centre to centre, 6×8 inch scantlings—required the thrust. Assuming each square foot of roofing to weigh 15 lbs., we have for the load on each rafter $15 \times 4\sqrt{20^2 + 30^2} = 60\sqrt{1300} = 2163$ lbs. The rafter itself weighs $\frac{1}{2} \times \frac{2}{3} \times 44\sqrt{20^2 + 30^2} = \frac{44}{3}\sqrt{1300} = 529$ lbs., and, therefore, the vertical pressure of a rafter $V = G = 2163 + 529 = 2692$ lbs., and the thrust $= \frac{1}{2}\,G\,\frac{b}{h} = \frac{1}{2}\,.\,2692\,\frac{20}{30} = 897$ lbs.

§ 23. *Compound Roofs.*—In many framings, as in mansard roofs, the rafter DE, Fig. 34, does not rest on a tie-beam, but on a second rafter CD, and this again on a third, and fourth, and so on. That the pressure of one beam may be completely transferred to the next in this case, it is necessary that they should have certain relative positions. These positions are determined by the conditions that any two beams abutting against each other should undergo equal horizontal pressures. The horizontal pressure of the rafter DE, is $H = \frac{1}{2}\ G\ cotg.\ \delta$, when $G =$ the weight, and δ its inclination. For the second beam or rafter DC: $H = \dfrac{\frac{1}{2}\,(G + G_1)}{tang.\ \delta_1 - tang.\ \delta}$, when G_1 and δ_1 denote weight and inclination of this second beam. Hence by equating the two values, we have:

Fig. 34.

$$G\,cotg.\ \delta = \frac{G + G_1}{tang.\ \delta_1 - tang.\ \delta}, \text{ i. e.}$$

$$tang.\ \delta_1 = tang.\ \delta + \frac{(G + G_1)}{G}\,tang.\ \delta = \left(2 + \frac{G_1}{G}\right)tang.\ \delta;$$

and, in like manner, for the inclination δ_2 of a third beam, seeing *that the horizontal thrust is everywhere the same.*

$$G \, cotg. \, \delta = \frac{G_1 + G_2}{tang. \, \delta_2 - tang. \, \delta_1}, \text{ hence}$$

$$tang. \, \delta_2 = tang. \, \delta_1 + \frac{G_1 + G_2}{G} \, tang. \, \delta$$

$$= \left(2 + \frac{G_1}{G} + \frac{G_1}{G} + \frac{G_2}{G}\right) tang. \, \delta = \left[2\left(1 + \frac{G_1}{G}\right) + \frac{G_2}{G}\right] tang. \, \delta,$$

and in like manner for a fourth:

$$tang. \, \delta_3 = tang. \, \delta_2 + \frac{G_2 + G_3}{G} \, tang. \, \delta$$

$$= \left[2\left(1 + \frac{G_1}{G} + \frac{G_2}{G}\right) + \frac{G_3}{G}\right] tang. \, \delta, \text{ \&c.}$$

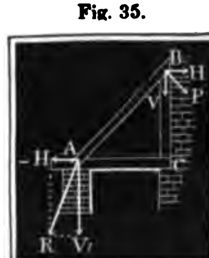
If each beam be of the same weight G, then
$tang. \, \delta_1 = 3 \, tang. \, \delta$, $tang. \, \delta_2 = 5 \, tang. \, \delta$, $tang. \, \delta_3 = 7 \, tang. \, \delta$, $tang. \, \delta_4 = 9 \, tang. \, \delta$, &c.

If, therefore, in this form of roof, the height EH, Fig. 34, corresponding to the first beam or rafter DE, be set off upwards repeatedly, and through the divisions 1, 3, 5, 7, &c., lines $D1$, $D3$, $D5$, $D7$, &c., be drawn, these lines give the inclinations of the other rafters. It is also evident, that the figure of this combination of rafters is that of a funicular polygon formed by the weights G_1, G_2, G_3, &c. (*see* Vol. I. § 144), and this coincidence is quite explained, if we conceive the two halves of the weight G of each beam collected at its ends D, C, B, A, &c., and pulling downwards, that is, if we assume the weight G acting at each of these points.

If we take the beams very short, and very numerous, the axis of such a framing becomes a catenary.

§ 24. *Supported Rafters.*—If the head of a rafter rests on a pillar BC, Fig. 35, the thrust of the rafter is less than when it merely leans on a vertical wall. In this case, according to § 21*, the pressure on the head of this pillar is:

Fig. 35.



$$P = G \frac{s}{l} \, cos. \, \delta = \tfrac{1}{2} G \, cos. \, \delta,$$

and the horizontal thrust:

$H = P \, sin. \, \delta = \frac{1}{2} G \, cos. \, \delta \, sin. \, \delta = \frac{1}{4} G \, sin. \, 2\delta.$

As the pillar supports a part of the weight $G = V = P \, cos. \, \delta = \frac{1}{2} G \, (cos. \, \delta)^2$, the beam does not, of course, press with its whole weight G on the foot A; but with a force:

$$V_1 = G - \tfrac{1}{2} G (cos. \, \delta)^2 = G\left[1 - \tfrac{1}{2}(cos. \, \delta)^2\right] = \tfrac{1}{2} G \left[1 + (sin. \, \delta)^2\right].$$

From this vertical pressure, and the horizontal thrust H, we get the angle ϕ, which the resultant R makes with the horizon, viz:

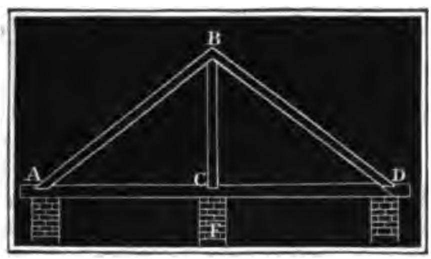
$$tang. \, \phi = \frac{H}{V_1} = \tfrac{1}{2} \cdot \frac{sin. \, 2\,\delta}{1 + (sin. \, \delta)^2}.$$

If we introduce the depth $AC = b$ and height $BC = h$, we get $H = \frac{b\,h}{b^2 + h^2} \cdot \frac{G}{2}$, while in the case of the beam simply leaning, we

had $H = \frac{b}{h} \cdot \frac{G}{2}$. If each unit of length of the rafter bears a load whose weight is γ, we have $G = \sqrt{b^2 + h^2} \cdot \gamma$, and therefore in the one case $H = \frac{b h \gamma}{2 \sqrt{b^2 + h^2}}$, and for the other $H = \frac{b \sqrt{b^2 + h^2}}{2 h} \cdot \gamma$, so that if the pillar *support* the rafter, the horizontal thrust is so much the less the lower the roof; while for roofs without such support, the thrust is greater as the roof is lower.

That the post BC may not be overturned by the horizontal force H, it is necessary to support it by a wall.

Fig. 36.



The relations of the forces now discussed, occur in the coupled roof, shown in Fig. 36, applicable in some cases, where the rafters are supported at the ridge by a central wall or column. The pillar takes up the weights $\frac{1}{2} G (cos. \delta)^2$, $\frac{1}{2} G (cos. \delta)^2$, and transfers, therefore, the vertical pressure $G (cos. \delta)^2$ to its support, and the horizontal thrust $H = \frac{1}{4} G \, sin. \, 2 \delta$. There is no side support required for the pillar, as the horizontal thrust is equal on each side.

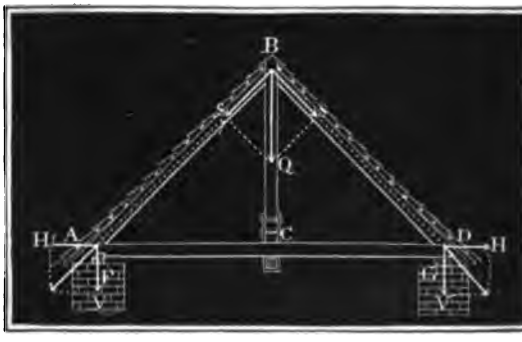
Example. For the roof in the example to § 22, the loading of one rafter $G = 2692$ lbs., $b = 20$ feet, $h = 30$ feet, therefore, $tang. \delta = \frac{3}{2}$, or $\delta = 56° \, 18' \, 36''$; and, therefore, when a pillar is put in, the horizontal thrust is:

$$H = \frac{2692}{4} \, sin. \, 112° \, 37' \, 12'' = 673 \, sin. \, 67° \, 22' \, 48'' = 621 \text{ lbs.}$$

The vertical pressure taken up by the pillar is $V = \frac{2692}{2} \, (cos. \, 56° \, 18' \, 36'')^2 = 746{,}3$ lbs.; and, therefore, the beam supports a strain of only $2692 - 746{,}3 = 1945$ lbs.

§ 25. *King-posts.*—Whilst in the cases just considered the posts relieve the tie-beam (or walls in the absence of a tie) of a part of the thrust of the rafters, the *king-post*, BC, Fig. 37, acts in a very

Fig. 37.



different way; it carries a part of the weight of the tie-beam AD, and transfers it through the rafters AB and DB to act as thrust on the side walls, or rather as tensile strain on the tie. The force Q acting through the king-post, may be deduced from the scantlings of, and kind of load acting on the beam AD. If the load be uniformly distributed, it may be assumed, that the one half is supported by the side walls, the other half hangs on the king-post; but if the load be applied at the centre of the tie, it must be considered as acting entirely on the king-post. The force Q on the king-post is decomposed into two others in the direction of the rafters, the value of each of which is $S = \frac{Q}{2\ sin.\ \delta}$; and if we combine these forces with those arising from the weights G, G of the rafters, we get the horizontal thrust in A and D:

$$H = \tfrac{1}{2}\ G\ cotg.\ \delta + S\ cos.\ \delta = \frac{G+Q}{2}\ .\ cotg.\ \delta,$$

and the vertical pressure at that point:

$$V = G + S\ sin.\ \delta = G + \frac{Q}{2}.$$

For bridges and roofs of great span, more complicated framings, with two or more posts, and termed *trusses*, are applied. Fig. 38

Fig. 38.

represents a *truss* with two posts, termed *queen-posts*, BC and B_1C_1, with a collar beam between them BB_1. The manner of calculating the strains in this framing is exactly similar to that for the simple couple with king-post. From the load on a queen-post Q, the horizontal thrust on the collar-beam tending to compress it, and acting on the side walls, if there be no tie, is $H = \frac{1}{2}\ Q\ cotg.\ \delta$, when δ is the inclination of the *rafters* or *braces* AB and A_1B_1 to the horizon. As this angle is frequently a small one, the thrust is considerable, and, therefore, care must be taken with the foot fastenings (*see* Vol. II. § 17). The scantlings of the braces and collar beams must be fixed by the rules in Vol. I. § 206, &c., so that they shall resist flexure and fracture, when exposed to forces

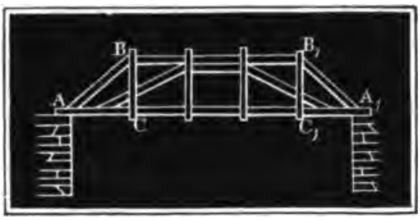
$$S = \frac{Q}{2\ sin.\ \delta},\ \text{and}\ H = \tfrac{1}{2}\ Q\ cotg.\ \delta.$$

The force Q depends on the loading of the bridge or roof. If the

load be uniformly diffused, we shall do best to assume that each post carries $\frac{1}{3}$, and each side wall $\frac{1}{6}$ of the load.

Example. Suppose the trussed bridge in Fig. 38, designed as one of two for a 60 feet span and 12 feet wide bridge: suppose each square foot of the bridge together with its load weighs 50 lbs., the weight of the bridge is $12 \times 60 \times 50 = 36000$ lbs., and the load on the queen-posts $= \frac{36000}{3} = 12000$. Therefore, for an inclination of the rafters of $22\frac{1}{2}°$, the horizontal pressure $= \frac{1}{2}\, 12000$ *cotg.* $22\frac{1}{2}° = 6000 \times 2{,}4142 = 14485$ lbs., and the thrust through each rafter $= \frac{6000}{sin.\ 22\frac{1}{2}°} = 15679$ lbs. The half of these strains come on the pieces of each of the two trusses, so that on each collar there would be 7242,5 lbs., and on each rafter 7839,5 lbs. If we take the resistance of wood (Vol. I. § 206) at 7400 lbs., and if we strain only to $\frac{1}{20}$ of the absolute strength, we get for the section of each collar beam $F = \frac{7242{,}5\ .\ 20}{7400} = \frac{1448{,}5}{74} = 19{,}6$ square inches, and for each brace or rafter $\frac{7839.5\ .\ 20}{7400} = \frac{15679}{74} = 21{,}2$ square inches.

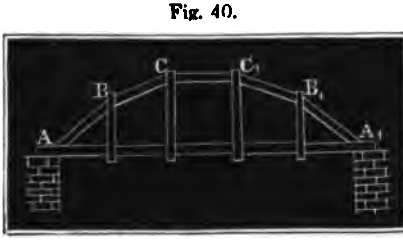
Fig. 39.



Remark. More composite trusses, as indicated in Figs. 39 and 40, are calculated in the same manner as the above. In each of these it may be assumed that each of the four posts or uprights carries one-fifth of the entire load, and that the remaining fifth rests immediately on the side walls. In the construction shown in Fig. 40, the directions of the different rafters are not optional, but dependent one upon the other. If Q be the weight on each post, and δ the inclination of the brace BC, and δ_1 that of AB, the horizontal thrust

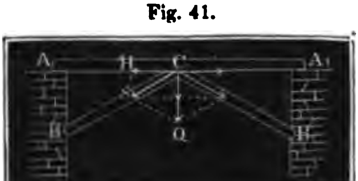
$$H = Q\ cotg.\ \delta = (Q + Q)\ cotg.\ \delta_1,\ \text{hence}\ cotg.\ \delta = 2\ cotg.\ \delta_1$$
$$\text{or}\ tang.\ \delta_1 = 2\ tang.\ \delta$$

Fig. 40.



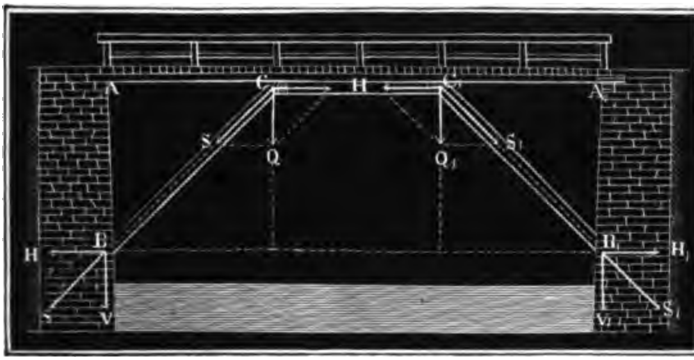
§ 26. *Timber Bridges.*—The framings in the foregoing section support the road-way or ceiling by suspension, but there are trusses applied for bridges, which support the road-way on the opposite principle of *sustaining* them. In these latter, the distribution of the pressure takes place exactly as in the former. In the simple case shown in Fig. 41, we have from the vertical force Q acting at the centre of the bridge AA_1, the horizontal thrust $H = \frac{1}{2}\ Q\ cotg.\ \delta$, and the strain on the *spear* or *strut* $BC = S = \frac{1}{2}\ \frac{Q}{sin.\ \delta}$, when δ is the inclination of the strut. In the example, Fig. 42, the forces are the same, but in this case Q may

Fig. 41.



be taken at $\frac{1}{3}$ of the whole load, whilst in the case Fig. 41, $Q = \frac{1}{2}$ the load. The piece CC_1 in Fig. 42 is termed a *straining cill.* If

Fig. 42.



there be a double set of struts or spears, as indicated in Fig. 43, there are four struts, and it may be assumed that each carries one-fifth of the whole load, or $Q = \frac{1}{5}G$. To prevent deflection of long spears, *braces* or *counter-braces* AD, A_1D_1 are added, particularly when there are several sets of spears. The distribution of the pressure in the case of spears of unequal length being used as in Fig. 44, is to be taken as exactly the same as in Fig. 43, only that in these the braces or suspending posts CD, C_1D_1 become the more requisite as the struts come to have considerable length. It is proper to take the weight of all the parts into calculation, and to reckon that half the weight of each part acts at its end.

Fig. 43.

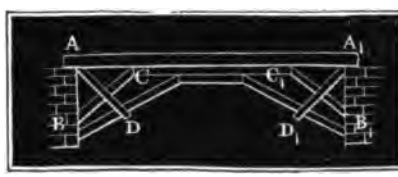


Fig. 44.

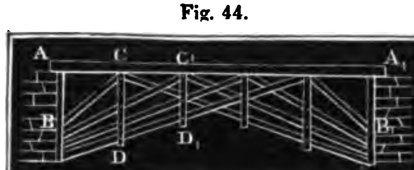
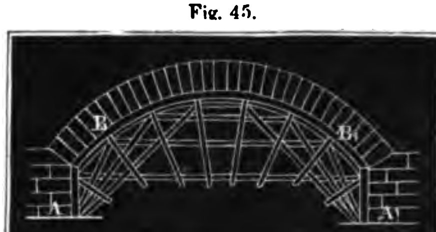
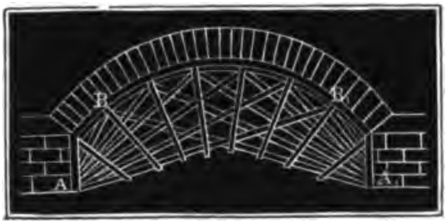


Fig. 45.



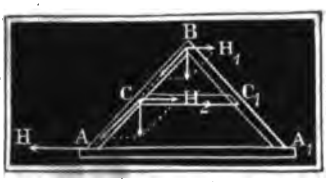
The centerings for bridges afford the most frequent application of the kind of framing we are now considering. Figs. 45 and 46 represent two such centres. The pressure which each simple frame ABB_1A_1 or ABA_1 undergoes and has to resist, may easily be determined by calculating the weight of the part of the arch bearing upon it.

Fig. 46.



If the two spears abutting on each other have different inclinations to the horizon, as in the construction shown in Fig. 46, the strain on them is of course unequal. If the angles of inclination of two such spears $= \delta$ and δ_1, and if the vertical pressure at the abutting joint $= Q$, the strain along the spears $S = \frac{Q\,cos.\,\delta_1}{sin.\,(\delta + \delta_1)}$, $S_1 = \frac{Q\,cos.\,\delta}{sin.\,(\delta + \delta_1)}$ and the horizontal thrust of both $= H = \frac{Q\,cos.\,\delta\,cos.\,\delta_1}{sin.\,(\delta + \delta_1)}$.

Fig. 47.



§ 27. *Roofs.*—In roofs, collar beams are applied to prevent deflexion of the rafters, as also *queen-posts*, *braces*, &c., and the nature of the forces may be traced, as in Figs. 47, 48, 49.

§ 28. *Posts.*—The strength of pillars and posts subjected to tensile or compressive strains, when these act in the direction of the axis, have been investigated, Vol. I. § 183 to § 206. It, however, not unfrequently happens, that the forces act

Fig. 48.

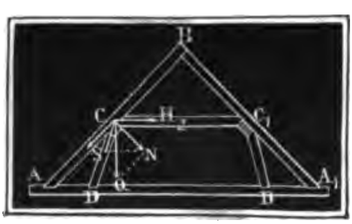


Fig. 49.

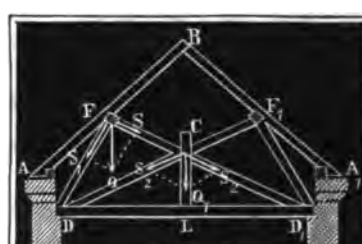
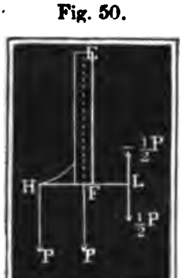


Fig. 50.



out of the axial direction, and we shall, therefore, examine this case. EF in Fig. 50, represents a suspending post to which a tensile strain P is applied excentrically. Let $F =$ the area l, the length EF of the post, $a =$ the leverage FH, or the distance of the direction of the force from that of the axis. Prolong FH in the opposite direction, and make $FL = FH = a$, and conceive that in L two equal and opposite forces $\frac{1}{2}P, -\frac{1}{2}P$ act: there results an axial force $FP = P$, and a couple $\frac{1}{2}P, -\frac{1}{2}P$. The former extends all the

fibres uniformly by a quantity $\lambda_1 = \frac{P}{F.E} . l$, but the latter extends the fibres unequally on one side, and compresses them unequally on the other. If the post be rectangular, with the sectional dimensions b and h, where h is in the same plane as a, the moment of the force:

$$Pa = \frac{\lambda_2}{l} WE = \frac{\lambda_2 b h^3}{12 l} E \text{ (Vol. I. § 191)},$$

but the extension or compression of the fibres at the distance 1 from the axis: $\lambda_2 = \frac{12 P al}{b h^3 E}$, and that of the extreme fibres $= \frac{h}{2} . \lambda_2 = \frac{6 P al}{b h^2 E}$, therefore the greatest extension:

$$\lambda = \lambda_1 + \frac{h\lambda_2}{2} = \frac{Pl}{E}\left(\frac{1}{F} + \frac{6a}{bh^2}\right) = \frac{Pl}{Ebh}\left(1 + \frac{6a}{h}\right).$$

But for the force K producing rupture: $\frac{K}{E} = \frac{\lambda}{l}$, hence the modulus of strength:

$$K = \frac{P}{bh}\left(1 + \frac{6a}{h}\right), \text{ and inversely:}$$

$$P = \frac{b h}{1 + \frac{6a}{h}} . K.$$

If the post be cylindrical, and its radius $= r$, we have (Vol. I. § 195).

$$P a = \frac{\lambda_2}{l} . \frac{\pi}{4} r^4 E, \text{ hence } \lambda_2 = \frac{4 P al}{\pi r^4 E},$$

and the longitudinal extension:

$$\lambda = \lambda_1 + r\lambda_2 = \frac{P}{\pi r^2 E} l + \frac{4 P al}{\pi r^3 E} = \frac{Pl}{\pi r^2 E}\left(1 + \frac{4a}{r}\right), \text{ hence}$$

$$P = \frac{\pi r^2 K}{1 + \frac{4a}{r}}.$$

If the force act at the periphery of the post, we have in the first case $a = \frac{1}{2} h$, and in the second $a = r$, and, therefore, for the rectangular section $P = \frac{bh}{4} K$, and for the cylindrical $P = \frac{\pi r^2 K}{5}$. Thus, theoretically, a rectangular post will carry only $\frac{1}{4}$, and a cylindrical one only $\frac{1}{5}$ when loaded in the direction of the side of what it will carry when fairly loaded. *Experiments* on cast iron give results of $\frac{1}{3}$ instead of $\frac{1}{4}$ for rectangular columns.

The same laws apply to the uprights AC, Fig. 51, but then λ must be taken as the *greatest compression.*

If the column be inclined, as in Fig. 52, and if its foot make an angle α, with the horizon, we may decompose P into two others $P_1 = P \sin. \alpha$, and $P_2 = P \cos. \alpha$, and in the equations for λ_1 and λ_2,

we must substitute $P \cdot \sin.\,\alpha$, for P, and besides this the extension λ_3 produced by the normal force $P\cos.\,\alpha$, has to be introduced. If we substitute $P\cos.\,\alpha$ for P, and l for a, we obtain $\frac{h\lambda_3}{2}$ for the greatest extension or compression produced by the force $P\cos.\,\alpha$, and hence for rectangular sectioned beams this extension or compression.

Fig. 51.

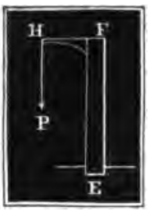
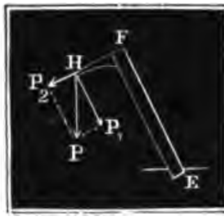


Fig. 52.



$$\frac{h}{2}\lambda_3 = \frac{6\,P\,l^2 \cos.\,\alpha}{b\,h^3\,E}, \text{ hence}$$

$$\lambda = \lambda_1 + \frac{h\,\lambda_2}{2} + \frac{h\,\lambda_3}{2} = \frac{Pl}{E}\left[\left(\frac{1}{b\,h} = \frac{6\,a}{b\,h^2}\right)\sin.\,\alpha + \frac{6\,l\cos.\,\alpha}{b\,h^2}\right]$$

$$= \frac{Pl}{E\,b\,h}\left[\left(1 + \frac{6a}{h}\right)\sin.\,\alpha + \frac{6\,l}{h}\cos.\,\alpha\right];$$

and therefore the tension:

Fig. 53.

$$P = \frac{b\,h\,K}{\left(1 + \frac{6a}{h}\right)\sin.\,\alpha + \frac{6\,l}{h}\cos.\,\alpha}.$$

If the arm FH be on the up-side of the beam, as shown in Fig. 53, we then have:

$$P = \frac{b\,h\,K}{\left(1 + \frac{6a}{h}\right)\sin.\,\alpha + \frac{6\,l}{h}\cos.\,\alpha}$$

and for round columns the expression becomes:

$$P = \frac{\pi\,r^2\,K}{\left(1 \pm \frac{4a}{r}\right)\sin.\,\alpha + \frac{4\,l}{r}\cos.\,\alpha}.$$

§ 29. If a loaded beam AB, Fig. 54, rests upon two uprights, the load P bears upon each in the proportion $\frac{l_2}{l}P$ on AD, and $\frac{l_1}{l}P$ on BE, when l_1, l_2, and l, represent the lengths AB, CA, and CB respectively.

Fig. 54.

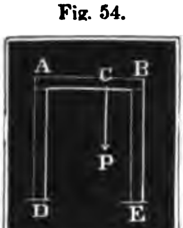


Fig. 55.

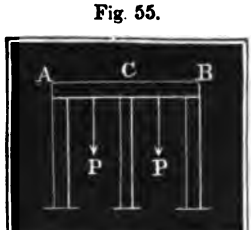
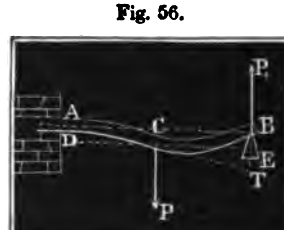


Fig. 56.



If a similar beam rests upon three or more uprights, the pressure on each can only be determined by aid of the theory of the elastic resistance of materials. If weights P and P act at the

centre of the lengths AC and BC, and if we assume that the one part AC is independent of the other part BC, the pressure on the centre upright $= P$, and that on each of the others $= \frac{1}{2} P$. But if we consider the beam as an entire piece, the circumstances are different.

When a beam fastened by one end into a wall AB, Fig. 56, supports a weight P at C, and is supported at the other end B, the beam forms an elastic curve, horizontal at A, but inclining upwards at B. For simplicity's sake, let us assume P as acting at the middle C, and put the length $AB = 2\,l$. The deflexion BT of the outer half CB is equal to the deflexion of the inner half $AD = BE$ plus the tangent distance TE. But according to Vol. I. § 189, the height $BT = \frac{P_1 l^3}{3\,WE}$, if P_1 be the force on the end B required. Again the deflexion:

$$AD = \frac{P\,l^3}{3\,WE} - \frac{P_1}{2\,WE}\left(\tfrac{16}{3}\,l^3 - \tfrac{11}{3}\,l^3\right) = \frac{P\,l^3}{3\,WE} - \frac{5\,P_1\,l^3}{6\,WE},$$

and the tangent distance:

$$TE = CE\,.\,tang.\,\alpha = l\left(\frac{P\,l^2}{2\,WE} - \frac{3\,P_1\,l^2}{2\,WE}\right) = \frac{P\,l^3}{2\,WE} - \frac{3\,P_1\,l^3}{2\,WE};$$

and hence it follows:

$$\frac{P_1}{3} = \frac{P}{3} - \frac{5\,P_1}{6} + \frac{P}{2} - \frac{3\,P_1}{2},\ \text{or}\ 16\,P_1 = 5\,P,\ \text{therefore}\ P_1 = \tfrac{5}{16}\,P.$$

According to this view of the matter, the support B bears $\frac{5}{16}\,P$, and the point of fixture A $\frac{11}{16}\,P$. The same relations obtain in the case of a beam supported by three uprights, when the ends A and B are free to move up, but the middle part C kept horizontal. The uprights under A and B carry, therefore, each $\frac{5}{16}$ of the weight P, whilst the centre post carries $\frac{22}{16}\,P$.

If the supports be inclined as shown in Fig. 57, there arises a horizontal thrust $H = \frac{1}{2}\,P$ $cotg.\,\delta$, with which the feet of the posts tend to spread. If, again, a beam resting upon two uprights be strengthened by two braces as shown in Fig. 58, we may, though only as an approximation, assume that at each end A, A_1 a pressure $-\frac{5}{22}\,P$ acts; whilst on each point C, C_1, there is a pressure of $\frac{16}{22}\,P$. If δ be the angle of inclination BCA of the braces BC, the horizontal thrust in C and B, $= \frac{16}{22}\,P$ $cotg.\,\delta$, and the thrust along the brace $\frac{16}{22}\,\frac{P}{sin.\,\delta}$. If, again, l be the whole length AD, and l_1 the part BD of the support measured up to the brace, the horizontal strain on the upright $= \frac{l_1}{l}\,.\,\frac{8}{11}\,P\ cotg.\,\delta$,

Fig. 57.

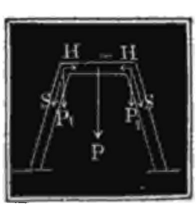
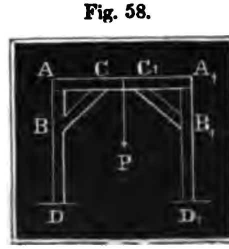


Fig. 58.



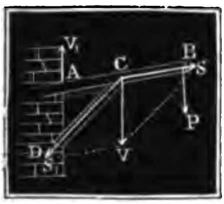
and, therefore, the column has not only to bear the vertical pressure $\frac{P}{2}$, but, likewise, a horizontal force $= \frac{l_1}{l} \frac{8}{11} . P\, cotg.\, \delta$, creating flexure round B. In order, therefore, to insure the sufficiency of such a frame, the formula:

$$\tfrac{1}{2} P = b\,h\,K : \left(1 + 6 . \tfrac{16}{11} \frac{(l - l_1)\, l_1}{lh}\, cotg.\, \delta\right),$$

must be satisfied.

§ 30. *Braces or Struts.*—Fig. 59 shows a case of frequent occurrence. Where a beam AB, fixed in a wall or otherwise at one end, loaded at the other, is strengthened by a brace or strut CD. Let AB the length of the beam $= l$, and the part $AC = l_1$, the inclination of the beam $= \alpha$, and that of the strut $= \delta$. From the load P there arises a vertical pressure in C *downwards:* $V = \frac{l}{l_1} P$, and a vertical pressure at A *upwards:* $V_1 = \left(\frac{l - l_1}{l_1}\right) P$. The first vertical pressure downwards resolves itself into two forces along the axes of the pieces:

Fig. 59.



$$S = \frac{V\, cos.\, \delta}{sin.\, (\delta - \alpha)} = \frac{l\, P\, cos.\, \delta}{l_1\, sin.\, (\delta - \alpha)}, \text{ and}$$

$$S_1 = \frac{V\, cos.\, \alpha}{sin.\, (\delta - \alpha)} = \frac{l\, P\, cos.\, \alpha}{l_1\, sin.\, (\delta - \alpha)}.$$

The case shown in Fig. 60, where the beam is supported by a *tie-brace*, is to be treated in a manner exactly similar to the above. In most cases, the beam AB is horizontal, or $\alpha = 0°$, then we have:

$$S = V\, cotg.\, \delta = \frac{l\, P}{l_1}\, cotg.\, \delta \text{ and } S_1 = \frac{l\, P}{l_1\, sin.\, \delta}.$$

The dimensions of the brace have to be determined in proportion to the strain S_1 acting on it, and that of the beam with reference to the strain S compressing it, and likewise the cross strain arising from P, acting with the moment $P\,(l - l_1)$. Hence (§ 28):

Fig. 60.

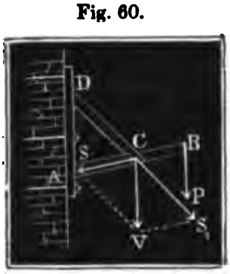
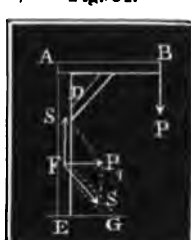


Fig. 61.



$$P = b\,h\,K : \left(\frac{l}{l_1}\, cotg.\, \delta = 6 \frac{(l - l_1)}{h}\right),$$

and by this equation, the section $b\,h$ of the beam must be determined. For the case shown in Fig. 61, we have to find the strain on the upright. The part DE of the upright is compressed by the

force P, and strained across by the moment Pl, therefore we must put $P = \dfrac{b\,h\,K}{1 + \dfrac{6\,l}{h}}$, in order to get the required section $b\,h$. The piece AD, on the other hand, is under a tensile strain $= \left(\dfrac{l - l_1}{l_1}\right) P$, whilst the cross strain is the same, as for lower part; we have, therefore, in this case:

$$P = \frac{b\,h\,K}{\dfrac{l - l_1}{l_1} + \dfrac{6\,l}{h}}.$$

If at the foot of the upright there be placed a strut FG, this would take up the strain $S = \dfrac{l\,P}{a\,cos.\,\alpha}$, if α be its inclination, and $a = EF$, and the force $S_1 = \dfrac{Pl}{h}\,tang.\,\alpha$ passes through the uprights. Hence the part EF of the upright is strained by a force $= P - S_1$ or $S_1 - P$, the former when $a\,cotang.\,\alpha < l$, and the latter when $a\,cotang.\,\alpha > l$, or according as the strut falls within, or beyond, the point of suspension.

Example. In the framing, Fig. 61, suppose $P = 1500$ lbs., $AB = 12$ feet, the upright $EA = 24$ feet, the inclination of the braces $= 45°$, and the horizontal projection of each $= 6$ feet; required the necessary strength for the frame.

The braces have strains:

$$S_1 = \frac{l\,P}{l_1\,sin.\,\delta} = \frac{12\,.\,1500}{6\,sin.\,45°} = \frac{3000}{0{,}7071} = 4243 \text{ lbs. to withstand.}$$

Taking 7400 as modulus of strength, we get, allowing 20 times absolute strength, the section of each brace $= \dfrac{4243}{7400}\,.\,20 = 11{,}5$ square inches. For the beam we may take according to Vol. I. § 198, $K = 12000$ lbs., for breaking across is here most likely to occur. Allowing 20 times the absolute strength, we have to put:

$$20\,.\,1500 = \frac{12000\,b\,h}{2\,.\,1 + \dfrac{6\,.\,l}{h}}, \text{ or } \frac{b\,h}{1 + \dfrac{18}{h}} = 5.$$

If now we make the depth of the beam double its breadth, we get:

$2\,b^2 = 5\left(1 + \dfrac{9}{b}\right)$, or $b^3 - \frac{5}{2}\,b = \frac{45}{2}$. From this we get the breadth of the beam 3,1 inches, and the depth 6,2 inches. For the upright, that is, for the centre part, by similar reasoning we get:

$$20\,.\,1500 = \frac{12000\,b\,h}{1 + \dfrac{6\,.\,12}{h}}, \text{ that is } \frac{b\,h}{1 + \dfrac{72}{h}} = \frac{5}{2}, \text{ or } b\,h = \frac{5}{2} + \frac{180}{h},$$

and if in this case we make $h = 2\,b$, we get $b^3 - \frac{5}{4}\,b = 45$, from which $b = 3{,}7$, and $h = 7{,}4$ inches.

§ 31. *Compound Beams.*—Beams laid upon one another, and united only by bolts, Fig. 62, have a resistance *equal only to the sum of the resistances of the individual beams.* If the beams only abut on each other, as in Fig. 63, and the butting joints be made to

Fig. 62.

Fig. 63.

break joint, the strength of *one beam is lost to the whole.* If the beams be morticed, and tenoned as in Figs. 64 and 65, well strapped together, the strength of the combination is almost equal to that of a solid beam of the same dimensions.

Fig. 64.

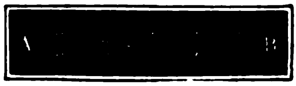
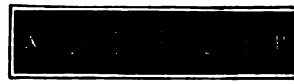
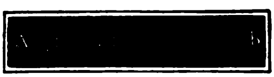


Fig. 65.



Beams are frequently *built* in this manner, to get great strength. The resistance of the elements of a beam increase, as their distance from the neutral axis. If, therefore, we separate two beams by thick tenons or wedges, and then strap or bolt them together, as in Fig. 66, their strength is considerably increased. If b be the breadth and h the depth, l the length and a the distance between the two beams, the strength of the combinations (Vol. I. § 200) is:

Fig. 66



$$P = \left(\frac{(a + 2\,h)^3 - a^3}{l\,(a + 2\,h)}\right) \frac{b\,K}{6}.$$

If, for example, $a = 2\,h$, then $P = 14\,\frac{b\,h^2}{l} \cdot \frac{K}{6}$, whereas $P = 4\,\frac{b\,h^2}{l} \cdot \frac{K}{6}$, if the two beams had only been morticed together.*

The same relations obtain in the beam, shown in Fig. 67, united by St. Andrew's crosses or *lattice-framing*. In like manner, we determine the strength of wooden beams, composed of curved pieces, as in the bridge, Fig. 68, but it must be strictly borne in mind, that wooden framings lose much of their strength by deflexion. A principal advantage of such constructions is, that they are more stiff, and less liable to vibrate than

Fig. 67.

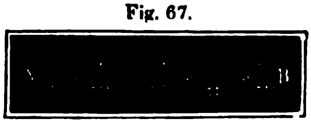
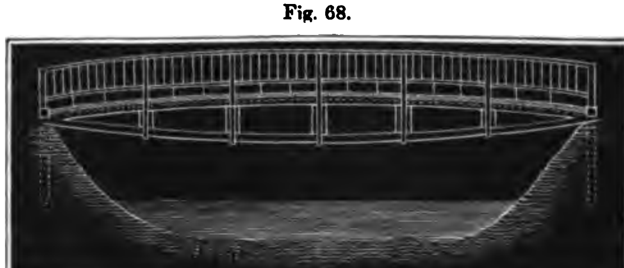


Fig. 68.

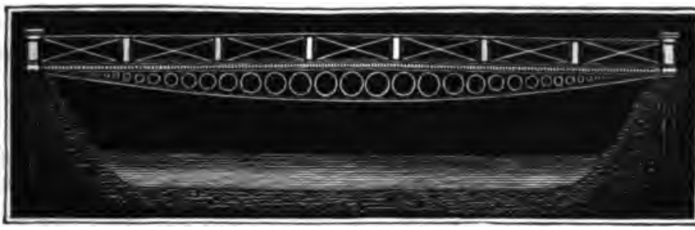


* Obvious as is the truth of this statement, and easy as is its application in practice, it is singular that so little use is made of it in the construction of timber bridges and other buildings in this country. It is evidently applicable to the double beam arches, often inserted in the so called arch and truss bridges.—Am. Ed.

simple beams; and that, as they act only vertically on their points of support, they require no *abutments*, properly so called.

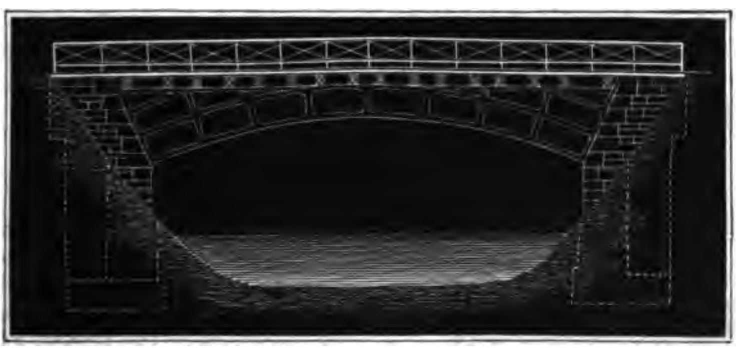
Curved beams, as shown in Fig. 69, have been frequently applied

Fig. 69.



in cast iron structures, and cast iron arches, as in Fig. 70, are a very usually employed bridge material. To judge of the strength of such

Fig. 70.



a structure, its line of resistance must be determined. If this fall everywhere *within* the arch, it shows that there is no cross-strain on the material, but only compression; but if the line of resistance fall *without* the arch, the weak point is where it runs furthest from the arch, and the resistance of the material to cross-strains, is that upon which the stability of the structure depends.

[*Proof of the strength of Complex Structures by means of Models.*—The plan of solving questions in practical mechanics and engineering by faithfully constructed models, presents the very obvious advantage of substituting the moderate cost of *experiment* for the often burdensome, sometimes ruinous, expense of *experience*. The conditions to be fulfilled in constructing models, so as to give reliable information in regard to the action or the stability of structures, may be stated as follows:—

1st. An entire correspondence must exist in the *model*, (at least, of all essential parts,) to the scale of dimensions and weights on which it is proposed to represent the *structure*.

2d. Identity not only in the nature, but also in the condition of materials employed in the model and structure respectively.

3d. Proportional accuracy in forming junctures; and proportional tension given by tightening screws, keys, wedges, and other mechanical means, by which the parts are compacted together.

In testing the model, modes of introducing, distributing, and withdrawing loads, conformable to those which practice will involve in regard to the structure, must be observed, so as to subject the model to shocks, jars, inequality of pressure and irregularities of application, at least proportional to those which the structure will be required to sustain.

Supposing the model of a bridge to have been constructed according to the above requirements, it might be used for either of the two following purposes:—

1. *To determine what weight the structure will bear when undergoing a given deflexion, or when on the point of breaking.*

2. *To ascertain whether the principle of construction be adequate to furnish a bridge of the proposed dimensions, and materials that can fulfil the specified duty.*

As a beam or bridge of uniform dimensions throughout will bear half as much weight accumulated at the centre as it could sustain if distributed throughout its length, the simplest mode of arriving at the result desired is to determine and apply to the centre of the model a weight which shall represent one-half the load supposed to come upon the structure.

The following formula applies to the loading of the model at its centre.

Let l = the length, in feet, of the model between the points of support; p the weight in pounds which the model is to sustain at the centre, representing a load uniformly distributed over its length; w = the weight of so much of the model as lies over the clear opening between its piers; r = the ratio of dimensions between the *structure* and the *model; P* = the load which the *structure* must be able to bear, when accumulated at the centre. Then it is evident that $r\,l$ = the length of structure between the piers. Since the relative resisting powers of similar beams or bridges are as the second powers of their corresponding dimensions, $\therefore r^2 : 1 :: $ resisting power of the *structure* : resisting power of the *model.* Hence, $r^2\,(p + \frac{1}{2}\,w)$ = the absolute resisting power of the structure. Also, since the weights of similar structures are as the third powers of their corresponding dimensions—or, what is the same thing, as the third powers of their ratios of dimensions—therefore $r^3\,w$ = the absolute weight of the structure; so that the weight P, which, by supposition, the structure can bear, accumulated at its centre, will be its absolute resisting power, diminished by half its own weight.

Hence, $P = r^2\left(p + \frac{w}{2}\right) - \frac{r^3\,w}{2} = r^2\,p - \frac{r^3\,w + r^2\,w}{2} = r^2\,p -$

$r^2\,(r - 1)\frac{w}{2} = r^2\left(p - \frac{w}{2} \times r - 1\right)$ [1].

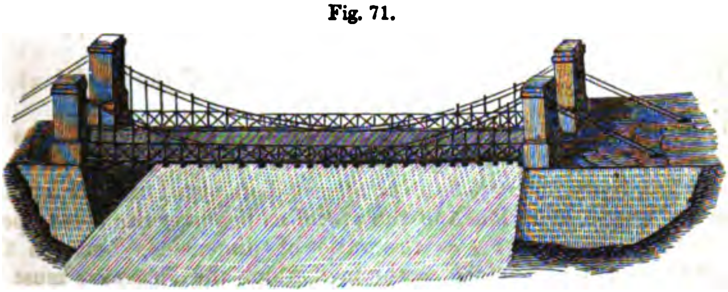
But as, by supposition, P is known, and it is desired to find p, the conversion of the last formula gives $p = \frac{P}{r^2} + \frac{w}{2}(r - 1)$ [2].

Example. It is required to construct, on a given plan, a bridge having a clear opening between the piers of 150 feet, and capable of sustaining two tons per foot of its length, or 300 tons in all, equally distributed over its surface. A model is made on the scale of one inch to the foot, and weighing 136.3 pounds, exclusive of the part which rests directly upon the abutments. It is required to find what number of pounds must be suspended from the centre of the model, in order to prove whether any bridge constructed on the plan, with the relative dimensions and of the materials used in the model, will bear the load above specified.

Substituting the values of the several symbols in the second of the above equations, viz: $p = \frac{P}{r^2} + \frac{w}{2}(r - 1)$, we obtain $p = \frac{300 \times 2240}{12 \times 12} + \frac{136.3}{2} \times (12 - 1) = 3082$ pounds; and twice this number, or 6164 pounds, is the weight which the model ought to bear, when distributed uniformly over its surface.]

§ 32. *Chain or Suspension Bridges.*—Suspension bridges involve considerations distinct from the principle of the stability of either stone, wood, or cast iron bridges, inasmuch as the road-way is suspended from chains or ropes, or is supported upon these. The former is the more frequent construction. Chains or cables drawn up with considerable force, between two or more piers or supports, pass over these to fastenings in rock or masonry, as shown in Fig. 71. The chains are formed of malleable iron bars, united by pins

Fig. 71.



or bolts: and cables of iron or steel wire, laid parallel or twisted together, are frequently employed instead of bar-chains. The

Fig. 72.

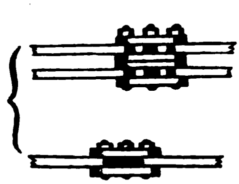
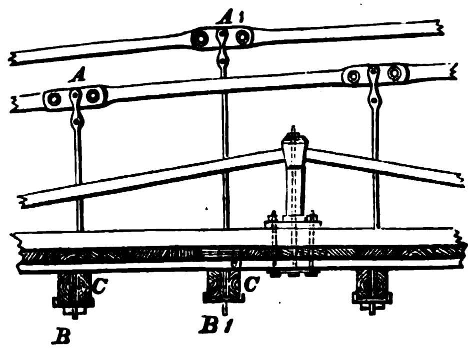


Fig. 73.



dimensions of the *links* or bars, depend upon those of the bridge. In large bridges they are made about 1 inch thick, from 3 to 9 inches deep, and from 10 to 16 feet long. Usually, several sets of bars are hung together, forming a compound chain united by coupling plates and bolts, as shown in Fig. 72 (or without coupling plates, according to Mr. Howard's patent plan). Wire cables are composed of wires of from $\frac{1}{20}$ to $\frac{1}{8}$ of an inch in diameter, and are made of any requisite diameter, varying from $\frac{1}{2}$ an inch to 3 inches. The suspending rods consist of wrought iron rods, or of wire ropes. The rods AB, A_1B_1, are hung by pins passing through the coupling plates as shown in Fig. 73, and suspending ropes are attached as shown in Figs. 74 and 75, by means of shackles with eyes, or by a simple loop. The cross-beams of the road-way C, C_1 are sometimes fastened to the suspending rods as shown in Fig. 74, sometimes as shown in Fig. 73. The rod goes either through the beam, and is then fastened by a *nut* resting on a metal plate, or *washer*, or a *stirrup*, or strap is put over the beam, a hook on the upper side of which goes into the eye of the shackle of the suspension rope, or into the loop formed on it. Upon the cross-bearers longitudinal beams are laid, and these are covered with three inch planking, and again three inch cross planking, according to circumstances, and upon this road-metalling, &c., is laid. In general there are two systems of chains, one above the other, on each side of the bridge, and hence the number of suspension rods is twice the number of joints in any one chain system. The distance from centre to centre of suspending rods is about five feet.

Fig. 74.

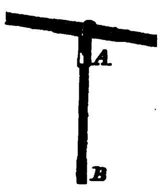
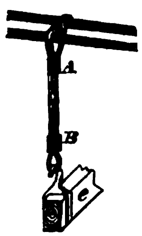


Fig. 75.



The *parapet* of the bridge ought to be framed so as to give the *greatest stiffness to the road-way.**

The width of road-way depends on the purposes which the bridge is to subserve. There should be 3 feet at least for a foot-path, and 7 to $7\frac{1}{2}$ for a carriage way. For a bridge for ordinary traffic, a total width of 25 feet between the parapets is sufficient.

§ 33. The versed sine of the arc of suspension bridges, is generally small in proportion to the cord, varying from $\frac{1}{8}$ to $\frac{1}{25}$, and, therefore, the strain on the chain is very great (Vol. I. § 144). The piers on which the chains pass, and the fastenings by which chains are held must withstand very considerable forces, and hence piers of great stability, and abutments, or rather *anchorage*, of great resistance must be provided. The span of suspension bridges is regulated by various circumstances. A series of smaller spans is often much more economical than one or more large spans to cover the same interval.

* See Appendix.

The Menai bridge in England, the two bridges at Fribourg in Switzerland, the bridge at Roche Bernard in France, the bridge over the Danube at Ofen, are examples of large spans of from 600 to 720 feet; whilst there are innumerable instances of less span in every country. If the chain be not equally strained on the two sides of the pier, which always occurs when one side only is loaded, the chain slides forward towards the side on which there is the greater load. As, however, there would arise considerable friction between the rope and the head of the pier, under the pressure of the resultant force being on it, the pier must have stability to counteract a force equal to this friction. To prevent this action, special contrivances are adopted for diminishing the friction. These means consist, either in passing the chains over rollers or pullies, Fig. 76, which reduces the sliding friction to a rolling friction on a small axle, or the chains pass over a sector which rocks on the head of the pier, inclining to one side or the other as external forces act upon it; or, lastly, the pier is made as a column rocking on its foot, or on a horizontal axis at its foot. That the resultant of the forces acting on the chain may press vertically on the pier head, and thus be least strained by it, it is necessary that the parts of the chain on each side of the pillar should have equal inclinations to the horizon. If this equality cannot be obtained, as is not unfrequently the case for the land piers of bridges, the piers must be considerably strengthened.

Fig. 76.

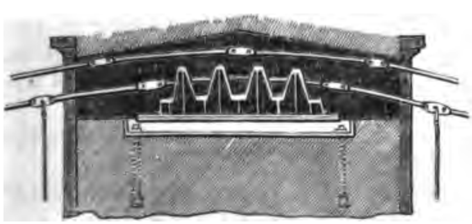
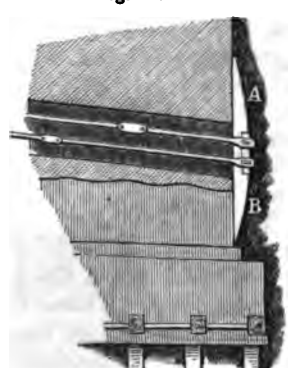


Fig. 77.



To fasten the ends of the chains to the land, various devices have been practised, the general plan of which is to carry the chains by wells or drifts into the rock or soil, and there to fasten them to broad iron or wooden piles, or planking as at *AB*, Fig. 77, which abut upon substantial retaining walls of masonry, or against an arch, or against the rock itself. The fastenings can thus be examined at any time, and adjusting wedges for compensating the influences of expansion and contraction be conveniently manipulated.

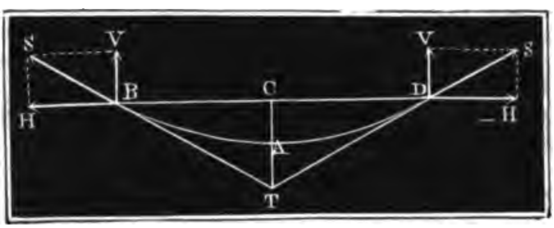
Remark. On the subject of suspension bridges, the most complete treatise is that of Navier, "Rapport et Mémoire sur les ponts suspendus, Paris, 1823." The papers of Mr.

Davies Gilbert, in the "Transactions of the Royal Society of London, 1826," are important in the history of these bridges. In Moseley's "Engineering and Architecture" there is a very elegant investigation of the properties of these structures. The treatise of Drewry on "Suspension Bridges, 1832," is a very excellent resumé of the general practice in respect to suspension bridges. The account of the suspension bridge over the Vilaine, at La Roche Bernard, by Leblanc, Paris, 1841, is very instructive. There is a treatise of Seguin, "Mémoire sur les ponts en fil de fer," worthy of attention. There are many memoirs in the "Annales des ponts et chaussées" on this subject; and, in the volume for 1842, there is an account of a bridge made of *ribbons* of hoop iron.

§ 34. The curve formed by the chain or cable of a suspension bridge, lies between the parabola and the catenary, and is very nearly an ellipse. The parabola approximates the curve in the loaded bridge, the catenary in the unloaded (compare Vol. I. § 144 and § 145, &c.). We shall consider the curve as a parabola, or the bridge in its loaded state.

If the two points of suspension *B* and *D*, Fig. 78, of a chain, be

Fig. 78.

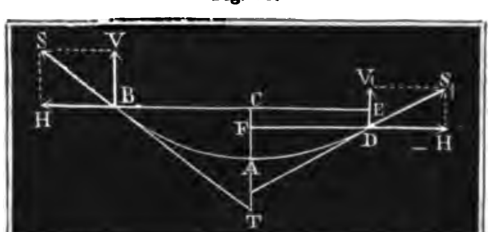


on the same level, and if $BD = 2\,b$, and AC the versed sine or height of the arc $= a$, and the angle $CBT = CDT = \alpha$, then

$$tang..\alpha = \frac{CT}{BC} = \frac{2a}{b} \text{ (Vol. I. § 144).}$$

If the points of suspension be at different levels, as in Fig. 79, the

Fig. 79.



apex of the curve is not in the centre, and the ends of the chain have different inclinations. If we put the co-ordinates AC and BC $= a$ and b, and the co-ordinates AF and $FD = a_1$ and b_1, we put the whole span $BE = s$, and the difference of $DE = h$, we have:

$h = a - a_1$, $s = b + b_1$, and $\frac{a}{a_1} = \frac{b^2}{b_1^{\,2}}$, we have, therefore, from h, s, and a:

1, $a_1 = a - h$, 2, $b = \dfrac{s}{1 + \sqrt{\dfrac{a_1}{a}}}$, 3, $b_1 = s - b = \dfrac{s}{1 + \sqrt{\dfrac{a}{a_1}}}$,

and for the angles of inclination α and α_1:

$$tang.\ \alpha = \frac{2\,a}{b},\ \text{and}\ tang.\ \alpha_1 = \frac{2\,a_1}{b_1}.$$

The length of the parts of the chain $AB = l$ and $AD = l_1$, is expressed by:

$l = b\left[1 + \frac{2}{3}\left(\frac{a}{b}\right)^2\right]$, and $l_1 = b_1\left[1 + \frac{2}{3}\left(\frac{a_1}{b_1}\right)^2\right]$, (Vol. I. § 147).

If we have the distance e between the suspension rods, their number for a length $BC = b$, is $n = \dfrac{b}{e}$; and if in the equation $x = \dfrac{y^2}{b^2}\,a$, we substitute for y the values o, e, $2e$, $3e$, $4e$, &c., we get for the lengths of the suspension rods:

$$0,\ \frac{e^2}{b^2}\,a,\ \frac{4e^2}{b^2}\,a,\ \frac{9e^2}{b^2}\,a,\ \&c.,\ \text{or}\ 0,\ \frac{a}{n^2},\ \frac{4a}{n^2},\ \frac{9a}{n^2},\ \&c.,$$

to each of which a few inches are to be added.

From the weight G of the loaded half of the chain AB, the horizontal tension of the whole chain:

$H = G\ cotg.\ \alpha = \dfrac{b}{2a}\,G$, and the entire tension on the end:

$$S = \frac{G}{sin.\ \alpha} = \frac{2\,a\,G}{\sqrt{b^2 + 4a^2}}.$$

If we know the modulus of strength of the chains and suspension rods, we can determine the sectional dimensions they should have. According to French experience, the greatest load that should be brought on chains, is 12 kilogrammes per square millimetre (or about 8 tons on the square inch), and for cables of iron wire 18 kilog. per square millim., or about 12 tons per square inch. The *suspension rods* are made much stronger in proportion, as they have to resist the *shocks* of loaded wagons, &c., passing along the bridge. The load on them is reduced to from $1\frac{1}{2}$ to 3 tons per square inch of section.

§ 35. *Sectional Dimensions of the Chains and Ropes.*—In order to determine the dimensions of the parts of a suspension bridge, we have to take into consideration, not only the weight of the road-way, but also the greatest weight of men, as troops, or of cattle, or of wagons, that can be brought to bear upon it. This has been taken as 42 lbs. per square foot of surface by Navier, but in the case of a dense crowd of persons, it might amount to 72 lbs. per square foot. Having assumed a certain maximum load, the dimensions of the cross and longitudinal beams have to be determined, and hence we find the entire weight of the road-way. If we put the sum of this constant weight, and the maximum load that may come on to the bridge $= G_1$, and the modulus of strength of the suspension rods $= K$, we get for the section of these $F_1 = \dfrac{G_1}{K}$. From this we have

the weight of these rods, which has to be added to that of the road and load, in order to put the total load on the chain G. If we put the section of the chains $= F$, and the specific gravity of the iron $= \gamma$, we have, retaining the notation as above, the weight of the chains:

$$G_2 = Fl\gamma = Fb\left[1 + \tfrac{2}{3}\left(\frac{a}{b}\right)^2\right]\gamma,$$

and hence the total load on one-half the bridge:

$$G = G_1 + G_2 = G_1 + Fb\left[1 + \tfrac{2}{3}\left(\frac{a}{b}\right)^2\right]\gamma,$$

and the strain at the point of suspension

$$S = \frac{G}{sin.\ a} = \frac{G_1 + Fb\left[1 + \tfrac{2}{3}\left(\frac{a}{b}\right)^2\right]\gamma}{sin.\ a}.$$

But for the necessary security $S = FK$ (where K is the modulus of strength), therefore:

$$FK\ sin.\ a - b\left[1 + \tfrac{2}{3}\left(\frac{a}{b}\right)^2\right]\gamma = G_1,$$

i. e., the section of the chains:

$$F = \frac{G_1}{K\ sin.\ a - b\left[1 + \tfrac{2}{3}\left(\frac{a}{b}\right)^2\right]\gamma}.$$

Example. The dimensions of the parts of a suspension bridge of 150 feet span, 15 feet deflexion, and 25 feet in width are required. Suppose 45 suspension rods on each side, we have then 44 equal parts of 3,409 feet each. The length of these rods, commencing at the centre would be 0, $\frac{15}{22^2} = 0{,}031$, $4 \cdot \frac{15}{22^2} = 0{,}124$, $9 \cdot \frac{15}{22^2} = 0{,}279$, $16 \cdot \frac{15}{22^2} = 0{,}496$, $25 \cdot \frac{15}{22^2} = 0{,}775$ feet, &c., or if we add to each 2 inches, the length becomes: 2, 2,37, 3,49, 5,35, 7,95, 11,30 inches, &c.

The maximum load on the half bridge, we shall take according to Navier $75 \times 25 \times 42$ lbs. $= 78750$ lbs., and if the road-way weighs a little less than a ton per foot of length $G_1 = 157500$, and the section of all the rods of one-half of the bridge: $F_1 = \frac{157500}{2190} = 72$ square inches. The whole bridge is suspended on 90 rods, and, hence the section of each rod is $\frac{72 \cdot 2}{45 \cdot 2} = 1{,}6$ square inches, or the diameter of the rods must be 1,427 inches. According to the rules for the quadrature of the parabola, the mean length of a suspension rod $= \frac{1}{3}$ that of the largest, therefore, $= \frac{1}{3} \cdot 15 = 5$ feet, and if as above, we add 2 inches to it, then it $= 5\frac{1}{6}$ feet, or 62 inches. Thus the volume of all the rods is $90 \times 62 \times 1{,}6 = 8928$ cubic inches, and the weight taken at 0,29 lbs. per cubic inch $= 2598$ lbs. The half of this added to the above-found weight of half the road-way gives $G = 158794 \cdot 5$ lbs., and, hence, according to the formula:

$$F = \frac{G_1}{K\ sin.\ a - b\left[1 + \tfrac{2}{3}\left(\frac{a}{b}\right)^2\right]\gamma},$$

if, $G_1 = 158794{,}5$, $K = 17500$, $b = 75 \times 12 = 900$, $\frac{a}{b} = \frac{15}{72} = 0{,}2083 \ldots$, $\gamma = 0{,}29$,

and $sin.\ a = \frac{2a}{\sqrt{b^2 + 4a^2}} = \frac{30}{\sqrt{75^2 + 30^2}} = \frac{1}{\sqrt{6{,}25 + 1}} = \frac{1}{\sqrt{7{,}25}}$

$$F = \frac{158794{,}5}{17500 \cdot 0{,}3714 - 900 \cdot 0{,}29\,(1 + \tfrac{2}{3} \cdot 0{,}2083^2)} = \frac{158794{,}5}{6499{,}5 - 268{,}5} = \frac{158794{,}5}{6231{,}0}$$

$= 25{,}48$ square inches, and, therefore, for 4 chains the section of each would be 6,37 square inches.

§ 36. *Elongation of Chains.*—The chains are elongated by the load, and, therefore, the deflexion is increased. Changes of temperature also, produce variations in the length of the chains. We must know the effects of both these. If the deflexion changes from a to a_1, the length.

$$l = b\left[1 + \tfrac{8}{3}\left(\frac{a}{b}\right)^2\right], \text{ and } l_1 = b\left[1 + \tfrac{8}{3}\left(\frac{a_1}{b}\right)^2\right],$$

and hence the elongation of the chain:

$$\lambda_1 = l_1 - l = \tfrac{8}{3}\left(\frac{a_1^2 - a^2}{b}\right) = \tfrac{8}{3}\frac{(a_1 - a)(a_1 + a)}{b},$$

or if Δ be the increase in the deflexion, and if we put as an approximation $a + a_1 = 2\,a$, $\lambda_1 = \frac{16}{3}\frac{a}{b}\Delta$, and, therefore, for the whole chain $\lambda = \frac{32}{3}\frac{a}{b}\Delta$, inversely $\Delta = \frac{3}{32}\frac{b}{a}\lambda$. From the weight G of the half bridge, the horizontal tension or tension at the apex, $H = G\, cotg.\, \alpha$, and the tension at the ends: $S = \dfrac{G}{sin.\, \alpha}$, therefore, the mean tension $= \dfrac{H + S}{2} = \dfrac{G\,(1 + cos.\, \alpha)}{2\, sin.\, \alpha}$, and the extension of the chains caused by this force $\lambda = \dfrac{(1 + cos.\, \alpha)}{2\, sin.\, \alpha} \cdot \dfrac{G}{FE} \cdot 2\, l$, (Vol. I. § 183), for which we may put as an approximation: $\lambda = \dfrac{2\,G\,b}{FE\, sin.\, \alpha}$. If we introduce this value into that for Δ, we get the increase in the deflexion for the loaded chains:

$$\Delta = \tfrac{3}{32} \cdot \frac{b}{a} \cdot \frac{2\,G\,b}{FE\, sin.\, \alpha} = \tfrac{3}{16}\,\frac{G}{FE\, sin.\, \alpha} \cdot \frac{b^2}{a},$$

or $sin.\, \alpha = \dfrac{2\,a}{\sqrt{b^2 + 4\,a^2}}$, or approximately $= \dfrac{2\,a}{b}$, we get

$$\Delta = \tfrac{3}{32} \cdot \frac{G}{FE} \cdot \frac{b^3}{a^2}.$$

Malleable iron expands 0,0000122 of its length for a rise of temperature of one degree of centigrade ($=$,0000068 for 1° Fahr.). This increase is, therefore, $0{,}0000122 \,.\, 2\,l\,t$ for the length of chain l, and a rise of t degrees of temperature, or $0{,}0000244\, l\,t$. Putting this in the expression for Δ, we get the increase of deflexion for a rise of temperature t:

$$\Delta = \tfrac{3}{8} \cdot \frac{b}{a} \cdot 0{,}0000244 \,.\, lt, \text{ or approximately } = 0{,}00000915 \,.\, t\,\frac{b^2}{a}.$$

In like manner the contraction is determined for decrease of temperature.

Example. Retaining the values of the example in the last paragraph, we get the increase of the height of the arc corresponding to the load, taking the modulus of elas-

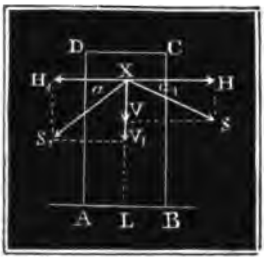
ticity of malleable iron = 29000000 (Vol. I. § 186), and adding 6841,8 lbs. the half-weight of the chains to the load 158794,5 lbs.

$$G = 158794,5 + 6841,8 = 165636,3 \text{ lbs.}$$

$\Delta = \frac{3}{8} \cdot \frac{165636,3}{25,48 \cdot 29000000} \cdot \frac{900^2}{180^2} = \frac{1397}{739} = 1,9$ inch. For a change of temperature of 20° C. this change of deflexion is:

$$0,00000915 \cdot 20 \cdot \frac{900^2}{180} = 0,4 \text{ inches.}$$

Fig. 80.



§ 37. *Piers and Abutments.*—The proportions of the piers and abutments form an important consideration.

If S and S_1 be the tension on the ends of the chain, Fig. 80, and α and α_1 the angles of inclination, the vertical pressure on the pier:

$$V_2 = V + V_1 = S \sin. \alpha + S_1 \sin. \alpha_1,$$

and the horizontal pressure, as the horizontal tensions counteract each other,

$$H_2 = H - H_1 = S \cos. \alpha - S_1 \cos. \alpha_1.$$

If, now, h be the height, b the breadth, and d the depth or thickness of a pier, the density of the masonry of which $= \gamma$, its weight is $b\,d\,h\,\gamma = G$, and the total vertical pressure $= V_2 + G = S \sin. \alpha + S_1 \sin. \alpha_1 + b\,d\,h\,\gamma$. In order, however, that the horizontal force $H_2 = H - H_1$ may not turn the pier on the edge B, it is requisite that the statical moment:

$$H_2 \cdot LX = H_2\,h = (S \cos. \alpha - S_1 \cos. \alpha_1)\,h$$

should be less than the statical moment:

$$(V_2 + G)\,BL = (S \sin. \alpha + S_1 \sin. \alpha_1 + b\,d\,h\,\gamma)\,\frac{b}{2},$$

i. e. it is requisite that:

$$b^2 + \frac{S \sin. \alpha + S_1 \sin. \alpha_1}{d\,h\,\gamma}\,b > \frac{2\,(S \cos. \alpha - S_1 \cos. \alpha_1)}{d\,\gamma}, \text{ or}$$

$$b^2 + \frac{V + V_1}{d\,h\,\gamma}\,b > \frac{2\,(H - H_1)}{d\,\gamma}.$$

For the sake of security, the greatest value of $S \cos. \alpha$ and the least value of $S_1 \cos. \alpha_1$ are to be taken, that is to say S is to be taken as completely loaded, and S_1 as unloaded. This formula assumes that the forces S and S_1 are entirely transferred to the pier head, which, of course, only takes place when the friction on the pier head exceeds the difference $S - S_1$ of the tensions. According to Vol. I. § 175, this friction is:

$$F = \left[\left(1 + 2 f \sin. \frac{\beta}{2}\right)^n - 1\right] S_1,$$

where f is the co-efficient of friction, n the number of links on the pier head, and β the central angle corresponding to *one link*, it is hence requisite that:

$$S - S_1 < \left[\left(1 + 2 f \sin. \frac{\beta}{2}\right)^n - 1\right] S_1, \text{ or}$$

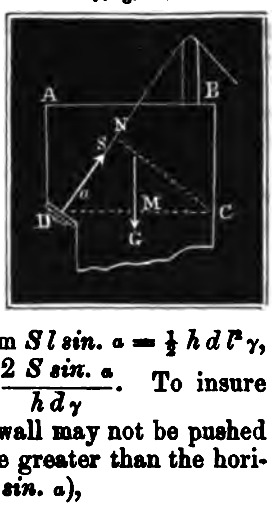
$S < \left(1 + 2 f \sin. \frac{\beta}{2}\right)^n S.$ Unless this condition be fulfilled, the chain will slide on the pier head, and therefore we have only to put $S = \left(1 + 2 f \sin. \frac{\beta}{2}\right)^n S_1$, or for ropes $S = e^{f\alpha} S_1$ (Vol. I. § 176), in the above formula. If the chain or cable be laid upon pulleys, this difference is much less, and, therefore, the requisite thickness of pier is less. If the radius of the pulleys $= a$, and the radius of the axes on which they turn $= r$, then:

$$S = S_1 + f \frac{a}{r} (S \sin. \alpha + S_1 \sin. \alpha_1),$$

for the friction reduced to that of the axis may be put $= f \frac{a}{r} (S \sin. \alpha + S_1 \sin. \alpha_1) = f \frac{a}{r} (V + V_1)$. If the rope passes over rollers, then the friction is so much reduced, that we may put $S = S_1$.

From the tension S on the land or back chains, we can determine the dimensions of the retaining wall AC, Fig. 81.

Fig. 81.



The strain S tends to turn the masonry AC round C, and acts with a leverage $CN = CD \sin. \alpha = l \sin. \alpha$, if α be the angle of inclination SDC of the rope to the horizon, and l the length CD of the wall. The height of the wall resists with the moment:

$$G \,.\, CM = h d l \gamma \,.\, \frac{l}{2} = \tfrac{1}{2} h d l \gamma,$$

where h is the height BC, d the depth, and γ the weight of the masonry. For equilibrium $S l \sin. \alpha = \frac{1}{2} h d l^2 \gamma$, and, therefore, the requisite width of wall $l = \frac{2 S \sin. \alpha}{h d \gamma}$. To insure stability this must be doubled. That such a wall may not be pushed forward, the friction $f (G - S \sin. \alpha)$ must be greater than the horizontal force $S \cos. \alpha$, or, $f G > S (\cos. \alpha + f \sin. \alpha)$, i. e. $l > \frac{S}{h d \gamma} \left(\frac{\cos. \alpha}{f} + \sin. \alpha\right)$, in which f may be taken $= 0{,}67$.

Example. For the suspension bridge mentioned in previous paragraphs, the vertical force of the loaded chain: $V = 165636{,}3$ lbs., and that of the unloaded: $V_1 = V - 78750 = 86886{,}3$ lbs., if now we suppose friction pulleys to be applied, the radius of each pulley being to that of its axis as $\frac{a}{r} = \frac{1}{4}$ and $f = \frac{1}{4}$, the friction at the pulleys would be $\frac{1}{4} \,.\, \frac{1}{4} \,.\, (165636{,}3 + 86886{,}3) = 15782{,}6$ lbs., or much less than the difference of the tensions, and therefore the chains would move, and the pulleys turn till the tension on the one had so far increased, and that on the other so far decreased that the difference would be only 15782,6 lbs. If now the height of the pier be 16 feet, the thickness 4 feet, and the weight of the masonry 130 lbs. per cubic foot, we have for the necessary width of piers:

$$b^2 + \frac{252522{,}6}{16\,.\,4\,.\,130}\,.\,b = \frac{2\,.\,15782{,}6\ cos.\ a}{4\,.\,130}, \text{ i.e. } b^2 + 30{,}4\,b = \frac{15782{,}6\,.\,0{,}9285}{260} = 56{,}36.$$

Therefore, $b = \frac{56{,}36 - b^2}{30{,}4} = 1{,}75$ feet. This would, in practice, be made 4 to 5 feet.

The requisite length of retaining wall, when $h = 16$ and $d = 16$ feet, is:

$l = \frac{2\ S\ sin.\ a}{h\,d\,\gamma} = \frac{2\,.\,165636{,}3}{16\,.\,10\,.\,130} = 15{,}9$ feet, which would be made 20 to 25 in practice.

STRENGTH OF MATERIALS.*

The strength of an engineer's work depends upon its *proportions*, the *materials* of which it is composed, and the *manner of putting them together*.

As to stability, a structure may yield, under the pressures to which it is subjected, either by the *slipping of certain of its surfaces of contact upon one another*, or by their *turning over upon the edges of one another*. The former case very rarely occurs.

The *strength of materials* depends upon their physical constitution, viz: *form, texture, hardness, elasticity, and ductility.*† The resistance of materials in buildings is tested in reference to various strains—compression—extension—detrusion—deflexion under a cross strain, and fracture under a cross strain.

A. *Compression.*—In prismatic pieces of *stone, wood*, or *cast iron*, which absolutely *crush* under a strain, *the strength is directly proportional to the transverse area of the piece.*

Pieces exposed to compression are not fairly *crushed*, but in some measure broken across, where their height is to their diameter or least lateral dimensions in the case of,

Stone, more than	as	6	to 1?
Wood, "	"	4	to 1
Cast iron, "	"	3½	to 1
Wrought iron,	"	2½	to 1

The manner in which materials yield under a crushing strain is very remarkable, as is exhibited by the experiments of Rondelet, Vicat, and E. Hodgkinson, the latter of whom has found, that the plane of rupture is always inclined at the same angle to the base of the column, when its height is within the limits above mentioned. The angle of rupture depends upon the nature of the material. In cast iron, for instance, it varies from 48° to 58° in different makes of iron, though confined to narrow limits for different prisms of the same make.—*See* "Report British Association," 1836, and Moseley's "Engineering," p. 550.

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* Professor Weisbach has treated this subject as it is usually given in elementary works on mechanics. Excepting as exhibiting approximately the laws of the phenomena, the "theory of the strength of the materials" has many practical defects. These we shall not here enumerate; but have put together, in as concise a form as possible, what we consider to be the most valuable part of our present knowledge on this subject to engineers or architects engaged in the execution of works.

† *See*, on this subject, Poncelet's "Mécanique Industrielle."

TABLE OF THE RESISTANCE OF MATERIALS TO CRUSHING.

Material		lbs. per sq. inch.
Granite, Scotch		10804 to 8184
" Cornwall		6292
Sandstone, Dundee		6490
" Derby		3110
Marble (white)		9583
Limestone (Portland)		6550
Stourbridge brick		1695
Deal	unseasoned	6780
	seasoned	7290
Beech	unseasoned	7730
	seasoned	9360
Oak	unseasoned	6480
	seasoned	10000
Mahogany		8198
Larch	unseasoned	3200
	seasoned	5568
Poplar	unseasoned	3100
	seasoned	5100
Cast iron, *good common*		109800
" " *Stirling's toughened*		145500
Wrought iron		56000?

The effect of *seasoning* or drying timber, in increasing its strength, is never to be lost sight of. In wrought iron, a strain of 28000 lbs. reduces the length, and causes a slight lateral bulging, corresponding to the slight reduction in length; that is to say, for a compressive strain of about ½ths of the absolute crushing-strain, wrought iron is quite "*crippled.*"

Stirling's process of *toughening* cast iron, consists in adding to it proportions of *malleable scrap*, varying according to the nature of the cast iron in its normal state.

Scotch hot blast, No. 1, will take 28 to 30 lbs. of scrap per cent.
" " " No. 2, " 20 " " "

Welsh and Staffordshire hot or cold blast iron require a less addition of scrap.

This process increases the strength of all cast irons, from 50 to 80 per cent.

The strength of pieces, such as pillars, that *break across*, but are not *crushed* under compression, may be calculated by the following formulas, as found by Mr. Hodgkinson's "Experimental Researches on the Strength of Pillars," published in the Phil. Trans., 1840, and in his edition of "Tredgold on Cast Iron," published 1846.

For stone: $b = \frac{ad^4}{l^2}$? For timber: $b = \frac{ad^4}{l^2}$.

For cast iron. Solid pillar, round ends $b = \frac{ad^{3,76}}{l^{1,7}}$.

" " flat ends $b = \frac{ad^{3,55}}{l^{1,7}}$.

The length 1 *being not less than* 30 d.

Hollow pillars, round ends $b = a\frac{D^{3,76} - d^{3,76}}{l^{1,7}}$.

The length 1 *being not less than* 15 d.

Hollow pillars, flat ends $b = a\frac{D^{3,55} - d^{3,55}}{l^{1,7}}$.

The length 1 *being not less than* 30 d.

Wrought iron, round ends $b = \frac{ad^{3,76}}{l^2}$.

" " flat ends $b = \frac{ad^{3,55}}{l^2}$.

When the length is from 30 *to* 90 *times the diameter.*

The laws indicated by the formulas do not hold good for shorter columns.

TABLE OF THE VALUES OF a, (D and d being in inches, l in feet, and the result b being the *crushing-weight in* lbs.

Granite	flat ends	25000?
Sandstone		15000?
Marble		24000?
Dantzic oak		24542
Red deal		17511
Cast iron solid pillar,	flat ends	98922
" " " "	round ends	33379
Hollow pillars,	flat ends	99318
" "	round ends	29074
Wrought iron	flat ends	299617
" "	round ends	95844

The numbers here given are co-efficients, and have no meaning, apart from the special position they occupy in the formulas.

In all pillars of cast iron, whose length is thirty times the diameter or upwards, the strength of those with flat ends seems to be *three times* as great as the strength of those of the same dimensions with rounded ends: when l is less than 30 d, the ratio of the strength of pillars of the same dimensions with flat and with rounded ends, is very variable.

When pillars are reduced in length below the proportion above indicated, there is a *falling off* of their strength, *nearly in proportion to the reduction in the length of the pillar;* and this obviously must be the case, as the strength to resist *flexure*, under a compressive strain, increases as the fourth power of the diameter, whilst the resistance to *crushing* increases only as the square of the diameter.

For pillars of less length than 15 times their diameter, there is a falling off in the resistance, on account of the change produced in the position of the molecules of the material by the great weight necessary to break them: Mr. Hodgkinson has, however, given a formula which includes this case, and by which the strength of the pillars, however short, may be deduced from the results of the formulas for long columns, when the *crushing strength* of the material is known. The formula is $y = \frac{b\,c}{b + \frac{3}{4}c}$ in which b is the strength of the pillar, as calculated by the rules for long pillars, and c the crushing weight of the material, and $y =$ the strength of the short pillar.

In *similar pillars*, the strength is nearly as the square (1,865 power) of the diameter, or of any other lineal dimension; and as the area of the section is as the square of the diameter, the strength is nearly as the area of the transverse section.

The strength of pillars not less than 30 times their diameter: that of cast iron with rounded ends being set = 1000

Wrought iron	is = 1745
Cast steel	= 2518
Dantzic oak, square ends . .	= 108,8
Red deal	= 78,5

In all long pillars, whose ends are firmly fixed, the power to resist breaking is equal to that of pillars of the same diameter and half the length, with the ends rounded or turned, so that the strain runs through the axis.

B. *Extension.*—When a tensile strain *passes up the centre* of a piece of stone, wood, or metal, the *resistance is proportional to the transverse area of the piece.*

TABLE OF THE RESISTANCE OF MATERIALS TO RUPTURE BY TENSILE STRAIN.

Stone. Portland	857 lbs. per sq. inch.
Fine sandstone . . .	215
Brick	275 to 300
Glass	3565
Hydraulic lime, best . . .	168
Good	142
Mean quality	100
Common lime	48
Timber. Deal	12857 to 11549
Beech	17850
Oak	9198 to 12780
Mahogany . . .	16500
Larch	9700 to 10220
Poplar	7200
Cast iron (Hodgkinson) . . .	13505 to 17136
" (Rennie)	19200
" (Cubitt)	27778?
" Stirling's toughened . .	28000
Wrought iron bars	65520 to 56000
Wire (hard) . .	128000 to 65360
Wire (annealed), half	the strength of hard.
Plates . . .	52100
Brass wire (hard)	98960 to 63000
Annealed . . .	49000
Gun metal (hard)	36368
Copper rolled	35000
" cast	19200
Ropes. Hemp	1 ton per lb. weight per fathom.
Wire, (Newall and Co.) .	2 tons per lb. weight per fathom.

In reference to the above table, it may be stated that it contains numbers which are the *mean values of the tensile strain,* as deduced

after a careful weeding of the experimental results that have hitherto been published.

[*Thermotension, or the Effect of Heat on the Tenacity of Iron.*— The following table exhibits the effect of heat on the tenacity of iron, both while actually hot and also subsequent to the application of a strain at high temperature. The comparisons are made on thirty-two different specimens of iron, the origin of which is designated in the first column of the table. The temperature at which either the "hot fracture" or the hot strain was made on each bar, and which produced the strengthening effect of "thermotension," is contained in the second column. The third contains the number of trials made on each specimen of iron to ascertain its strength in its ordinary state and temperature, as it came from the hammer or the rolls, and before being put under strain at a high temperature. Column fourth shows the number of times the specimen was broken, or at least strained, at the temperature marked in column third. Column fifth gives the number of fractures made on the specimen to obtain the average strength after being heated, strained, and then cooled again to ordinary temperature. Columns six, seven, and eight, contain the absolute strength given in the three different states respectively. Column nine exhibits the per centage increase of strength by treatment with thermotension, and ten, the difference in strength between the iron at ordinary temperature in its original state, and that which it possessed while heated as in column third. In three cases only does it appear that the strength had been diminished by heating up to the point at which the trials were made. One of those trials was at 766°, one at 662°, and the third at 552°. The average temperature at which the effect was produced was 573.7°, at which point the tenth column shows that the strength of thirty varieties of iron, was 5.9 per cent. greater than at ordinary temperatures, say at 60 or 80 degrees.

It also appears that the average gain of tenacity in thirty-two samples of iron, by the process above mentioned, was 17.85 per cent., ranging from 8.2 to 28.2 per cent. In a report by the Editor to the Bureau of construction, equipment, and repairs of the Navy Department of the United States, it is proved that the average gain of *length* of bolts of iron treated at the Washington Navy Yard, by this same process, was 5.75 per cent., and the gain of *strength* 16.64, making together the gain of *value* 22.4 per cent. The addition of 5.75 to 17.85, gives 23.6 per cent. for the total gain of value. In many instances the experiments proved the gain of *length* to exceed 7 per cent. The total elongation of a bar of iron, broken in its original cold state, is from two to three times as great as the *same force* would produce upon it if applied at a temperature of 573°, which force will, moreover, not break the bar at that temperature.

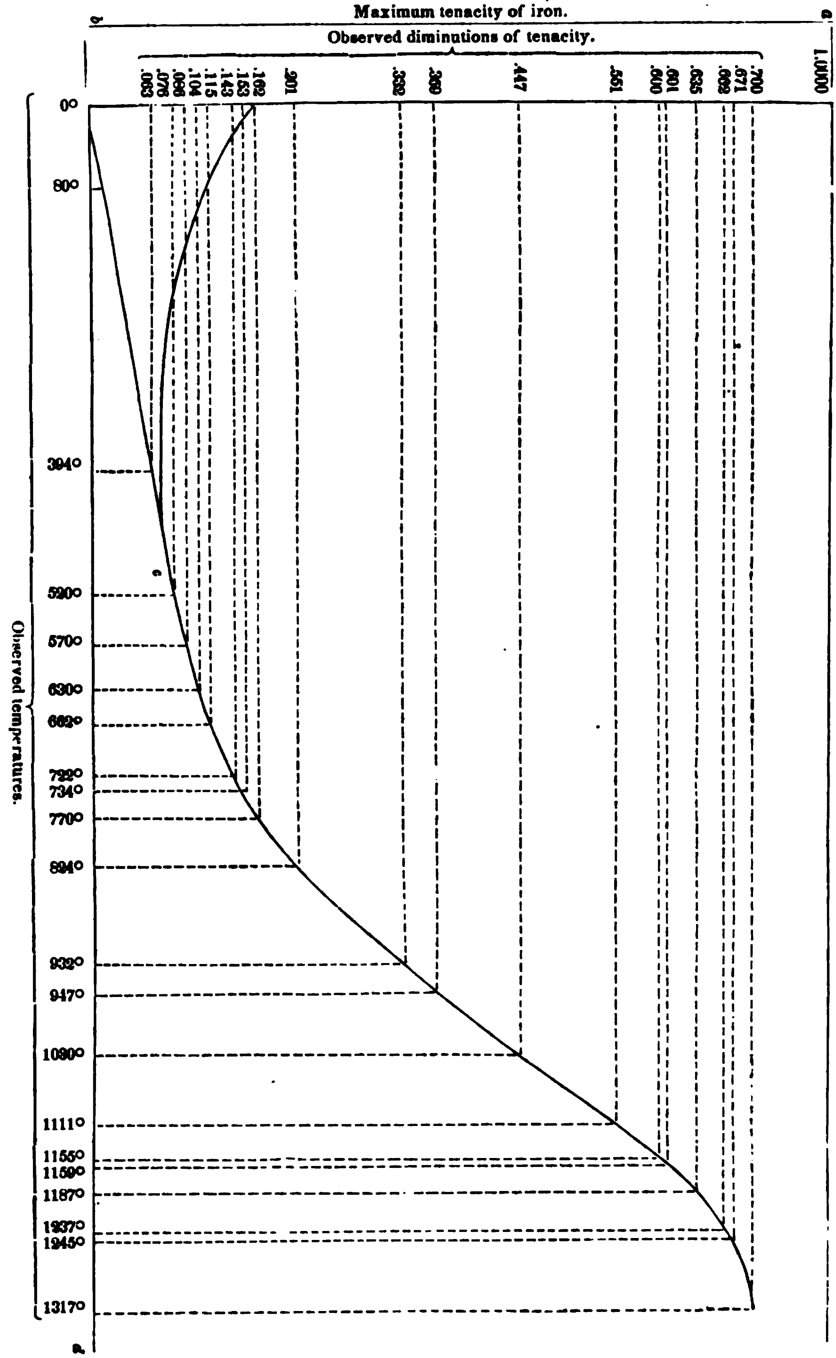
TABLE EXHIBITING THE EFFECT OF HEAT ON THIRTY-TWO VARIETIES OF MALLEABLE IRON.

NAME AND ORIGIN OF THE SPECIMEN OF IRON TRIED.	Temperature of iron when proved hot.	No. trials for average ordinary strength.	No. of trials at high temperatures.	No. of trials after heating and straining.	Average strength in ordinary state and at ordinary temperature.	Strength exhibited while hot.	Average strength after applying thermotension.	Gain of tenacity per cent. by the treatment with thermotension.	Diff. per ct between original strength and that shown when the bar was hot.
Salisbury (Conn.), gun bar	554°	4	1	5	59.271	60 459	65 090	8.2	+ 2.0
Maramec (Mo), bar iron	528	7	1	3	63.775	54.273	69.044	9.8	+ 0.9
Phillipsburgh (Pa). wire	500	4	2	4	79,720	80.4[illegible]	84 187	10 4	+ 0.9
Ellicott's Baltimore boiler plate	770	3	1	5	56.644	56,644	63.132	10.7	+ 0.0
" " " "	662	4	1	7	58.899	58,181	64,820	10 9	— 1 2
Salisbury (Conn.), gun bar	560	3	1	7	59.654	60.323	66,638	11 7	+ 1.1
" " "	590	4	1	9	59 032	62.952	67.384	13.6	+ 6.6
Swedish bar iron	530	2	1	3	58.012	59,775	66.334	14.3	+ 3 0
Nashville (Tenn.), bar iron	580	7	1	5	54,934	58 451	62.600	14 5	+ 6.4
Salisbury (Conn), gun bar	572	5	1	10	58 305	58.895	63.558	15.0	+ 1.0
Ellicott's Baltimore forged bar	394	1	1	1	57.182	63,322	66.960	15.3	+10.7
Spang & Son,* hammered plate	766	3	1	1	57.664	54.819	66.500	15.8	— 4.9
Blake & Co.,* hammered plate	572	6	1	4	60 532	62,278	66,941	16.3	+ 2.8
Salisbury (Conn.), gun bar	580	4	1	5	55.977		65,883	17.0	no hot frac.
" " "	564	4	2	8	54,644	60.215	64 363	17.6	+10.2
" " "	576	5	2	5	58 299	64 278	68.988	18.4	+10.0
" " "	630	4	1	6	57.433	60,010	67.569	18 7	+ 4.5
English "best-best" cable bolt	560	3	1	10	62.466		71,000	19.3	no hot frac.
Spang's Pittsburgh hamm'd plate	552	2	1	3	56 762	55 932	62,736	19 4	— 1.4
Nashville (Tenn.), bar iron	560	7	1	6	52 729	58.534	62,127	19 5	+11.0
Mason & Miltenberger,* piled	574	4	1	2	55 426	60.083	68,329	19 5	+ 8.4
Nashville (Tenn.), bar iron	562	4	1	2	52,194	59,623	62,433	19.8	+14 2
Ellicott's Baltimore boiler iron	553	2	1	3	61,519	66,450	73.898	20.1	+ 8 0
Schaenberger's Pittsburgh boiler	630	1	1	3	53.803	56,159	64,926	20.6	+ 4.4
Maramec (Mo), bar iron	564	5	1	9	49.974	52.158	58.126	20.6	+ 4.3
Nashville (Tenn.), bar	578	5	1	5	52,406	59.192	62,951	21.1	+12 9
Russia sable bar	534	5	1	3	76.071	77.161	92 470	21 5	+ 1.4
Maramec (Mo.), bar	576	6	1	4	43 386	50.067	53.368	23 0	+15.4
Ellicott's hammered bar	394	1	1	1	53.176	56.570	69.767	25 8	+ 6.4
Salisbury (Conn.) gun bar	575	3	1	6	52 673	60.988	66,685	26 6	+15.3
Maramec (Mo). bar	574	5	2	4	45.586	51,437	58,252	28 1	+12.8
Blake's Pittsburgh hamm'd plate	564	6	1	4	52.937	58,284	65,425	28 2	+10 1
Mean,	573.7	129	36	153			Mean,	17.85	+ 5.9

Fig. 82 represents the tenacity of wrought iron at various temperatures from 0° up to 1317° as measured in parts of the total maximum tenacity, the line *a b* representing that maximum, and the line 0°*d* (indefinite towards *d*) being the scale of observed temperatures, in degrees Fahrenheit marked below it. The vertical dotted lines, or ordinates of the curve, therefore, exhibit temperatures, and the corresponding horizontal ones, or abscissas, show diminutions from the maximum strength, at the temperature observed. Thus, at a temperature of 1030°, the diminution from maximum tenacity is .4478, and, consequently, the remaining strength is 55.22 per cent. At 1187° the diminution is .6852, and the remaining strength 36.48 per cent., and at 1245° (a dull red heat in daylight) the diminution is .6715, and the remaining cohesion only 32.85 per cent., &c.

* Of Pittsburgh.

Fig. 82.


Maximum tenacity of iron.
Observed diminutions of tenacity.
Observed temperatures.
b
a
1.0000
.063
.076
.096
.104
.115
.143
.153
.162
.201
.332
.369
.447
.551
.600
.601
.635
.668
.671
.700
0°
80°
394°
520°
570°
630°
662°
722°
734°
770°
824°
932°
947°
1030°
1111°
1155°
1159°
1187°
1237°
1245°
1317°

For a more full exposition of the effect of heat on the tenacity of iron under direct tension, and for investigations of the relation between temperature and tenacity, reference may be had to the "Report on the Strength of Materials for Steam Boilers," page 212—218.

At page 75 of the same report, will be found the law of tenacity as affected by temperature for rolled copper. In that metal no increase of strength takes place from increase of temperature in any part of the scale; and the law eliminated from about 180 comparisons of different experiments on several specimens of copper, is, that the diminutions of strength by augmentations of temperature follow the principle of a parabola, of which the ordinates representing the elevation of the temperature above 32°, have to the abscissas representing the diminutions of tenacity, a relation expressed by saying, that *the third powers of the temperature are proportional to the second powers, of the diminution of strength which they produce.* This law was ascertained in the following manner: Putting $t =$ any observed temperature above 32°; $t' =$ any other observed temperature above the same point; $d =$ the diminution of tenacity by the former temperature and $d' =$ that by the latter: also making $x =$ that power of the temperature according to which the diminution of tenacity takes place; we have, by the supposition $t^x : t'^x :: d : d'$, or $\frac{t^x}{t'^x} = \frac{d'}{d}$.

From this we derive the expression $x = \frac{log.\ d' - log.\ d}{log.\ t' - log.\ t}$.

Example. At a temperature of 1016° the tenacity of a bar of copper was found to have been diminished 66.91 per cent. below its strength at 32°; at the temperature of 492° it was 21.33 per cent. below what it was at 32°; according to what power of the temperature did the tenacity vary?

Here $x = \frac{log.\ .6691 - log.\ .2133}{log.\ (1016-32) - log.\ (492-32)} = 1.50$; hence $t^{1'5} : t'^{1'5} :: d : d'$, or $t^3 : t'^3 :: d^2 : d'^2$.

Transforming this into an equation, we get $\left(\frac{t'}{t}\right)^3 = \left(\frac{d'}{d}\right)^2$, and $\frac{d'}{d} = \left(\frac{t'}{t}\right)^{\frac{3}{2}}$, or $d' = d\left(\frac{t'}{t}\right)^{\frac{3}{2}}$. From this $\frac{3}{2}\,(log.\ t' - log.\ t) + log.\ d = log.\ d'$; by which, knowing the diminution d at any one temperature t, we are enabled to calculate what it will be at the temperature t'.]

In reference to cast iron, the first or lower numbers (p. 70) are the results of Mr. Hodgkinson's experiments; the higher number is the result of numerous experiments made for Mr. Thomas Cubitt by Mr. Dines.* This difference is chiefly of importance in respect of there being a discrepancy so wide, between results stated by two careful experimenters. In reference to the experiments on Mr. Morries Stirling's toughened iron, they were made by the same direct means as were all Mr. Hodgkinson's experiments. The tensile strain of cast iron is seldom brought directly into action; and the part it plays in the resistance to cross strains is evidently not that for which the direct strength shown by Mr. Cubitt's experiments can be attributed to it.

* See Mr. Henry Law's edition of Gregory's "Mathematics for Practical Men," p. 375.

The elongation of wrought iron, under a given tensile strain, may be judged of from the following experiment.*

Load per square inch in producing an elongation of				Load per square inch, producing fracture.	Total elongation divided by original length.
$\frac{1}{400}$	$\frac{1}{100}$	$\frac{1}{50}$	$\frac{1}{20}$		
lbs. 34700	lbs. 40980	lbs. 46124	lbs. 52122	lbs. 56834	lbs. .086

According to Vicat, the elongation of iron wire for a load of 1428 lbs. per square inch, or $\frac{1}{40}$ the breaking strain, amounts to 0,000057.

Mr. E. Hodgkinson's experiments have proved, in like manner, that no material is so elastic as to recover itself perfectly from even very small loads allowed to act for a considerable time, and the defect of elasticity is nearly as the square of the weight applied.

The *modulus* or *co-efficient* of elasticity, is a term first suggested by Dr. Thomas Young, to denote the measure of the elastic reaction, or the energy of the resistance of any substance, and is represented thus: $E = \frac{P}{A\,i}$.

Where E is the co-efficient of elasticity, P the weight in pounds, producing the proportional elongation $i \left(= \frac{l}{L}\right.$ where $l =$ the elongation, and L the original length) in a bar with a base of sectional area A.

Rigidity is expressed by the ratio $\frac{EA}{L}$.

Thus, the *elastic resistance* of a prism of any material, is really only the *rigidity* referred to the unit of length of the prism.

[* In the report of the Committee of the Franklin Institute, on the materials for steam boilers, p. 219–20, will be found very numerous observations on the elasticity of iron, of which the following may be cited as the results of direct measurement.

Bar 49 Boiler plate from Juniata blooms.	Recoil, when relieved from strain, in parts of original length.		Breaking weight in lbs. per square inch.	Total elongation after fracture.
	$\frac{1}{468}$ 51.030 lbs. per square inch.	$\frac{1}{348}$ 54.860 lbs. per square inch.	57.565	6.9 per cent.
Bar 226.	$\frac{1}{500}$ 43.800 lbs. per square inch.		49.053	6.25 per cent.
Bar 228.	$\frac{1}{313}$ 34.804 lbs. per square inch.		40.643	
Bar 230.	$\frac{1}{480}$ 47.155 lbs. per square inch.		49.368	

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TABLE OF DATA CONNECTED WITH THE ELASTIC RESISTANCE OF MATERIALS.

Name of material.	Tensile strain per square inch for limit of elasticity.	Proportional elongation for strain of limit of elasticity.	Ratio of strain in column 1 to that causing rupture.	Modulus of elasticity.
				lbs.
Oak	2856	0.00167	0.23	1,713600
Yellow pine	3332	0.00117	0.33	1,856400
Red pine	4498	0.00210	0.44	2,142000
Larch	2470	0.00192	0.30	1,285200
Beech	3355	0.00242	0.30	1,385160
Bar iron, ordinary quality . . .	17,600	0.00062	0.30	28,400000
" " Swedish hammered } selected .	24,400	0.00093	0.44	29,365000
" " English rolled } selected .	18,850	0.00072	0.37	29,465000
Wire, No. 9, unannealed . . .	47,532	0.00165	0.49	28,825000
" " annealed	36,300	0.00129	0.58	28,081000
Steel plates, tempered blue . . .	93,720	0.00222	0.67	42,600000
Steel wire of commerce	35,700	0.00120	0.50	29,500000
Cast iron	. . .	. . .	. . .	17,000000 to 13,000000

NOTE.—By "Limit of Elasticity," is meant the limits within which displacement of the parts of materials under strain may be called into play without *permanent palpable derangement, or crippling.*

C. Detrusion is the resistance that the coherence of the particles of materials opposes to their sliding on each other, under a detrusive strain.

The resistance to detrusion, or the "force necessary to *shear* across" any material, is called into play at the joints, and in the bolts of framings of timber and iron, and the rivets of steam boilers, &c.

The resistance of deal to detrusion in the direction of fibre is 592 lbs. per square inch.

The resistance of cast iron to detrusion is about 73000 lbs. per square inch, as deduced from experiments on crushing.

The resistance of wrought iron to detrusion, or to a force "shearing it across" is 45000 to 50000 lbs. per square inch, or from 70 to 80 per cent. of the resistance to a direct tensile strain.

D. Deflexion.—When a *beam* is deflected by a cross strain, the side of the beam which is bounded by the concave surface is *compressed*, and that bounded by the convex surface is *extended.* The surface at which extension terminates and compression begins, is termed the neutral surface.

The property of *elasticity*, inherent in all substances in a greater or less degree, causes them to resume their original form, very nearly, when, under forces of compression, extension, or deflexion, they have undergone a limited change of form. Up to this limit, the amounts of extension and compression for a given cross strain

are nearly equal, and, therefore, the neutral surface lies very nearly, if not accurately, in the centre of gravity of the cross section of the beam.

Beyond this limit the position of the neutral surface changes, as the flexure increases; because, in stone and cast iron at least, the resistance to compression is greater than the resistance to extension, whilst the amount of deformation, under the compressive strain, is less than under an equal tensile strain. In wrought iron, as it possesses great *ductility*, this limit occurs much later than in cast iron. In timber, the resistance to extension is greater than that to compression, and its want of homogeneity renders the limit alluded to very variable.

The general law of deflexion is, that it increases, *cæteris paribus*, directly as the cube of the length of the piece, and inversely as the breadth and cube of the depth.*

E. Fracture.—The theory of deflexion, which gives the displacements of the parts of beams before the conditions even of *crippling*, has few practical applications; while equations for the resistance to fracture, which is what is essential in practice to be known, are more simply established.

The hypothesis for the theory of resistance of materials to fracture or rupture, propounded by Galileo, consists in placing the horizontal axis of equilibrium at the lowest point of the section of rupture, or in supposing the material incompressible; and he considered the *internal force developed at each point of the section as constant for every point.*

The hypothesis commonly attributed to Mariotte and Leibnitz, consists in like manner in placing the horizontal axis of equilibrium at the lowest point of the section, and in supposing the internal force developed at each point *proportional to the distance of that point from the axis of equilibrium.*

The hypothesis now generally adopted, consists in admitting that the resistance of each point, at the instant rupture is going to take place, continues *proportional to the extension and compression*, and, therefore, *that the axis of equilibrium*, or *neutral surface*, *has the same position as in the case of a very small deflexion.*

Experiments have proved that none of these hypotheses is true, and, that, according to the physical constitution of the material, the formula deduced from the one or the other may be taken as representing experiments. Experiments on cast iron are best represented by the deduction from Galileo's hypothesis; those on stone, by Mariotte's, and those on timber and wrought iron, by the modern hypothesis, announced by Hooke, and first developed by Dr. T. Young.

The formula commonly employed for reducing experiments, or for calculating dimensions by aid of experiments, on beams of uni-

* For the most complete development of this subject, the student is referred to Mr. Moseley's work, "Engineering and Architecture," Part V.

form rectangular section, fixed at one end and loaded at the other, is $W = \frac{f\,b\,d^2}{n\,l}$.

On Galileo's hypothesis	$n = 2$
On Leibnitz and Mariotte's	$n = 3$
On Young's hypothesis	$n = 6$
The mean of experiments gives for cast iron	$n = 2.63$
" " stone	$n = 3$
" " wrought iron and wood	$n = 6$?

To answer the imperfection of the theory, however, f^1 is substituted for f; or for the resistance to a direct tensile or compressive strain there is substituted a co-efficient of the composite resistance to fracture, under a cross strain.

The most convenient general formula in use for calculating the resistance to fracture under a cross strain is $W = \frac{f^1\,I}{l\,c_1}$.

Where $W =$ the breaking weight, $I =$ the moment of inertia of the cross section of the beam, round an axis passing through its centre of gravity, c_1 the distance of the neutral surface, from the side at which the material gives way; and l the length. The beam is supposed fixed in the circumstances above mentioned.

For a beam supported at each end, and loaded in the middle, this becomes $W = \frac{4\,f^1\,I}{l\,c_1}$, and for beams of triangular section: $W = \frac{2}{3}\frac{f^1\,b\,d^2}{l}$.

For a beam supported at each end, if the load be uniformly distributed over it, we have $W = \frac{8\,f^1\,I}{l\,c_1}$, and for beams of rectangular section, $W = \frac{4}{3} f_1 \frac{b\,d^2}{l}$.

If the weight of the beam G be taken into account, the above formulæ become respectively $W + \frac{1}{2}\,G = \frac{2\,f^1\,I}{l\,c_1}$, and $W + G = \frac{8\,f^1\,I}{l\,c_1}$.

For the forms of transverse section commonly met with in practice, the values of I in terms of the breadth b, and depth d, of the beam, are as follows:

1. Rectangular section.	$I = \frac{1}{12}\,b\,d^3$.	$c_1 = \frac{1}{2}\,d$.
2. Circular section.	$I = \frac{1}{4}\,\pi\,r^4$.	$c_1 = r$.
3. I shaped and hollow rectangular, b_1 and d_1, being the breadth and depth of hollow.	$I = \frac{1}{12}\,(b\,d^3 - b_1\,d_1^3)$.	$c_1 = \frac{1}{2}\,d$.
4. Hollow cylinder, or annular section, $r_1 =$ radius of hollow.	$I = \frac{1}{4}\,\pi\,(r^4 - r_1^4)$.	$c_1 = r$.
5. Inverted ⊥ (Mr. Hodgkinson's for cast iron). When $\mathcal{A}_1$ $\mathcal{A}_2$ $\mathcal{A}_3$ are the *areas*, and d_1 d_2 d_3 the *depths* of the top flange, the bottom flange and the uniting rib respectively.	$I = \frac{1}{12}\,(\mathcal{A}_1 d_1^2 + \mathcal{A}_2 d_2^2 + \mathcal{A}_3 d_3^2)$. $+ \frac{1}{4}\,(d_1 - d_3)^2 \mathcal{A}_1 - (d_2 + d_1)^2 \mathcal{A}_2$. $- \frac{1}{4}\left(\frac{(d_2 + d_3)\,\mathcal{A}_2 - (d_1 + d_3)\,\mathcal{A}_1}{\mathcal{A}}\right)$.	c_1 depending on the form of the beam.

The following table contains values of f^1, or *modulus of rupture*, being deductions from experiment by the formula $f^1 = \frac{3\,w\,l}{2\,b\,d^2}$, all dimensions, that is, l, b, and d, being in inches.

Name of material.	Modulus of rupture. lbs.	Working load. lbs.
Stone (Rochdale)	2358	235
" Yorkshire flag	1116	112
" Caithness slate	5142	514
Beech	9336	1550
Birch	9624	1600
Deal (Christiania)	9864	1640
" Memel	10386	1700
Fir	6700	1100
Larch	6894	1150
Oak, English	10000	1700
" Dantzic	8742	1500
Cast iron	30000 to 46900	5000 to 8000
" Hot blast mean	36900	6000
" Cold blast mean	39987	6500
" Stirling's toughened	46750	7800
Wrought iron	54000	9000

The following table, drawn up by Mr. Hodgkinson, gives the relation between the resistances to *crushing*, rupture by *tension*, and by *cross strain*.

Material.	Assumed resistance to crushing per square inch.	Mean resistance to rupture by extension per square inch.	Mean transverse strength of a bar, 1 inch square and 1 foot long.
Timber	1000 or 1	1900 or 1.9	85.1 or 0.045
Cast iron	1000 or 1	158 or 0.16	19.8 or 0.02
Stone	1000 or 1	100 or 0.1	9.8 or 0.01
Glass	1000 or 1	123 or 0.125	10. or 0.01

From this table we get an idea of the extent to which the mutual dependency of the fibres or particles of the material comes into play when the pieces are bent.

This table indicates, too, that the resistance of the same area of cross section must vary according to the disposition of the material compressed and extended in the section. Mr. Hodgkinson has proved, in reference to this, that for cast iron, one mode of disposing the iron in the section gives a greater strength per square inch of the section than another, in the ratio of 40 to 23, and the principle holds in other materials.

For the inverted ⊥-shaped girder, the strongest form is that in

which the bottom flange is six times the area of the top flange. When, in these girders, the *length*, *depth*, and *top flange* are constant, and the thickness of the vertical rib between the flanges small and constant, *the strength is nearly in proportion to the area of the bottom flange.* Again, in beams of this form which vary only in depth, the strength is nearly as the depth.

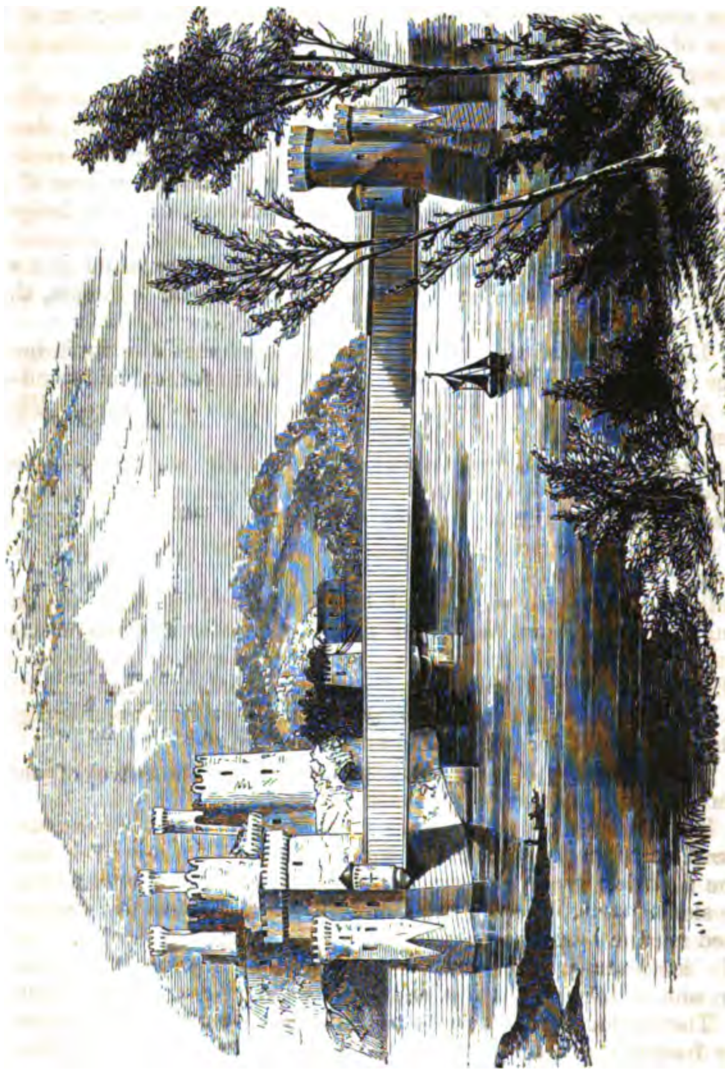
Mr. Hodgkinson has hence deduced the following simple rule for calculating the strength of cast iron beams *approaching the form of greatest strength*, viz: $W = \frac{2.166\, a\, d}{l}$ in which $W =$ the breaking weight in tons; $a =$ the area of bottom flange at centre of length in square inches, $d =$ the depth of the beam in inches, and l its length in feet.

As it is very usual to express the load a girder or beam has to bear, in terms of its length, or $W =$ to $w\, l$, (as, for example, the girders of railway bridges have to be of dimensions to bear a strain of 2 tons per foot of their length,) Mr. Hodgkinson's formula may be converted into the following very simple one for calculating the area of the bottom flange, viz: $a = \frac{w\, l^2}{2{,}116\, d}$ in which w is *the weight per foot* of the girder, of the load upon it. Further, as d is generally a simple fraction of $l = x\, l$, we may make the formula $a \doteq \frac{w\, x\, l}{25{,}992}$. For example, it is a usual and generally convenient proportion to make $d = \frac{1}{18}\, l$, and hence, for railway girders, in which $w = 2$, $a = \frac{l + 16}{12.99}$, which we may put $a = \frac{l + 16}{13}$. Engineers now make girders of proportions such as to bear 6 times the greatest load likely to come upon them. Hence, as there are generally 4 girders to take the load in a railway bridge, our formula may be written $a = \frac{l + x}{8.662}$ for the area of the bottom flange (at its centre) of each girder.

Open cast iron girders are bad in principle. Of all systems of framing girders or beams, the principle of perfect continuity of the component parts, involved in Mr. Fairbairn's patent *malleable iron girders*, is the best.

Without entering further into an examination of this subject, it appears that the present is a fitting place to give a concise account of the so-called "TUBULAR BRIDGES," now being erected by Mr. Robert Stephenson for crossing the Conway, and the Menai straits on the line of the Chester and Holyhead railway. The problem of passing both these points with the "Holyhead road," was solved by Telford in 1825, by the erection of the well known Conway and Menai suspension-bridges. Suspension-bridges have been rejected as inapplicable to railways, and Mr. Stephenson has proposed, nay, has already completely settled the practicability of carrying out the *girder system* to meet the case. A girder to span 462 feet is an

Fig. 83.



original and bold conception; and now that it may be said *to have been executed*, an attempt, if only imperfect, to sketch the progress of engineering art in the direction that has led to this master-piece, cannot but be useful.

The circumstances demanding or necessitating the erection of a bridge of great span, occur but seldom, and the double condition of erecting the bridge without centering, still more rarely.

The deep and rapid rivers of Switzerland, seem first to have called forth constructive skill for this purpose. In the year 1757, Jean Ulrich Grubenmann, born at Taffen, in the canton Appenzell, erected the celebrated bridge at Schaffhausen, over the Rhine, in lieu of a stone bridge that had been swept away by the stream. In designing his bridge, Grubenmann took advantage of a rock about midway across, for the erection of a pier to support the ends of two frames or *compound girders of carpentry*, the one of 170 feet, the other 193 feet clear-bearing, or span.

In 1778, Grubenmann and his brother constructed the Wettingen bridge over the Limmat, on the same principle that had guided them so successfully to the erection of that at Schaffhausen. This bridge had a clear span of 390 feet.*

To Chretien von Michel, an engraver at Bale, we are indebted for the preservation of a record of the details of construction of these two bridges, viz.: "Plans, coupes et élévations des trois Ponts de Bois les plus remarquables de la Suisse, publiés d'après les dessins originaux, Basle, 1803."

Both these bridges were burnt by the French in 1799, the one having stood 42 years, the other 21 years. Over the one, stones weighing 25 tons each had passed; and over the other a division of the French army with its artillery, in extreme haste. ("Emy, Traité de la Charpente.") The points of construction in Wittingen bridge, to which we would direct especial attention, are:—

1. The *continuity* of the framing, especially in its vertical plane, as perfect as the nature of the materials allow.

2. The introduction of a *roof* as an integral part of *the constructive strength of the* bridge, and of the disposition of the greater *mass* of the timber towards the top and bottom, while the intermediate more slender part, or *rib*, is stiffened at every 15 feet by strongly framed uprights on the outside and inside. The timbers are laid nearly horizontally, accurately bedded on, and indented into each other, and bolted together by numerous wrought iron through-bolts.

3. The circumstance that the two side frames of each were raised ready framed into their positions. This latter is an inference from the fact, that powerful *screw-jacks* placed on a scaffolding, supported

* [The single arch wooden bridge, built by Lewis Wernwag, over the river Schuylkill, at Fairmount, Philadelphia, had a span of 340 feet 4 inches, and a rise of the arch in the centre of nearly 19 feet or above $\frac{1}{18}$th of the chord line. This bridge had a triple beam arch of timber surmounted by king-posts and truss braces, with longitudinal ties above, the whole being strengthened by screw-bolts. See a figure of it in Rees' Cyclo., Amer. Edition, vol. 34.—Am. Ed.]

on piles ("des *vérins* placés sur des échafaudages établis sur *pilotes*"), were used in raising the bridge at Schaffhausen, and that the Limmat, near the convent of Wettingen, is of great depth.

Fig. 84.

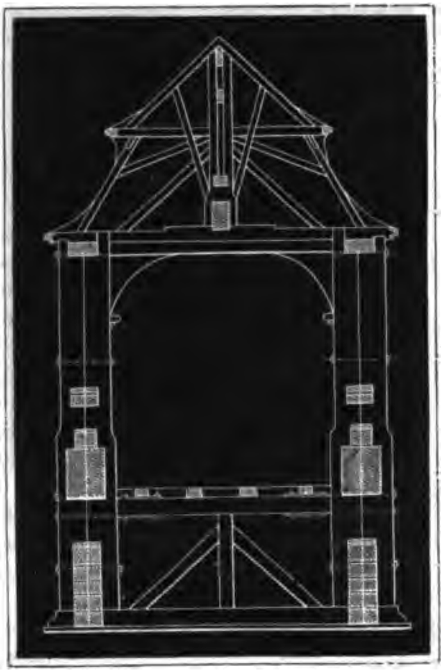


Fig. 85.

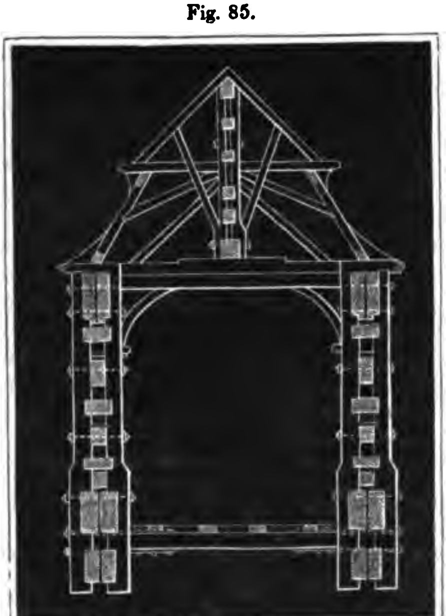


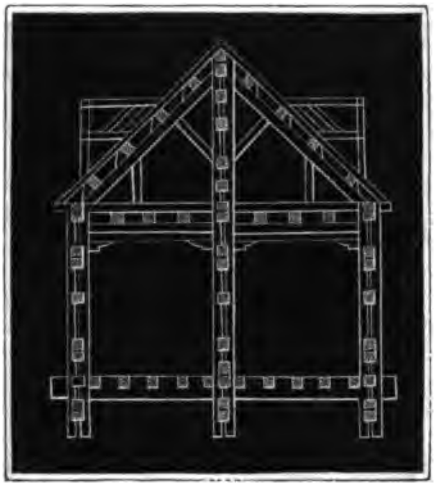
Fig. 84 is a section of the bridge of Wettingen at the ends, and Fig. 85 a section at the centre. They sufficiently illustrate what we have said above in reference to the principle of continuity, and the disposition of the roof and timber of the frames generally, in reference to the strength of the bridge.

At the period when the Wettingen bridge was erected by the Apenzell carpenter, the science of the strength of materials had scarcely begun to be formed. Galileo's theory, partially corrected by the hypothesis of Hooke and Leibnitz, and by the experiments of Mariotte and Buffon, began to attract notice; but our present knowledge of the mechanism of the transverse strain, resulting from the later experiments of Duhamel, Rondelet and Barlow, and the theories founded upon them were undeveloped. Yet we find the essential elements of these theories fully recognized in the construction of the bridges erected by the brothers Grubenmann. Art is the mother of Science.

This was the largest bridge ever *erected* on Grubenmann's principle; but, in 1772, there was exhibited, at the Hotel d'Espagne, rue Dauphine, a model of a bridge designed by one M.

Claus, for Lord Hervey. This was the model of a bridge 900 feet span, to be thrown across the Derry. The model was 20 feet long, or $\frac{1}{45}$ of the full size. The engravings were executed by Lerouge, and Fig. 86 is taken from the plate. It is a transverse section of the bridge at about $\frac{1}{3}$ of the span from the abutment or pier. The scale being about $\frac{1}{183}$.

Fig. 86.



Grubenmann's principle is adopted. The frames are here again nearly continuous. They consist of beams laid *nearly horizontal*, indented into each other, bolted together by innumerable long wrought iron bolts, forming the side ribs, and these were stiffened laterally by uprights. The floor and roof are so framed with the trusses or ribs, as to form one great double *box*, or hollow girder, nearly every pound in the weight of which is available towards the absolute strength of the whole.

This bridge was never executed; but we see in it a still more perfect adoption of the plan of making the floor and roof a part of the framing, and also a recognition of the fact that wood has double the resistance to extension, that it has to compression; and, hence, the timbers of the upper part are arranged conformably to this fact. This was clearly recognized by Grubenmann, but not so perfectly worked out in the construction of his bridges, as was done by Claus. The introduction of a roof, as an integral part of the structure, is, of course, limited to cases in which the span is such as necessitates a *depth* of girder of 16 to 18 feet at least. The proportion of the depth to the length of the bridge of Wettingen is nearly $\frac{1}{12}$. (For further details of the construction of the model, *see* "Emy, *Traité de la Charpente*, Vol. II. p. 398, and plate 134.)

In Great Britain, the problem of erecting bridges of wide span had scarcely ever been mooted till about the beginning of this century, when the joint influence of the inventions of her Dudleys, Brindleys, Hargreaves, Arkwrights, Smeatons, Watts, Corts, Wyatts, Mylnes, Rennies, Telfords, so rapidly developed the long latent industrial genius of the country, that in the short space of half a century, from being as low as any, she became the first in the scale of nations for perfection in internal communication, manufacturing skill, and in productiveness of the useful metals, especially *iron*.

In the year 1800, the subject of replacing Old London Bridge,

occupied the attention of nearly every engineer of eminence, and of many men of acknowledged scientific attainments. At this period, the success of the Wearmouth Bridge, designed by Mr. Wilson, in 1793, and erected in 1796, by Rowland Burdon, and of that of Buildwash, erected by Telford, 1796, seems to have drawn the attention of the most distinguished engineers to this material, as that best facilitating the execution of bridges of great span. The wonderful progress of the iron trade at this period, also, had its influence. The question of rebuilding London Bridge was shelved at this period; but Messrs. Telford and Douglas gave in designs for spanning the Thames by a single arch of 600 feet span, and the practicability of the design was supported by the opinions of Playfair, Robison, Watt, Southern, and others. In 1808—12, Staines Bridge was erected by Mr. Wilson, that of Boston by Mr. Rennie, and that at Bristol by Mr. Jessop. Vauxhall Bridge was commenced, 1813, by Rennie, finished 1818, by Mr. Walker. The magnificent Southwark Bridge was erected 1814 to 1818, by Messrs. Rennie, father and son.

The principle of construction adopted in all these, was that of the *arch*. The cast iron was framed so as to render the structure as strictly analogous to that of an arch of voussoirs as possible. We shall here only notice that the adoption of this principle involves a prodigious expenditure of cast iron, to insure the lateral stability, essential in the voussoir principle, beyond what is necessary for the vertical strength required to bear the load.

The use of cast iron as the framing of machinery, floor-girders, lock-gates, swivel-bridges, &c. &c., became more and more usual in the construction of works executed after 1808, at which period Brunel, by demonstrating the practicability by using cast iron as the framing of his block-machinery, gave new confidence in adopting the recommendation of Smeaton, on this subject, made 50 years earlier.

In 1817, Barlow's Essay on the "Strength of Timber, Iron, and other materials," was published, and English engineers were thus put far on the way of making "principles of science rules of their art." A few years afterwards, Tredgold's Essay "On the strength of cast Iron and other Metals," was published; and this remarkable work of a most remarkable man, together with Barlow's work, had—all engineers will admit—a powerful influence in extending the rational use of iron in construction. Ten years later, Mr. Eaton Hodgkinson, of Manchester, began a course of inquiry on the strength of iron, which, while it has earned for him and his coadjutor, Mr. Fairbairn, a high reputation for scientific knowledge and skill, has, even more directly than the earlier works mentioned, contributed to the present important position of iron as a material in construction.

During this period, too, the dependence of England on Russia and Sweden, for malleable iron, was put an end to, by the improvements and vast extension of the Welsh and Staffordshire rolling-mills, which, towards 1810, began to stock the markets with iron, equal

for all ordinary purposes to that which, up to this period, had been chiefly supplied by foreigners.

It is a distinguishing element in the engineer's art, to adopt the material best suited, economically speaking, to the work he has to accomplish.

In 1806, the price of bar iron, larger size, was £20 per ton; in 1816, it was £10 per ton; in 1828, it was £8 per ton; and in 1831, it was £5 to £6 per ton.

Thus this material has gradually come into the domain of applications in construction, from which its high price had long excluded the consideration of its qualifications. Roofs of great span began to be formed of combinations of cast and malleable iron. The eligibility of the one to resist strains of compression, and of the other to resist tensile strains, became familiar to those engaged in practical construction.

In 1825, a new engineering era had arisen. As the genius of Brindley, under the mighty influence of the policy of a Chatham, had created the inland navigation of England, the genius of a Stephenson, under the influence of the policy of a Huskisson, created the railway system. Steam navigation advanced from mere essays to a system of vast importance. The demands of the ship builder, the locomotive maker, the railway engineer, gave rise to new exertions of the iron masters. Blooms were puddled, of sizes hitherto deemed impracticable. It became usual to have bars rolled, and pieces forged of sizes exceeding those which, within a few years, had been deemed wonderful or isolated examples. In this respect, the complacent dictum of a celebrated engineer, that "no difficulty can arise in engineering or mechanical art, that is not certain to be overcome," has been fully borne out.

In the construction of the London and Birmingham Railway, the Great Western Railway, the Midland Counties Railway, and others, the engineers made ample use of cast iron, and examples of girders of 50, 60, even 70 feet in length are to be found on these lines of railway. The scientific principles of construction of such girders were not at once recognized or learned, and we consequently find excess of iron in most instances, and mistaken construction in others. There was no time for gathering exact knowledge, though extant. A limited experience of successful cases led to endless repetitions of girders of not very happy proportions, and "trussed" in the wrong direction. The outcry made in England on the subject of hot blast iron being so inferior in quality, so treacherous, &c. &c., the consequent high price demanded for castings of what was termed good iron, had considerable influence in limiting the applications of iron in railway bridges. Stone and brick were preferred for the few bridges of great span erected. Suspension bridges were tried and failed. Of the wooden bridges erected, that over the Tyne at Scotswood, by Mr. Blackmore, deserves mention as involving the best principles of construction. The path so well opened up by Grubenmann had long been lost. The system of the Bavarian engineer,

Wiebecking, and applied by him successfully to the bridge at Bambery, 215 feet span, and others, were extensively made known by his published writings, whilst the better principle of Grubenmann was overlooked. The essential part of Wiebecking's system consists in putting the main *strength* of the frame in arches of curved timbers trenailed together, on to which the rest of the timbers of each truss is framed, *suspending the horizontal ties*, from which the road-way is supported. Wiebecking's system, with certain modifications, was adopted in France by M. Emmery, about 1830, and by the Messrs. Green, of Newcastle, about 1840. In imitation of Wiebecking's plan, too, the *bow and string* fashion of open cast iron girders was adopted, small as is the analogy between *wood* and *iron*. Beginning with the bridge over the Regent's Canal at Camden Town, this fashion of girder has been many times repeated, on various scales; and is in execution even at the present moment, for spans of 120 feet, in the high level bridge at Newcastle-upon-Tyne.

In the mean time, in America, Town's lattice frame bridges, and Long's diagonal frame bridges, had been invented, and railway bridges of 150 to 180 feet clean span, had been executed according to each system. In the largest application of Long's system, the depth of the frame is about 20 feet, and the sides and floor, and roof are connected together, so as to form one *box-like girder*. The diagonal framing, even when carried out in the form of lattice work, makes but an imperfect continuity in the framing, or ribs connecting together the top and bottom rails or flanges; but this is *the principle* aimed at, and the bridges are to be considered as very successful engineering. They have been adopted in England, in a few cases, the largest being that of an occupation bridge on the Birmingham and Gloucester railway; but wooden structures are avoided in that country, on account of the extreme variations in the hygrometric state of the atmosphere.

Of the many lattice bridges erected in America, the most interesting in reference to our subject, is the iron tubular lattice bridge in the great hotel, Tremont House, at Boston. This is *an elliptical tube of lattice or trellis work*, the height being 7 to 8 feet, the minor axis of the ellipse being 4′—6″, the span about 120 feet. The top is stiffened by a longitudinal bar. The flooring of wood on the bottom, is about three feet 6 inches wide, and helps to stiffen the whole. This foot bridge had been several years in use in 1848, and its perfect rigidity, it may be here mentioned, at once suggested the applicability of the plan for carrying a railway across the Menai straits.

Among the circumstances concurring to the result consummated by Mr. Stephenson, the success of iron ships of enormous dimensions, in resisting the strain they have to undergo, is certainly a prominent one. The Great Britain steam-ship, for example, is 258 feet in length. It is mainly composed of sheet and angle iron, of less than half an inch in thickness; it is thus, like other iron ships, a mere shell; and yet from its perfect *continuity*, and the nature of

the materials, has, unimpaired, withstood lateral strains under which a vessel, on almost any other construction, must have broken up.

Such was the state of preparation of engineers' minds for solving the problem of carrying a railway across the Menai straits by girders, when, early in 1845, Mr. Stephenson's "aërial tunnel" was spoken of. On the 5th of May, 1845, he announced his plan before a committee of the House of Commons.

Few inventors can explain the development in their minds of an original conception. Invention in art consists of two distinct intellectual efforts—first, in seizing the ideal conception of the object to be made for a given end; and second, in the contrivance of the suitable arrangement of materials (or of mechanism, in the case of a machine) for that object. The nature of the first conception seems always to depend on the existing state of analogous objects, and, hence, the two parts of the process are generally intimately connected, though not inseparable. In Mr. Stephenson's case, the two processes seem to have been separated. For as early as April, 1845, Mr. Eaton Hodgkinson and Mr. Fairbairn seem to have been consulted as to experiments on the strength of cylindrical tubes of riveted sheets of iron, and as to the necessity of a combination of the girder plan with suspension chains, for his great bridges. We learn from a communication of Mr. Hodgkinson's to the Mechanical Section of the meeting of the British Association, held at Southampton, in 1846, "that a number of experiments were made upon *cylindrical* and *elliptical* tubes, and a few upon rectangular ones;" but, inasmuch as a girder has to resist in its vertical direction much more than in its horizontal, the oblong rectangular form should have immediately suggested itself as the best; and, therefore, these first experiments were works of supererogation.

Mr. Hodgkinson's experiments were, therefore, at once directed to ascertaining what should be the distribution of the metal in hollow rectangular girders, to secure a maximum of strength with a minimum of weight. Mr. Hodgkinson, whose investigations, published in 1840, had proved experimentally that hollow columns have a greater resistance to compression than the same weight of material in a solid column (as the usual theory had indicated, and the practice of Wiebecking and Gauthey thirty years earlier, and of Polonceau, in 1839, had testified), now made further experiments to ascertain the relative resistance of circular and rectangular tubes, with the object of disposing of the malleable iron, of which the girders were to be made in this hollow form, on the upper side, *i. e.*, the part *compressed* by the strain.

As might have been anticipated, the "*buckling*" of the plates on the top had to be prevented by particular contrivances, or by greatly increasing their substance beyond that of the bottom or extended side.

The following are some of the leading results of Mr. Hodgkinson's experiments.

Experiments on two similar tubes.

Length of tube.	Weight of tube.	Distance between supports.	Depth of tube.	Breadth of tube,	Thickness of metal in 16ths of an inch.	Breaking weight in tons.	Ultimate deflexion.
	cwt. qu.		feet		Top Bottom Side		Inches
31′—6″	20—3		2	1′—4″	6 4 2	26,1	2⅝
47—0	61—1	45	3	2′—0	9 6 3	65,5	3¼

This breaking weight in tons is in *excess* of the results deduced from the usual formula, when the value of I (the moment of inertia), is calculated by our formula 5 (page 79), when f^1 is taken = 56000. To ascertain the power of such tubes to resist a lateral strain—as from the action of wind—the smaller of these two tubes, after being well repaired, was laid on its side and broken. The mean of two experiments gave 15,2 tons as breaking weight, which is about 25 per cent. above the result of calculation by our formulas, when the value of f^1 is taken as indicated. Experiments on the strength of sheet iron, however, give the tensile resistance as high as 62000 lbs. per square inch, and if we introduce this as the value of f^1, the experimental results would almost exactly correspond with the received theory.

Mr. Hodgkinson's experiments on the resistance of sheet iron tubes to compression, show (as his experiments on cast iron columns made in 1839, had previously done, and as Euler's theory indicates), that rectangular tubes are weaker than square ones, and both of these much weaker than cylindrical tubes; so much so, indeed, *that the substitution of cylindrical for square or rectangular tubes, would*, according to Mr. Hodgkinson's experiments, *effect a saving of one-fourth of the metal in the top.*

Mr. Fairbairn, at the same meeting of the British Association, September, 1846, made the following communication of "Experiments on the Tubular Bridge, proposed by *Mr. R. Stephenson*, for crossing the Menai straits. These experiments, says Mr. Fairbairn, have put us in possession of facts, which greatly increase our knowledge of the properties of a material, whose powers, when it is properly put together, are but imperfectly understood; for exclusive of the rapidly increasing use of wrought iron in the construction of ship-boilers, &c., its application to bridges of the tubular form is perfectly novel, and originated with Mr. Robert Stephenson. Experiments of the most conclusive character were those made on a model tube on a large scale, containing nearly all the elements of the proposed bridge, and the various conditions with regard to form and construction, which had been developed by the previous inquiries (above alluded to). It occurred to Mr. Fairbairn that the strongest form would be that, wherein the top and bottom consisted of a series of pipes, with riveted plates on their upper and under sides. This form of top, says Mr. Fairbairn, would possess great rigidity, and is well adapted to resist the crushing forces to which it is subjected; and the bottom section appeared equally powerful to resist tension.

Mr. Fairbairn thought that this is the strongest form that could be devised; but practical difficulties present themselves in its construction, as an easy access to the different parts for the purposes of painting, repairs, &c., is absolutely necessary. The scale of the model tube was exactly one-sixth of the length, breadth, depth, and thickness of metal of the bridge intended to cross one span of the straits, 450 feet, (since increased to 462 feet.) In each of the experiments, the weights were laid on at the centre, about one ton at a time, and the deflection was carefully taken as well as the defects of elasticity after the load was removed.

"The rectangular model tube, Fig. 87, was 80 feet long, 4′—6″ deep, 2′—8″ wide, 75 feet between the supports. The thickness of the plate: bottom .156 inch, sides .099 inch, top .147 inch, sectional area of bottom 8,8 inches, weight of the tube 4,86 tons = 10,889 lbs. First experiment, breaking weight 79,578 lbs = 35½ tons. Ultimate deflexion 4,375 inches, permanent set under strain of 67,842 lbs. .792 inch. With the strain of 35½ tons, the *bottom* was torn asunder, directly across the solid plates, at a distance of 2 feet from the centre of the shackle, from which the load was suspended. One of the principal objects of this inquiry was to determine the ratio between the top and bottom of the tube. From the experiments immediately preceding this, it appeared that the ratio of the area of the top to that of the bottom, in a *rectangular tube* (of thin sheet iron), should be as 5 to 3.

Fig. 87.

"The plates forming the top of the model tube were somewhat thicker than intended, and consequently gave (as former experiments indicated) a preponderating resistance to that part. To obviate this disparity, two additional strips, 6½ by $\frac{5}{16}$, weighing about 4 cwt. were riveted along the bottom, extending 20 feet on each side the centre. This raised the area of the bottom to nearly 13 inches, being about the ratio of 5 to 3, or 23,5 to 13. With these proportions, and having repaired the fractured part by introducing new plates, the experiments proceeded as before.

"Second experiment. Breaking weight 97,102 lbs. = 43,3 tons. Ultimate deflexion 4,11 inches. In this experiment the tube failed by one of the ends giving way, which caused the sides to collapse. The weak point in this girder was evidently a want of stiffness in the sides. To remedy this evil and keep them in form, vertical ribs, composed of light angle iron, were riveted along the interior of each side at distances of 2 feet; and, having again restored the injured parts, the tube was a third time subjected to the usual tests.

"Third experiment. Breaking weight 126,138 lbs. = 56,3 tons, ultimate deflexion 5,68 inches. The tube was torn asunder through

the bottom plates. The cellular top gave evident symptoms of yielding to a crushing force by the puckerings of *each side*, which gradually enlarged as the deflection increased. These appearances became more apparent as the joints of the plates on the top side *had sheared off a number of the rivets*, and the holes had slid over each other to an extent of nearly $\frac{3}{10}$ of an inch."

On Mr. Fairbairn's most admirably stated facts, we shall only remark, that a *cellular bottom* would probably be found to be the *weakest* and not the strongest form in which the iron could be distributed there; for there is no tendency to *buckle* in the bottom; and to resist the transverse strain of passing loads (in the actual bridge), the separation of the plates composing the bottom, should only be such as to allow of the introduction of connecting plates or joists to stiffen it, so as to make the *bottom* a roadway. Again, the ratio of the areas of the *top* and *bottom* above deduced, is evidently not an *absolute* quantity, but refers only to the particular *form of cells* adopted in these experiments. Theory and experiment indicate this to be the true view of the case.

These experiments were used in determining the dimensions of the bridges already erected, and now in construction. In reference to the Conway Bridge, the first tube of which was erected in March, 1848, the following particulars are taken from the Civil Engineers' and Architects' Journal, for June, of this year.

Fig. 88.

"Fig. 88 exhibits a transverse section of one of the tubes. Fig. 89 is a side elevation, of 12 feet in length, of the tube, resting on the masonry. The tube consists of sides *a*, *a*, of wrought iron plates, from 4 to 8 feet long, and 2 feet wide, by ½ inch thick in the centre, and ⅝ths of an inch thick towards the end of the tube, riveted together to T-angle-iron ribs, placed on both sides of the joints, and angle-gussets at the feet of the ribs to stiffen them; a ceiling (or top flange), composed of 8 cells or tubes *b*, each 20½ inches wide, and

21 inches high; and a floor containing 6 cells or tubes *c*, 27½ inches wide, and 21 inches high. The whole length of the tube is 412 feet; it is 22 feet 3½ inches high at the ends, and 25 feet 6 inches high in the centre, (including the tubes at top and bottom, running the whole length,) and 14 feet wide to the outside of the side plates. The upper cells are formed of wrought iron plates, ⅝ inch thick in the middle, and ½ inch thick towards the ends of the tube, put together with angle-iron in each angle of the cells; and over the upper joints is riveted a slip of ½ inch iron, 4½ inches wide. The lower cells consist of ⅜ inch iron plates for the divisions, and the top and bottom of two thicknesses of plate, each 12 feet long, 2 feet 4 inches broad, and ½ inch thick in the centre, and ¼ inch thick at the ends, and so arranged as to break the joint; and a covering plate of ½ inch iron, 3 feet long, is placed over every joint on the underside of the tube. The external casing is united to the top and bottom cells by angle-iron, on both the inside and outside of the tube. The ends of the tube, where it rests on the masonry, are strengthened by cast iron frames *d*, to the extent of 8 feet of the lower cells. The tube was constructed on a platform erected on the shore of the river, close to where it was to cross; and, when finished, six pontoons were placed under the tube at low water, and at high water they lifted the tube off the piles upon which the stage was erected. It was then floated to its destination, and placed between the two towers, part of the masonry being left undone until the tube was put into its proper position, and as it was raised by means of hydraulic lifting presses, the masonry was built up under the tube. In order to allow of the free expansion and contraction of the tube, the ends rest on 24 pairs of iron rollers *i*, connected together by a wrought iron frame, and placed between two cast iron plates *j*, *k*, 12 feet long by 6 feet wide, and 4 inches thick. The lower plate is laid on a flooring of 3 inch planks *l*, bedded on the stone-work. Fig. 88, *h*, *h*, are uprights, into which are fitted the cross-lifting girders for attaching the chains of the hydraulic presses."

Fig. 89.

The weight of each tube of the Conway Bridge has been stated to be 1300 tons, but whether this is the weight including the fixtures for the rails, or of the tube *per se*, is not recorded in the papers to which we have had access. The total length being 420 feet, the weight may be stated as not less that 62 cwt. or 3,1 tons per "foot running." It is difficult to conceive anything more admirable than this final result, when we learn that, under the passage of the heaviest goods-trains, there is no sensible motion, by deflection, among the parts of the tube-girders. No method of construction, hitherto adopted for large spans, could have accomplished this absolute security with so small a weight of materials.

In what precedes, we have endeavored to trace the progress of a particular part of the engineer's art, with a view to encourage young engineers to look upon their art as capable of being formed into a science; and we yet venture to add, in reference to what we have said as to the separate intellectual efforts involved in Mr. Stephenson's invention—that the adoption of the results of the abovementioned experiments, the execution of the designs ultimately determined upon, and the erection of the tubes, are details requiring the highest order of skill and practice in the execution of works; but, *the "keeping hold of the original idea,"* until brought to the form of ascertaining, experimentally, the best shape or arrangement of the materials, is certainly the essence of invention—marks out the Engineer-in-chief's work unmistakably—is the element in the grand result that commands the homage paid to engineering genius, by the less gifted of the profession, and secures to itself, envy or jealousy notwithstanding, the due meed of fame and public applause. The co-operation of Mr. Hodgkinson, Mr. Fairbairn, and Mr. Clark, has doubtless been of very great service to Mr. Stephenson, and the Holyhead Railway Company, in working out the details of a design, in proposing which, however, they had no share; but it seems impossible to associate the names of others with that of Stephenson in this work, further than we associate the names of Davies, Gilbert, Rhodes, and Provis, with that of Telford, in connection with the Menai Suspension Bridge. Just as well might "all but the original idea" of the block machinery be claimed by Maudslay, who made it for Brunel. The same of the safety-lamp, by the tinsmith, who made the first for George Stephenson. The same of the hot-blast, by Mr. Wilson or Mr. Condie, who first applied it for Mr. Neilson; and the same of many other cases, in which, from imperative circumstances, the inventor has found himself necessitated to delegate to others the actual execution of his "original idea."

Whilst the experiments on the Tubular Bridges were in progress, letters-patent were granted to Mr. Fairbairn, for "improvements in constructing iron beams;" in which he claims "the novel application and use of plates or sheets of iron, united by means of angle iron and rivets, or by other means, for forming or constructing, by such combination, hollow beams or girders for the erection of bridges or other buildings."

Although the *principle* of Mr. Fairbairn's patent is perfect, the limits of its economical application, is, as far as we can judge, to cases of great span, and where extreme strains are likely to be suddenly brought upon the structure.

The system successfully applied by Wiebecking, for wooden bridges, and in cast iron, by Reichenbach and Gauthey, and, latterly, to a certain extent, by Polonceau, in the beautiful Pont du Carrousel, over the Seine, has recently been revived in malleable iron. On the extension of the London and Blackwall Railway, and elsewhere, Mr. Locke has introduced a hollow sheet iron *arch*, with suspended tie and diagonal bracings, to carry the roadway. We object to this form of bridge, as being a retrogression in the *principle* of construction, and consequently offering no economical advantage, but the contrary.*

[STRENGTH OF CYLINDRICAL STEAM BOILERS, TUBES, AND FIRE-ARMS.

It has been generally supposed that the rolling of *boiler-plate* iron, gives to the sheets a greater tenacity in the direction of the length, than in that of the breadth. Supposing this to be correct, it has frequently been asked how the sheets ought to be disposed in a cylindrical boiler of the common form, in order to oppose the greatest strength to the greatest strain. It has also been asked whether the same arrangement will be required for all diameters, or whether a magnitude will not be eventually attained, which may require the direction of the sheets to be reversed?

To determine these questions in a general manner recourse must be had to mathematical formulas, assuming such symbols for each of the elements as may apply to any given case of which the separate data are determined either by experiment or by the conditions of the case. The *principles* of the calculation require our first notice.

1. To know the force which tends to burst a cylindrical vessel in the longitudinal direction, or, in other words, to separate the *head* from the curved *sides*, we have only to consider the actual area of the head, and to multiply the number of units of *surface* by the number of units of *force* applied to each superficial unit. This will give the total *divellent* force in that direction.

To counteract this, we have, or may be conceived to have, the tenacity of as many longitudinal bars as there are lineal units in the circumference of the cylinder. The united strength of these bars constitutes the total retaining or *quiescent* force; and, at the moment when rupture is about to take place, the *divellent* and the *quiescent* forces must obviously be equal.

2. To ascertain the amount of force which tends to rupture the

* Of all the designs for iron bridges hitherto planned and executed—when we consider the situation, the extent, the elevation above the water, and the scenery by which it is surrounded—the most imposing is that of the wire bridge over Niagara river, a short distance below the Falls. This bridge is still in progress. It was planned and executed by Mr. Charles Ellet, Jr.—Am. Ed.

cylinder along the curved side, or rather along two opposite sides, we may regard the pressure as applied through the whole breadth of the cylinder upon each lineal unit of the diameter. Hence the total amount of force which would tend to divide the cylinder in halves by separating it along two lines, on opposite sides, would be represented by multiplying the diameter by the force exerted on each unit of surface, and this product by the length of the cylinder. But even without regarding the length, we may consider the force requisite to rupture a *single band* in the direction now supposed, and of one lineal unit in breadth; since it obviously makes no difference whether the cylinder be long or short in respect to the ease or difficulty of separating the sides. The *divellent* force in this direction is, therefore, truly represented by the diameter multiplied by the pressure per *unit of surface.* The retaining, or *quiescent* force, in the same direction, is only the strength or tenacity of the two opposite sides of the supposed band. Here, also, at the moment when a rupture is about to occur, the *divellent* must exactly equal the *quiescent* force.

3. In order to estimate the augmentation of *divellent* force consequent upon an increase of diameter, we have only to consider, that, as the diameter is increased, the product *of the diameter, and the force per unit of surface,* is increased in the same ratio. But unless the thickness of the metal be increased, the *quiescent* force must remain unaltered. The *quiescent* forces, therefore, continue the same—the *divellent* increase with the diameter.

4. Again, as the diameter of the cylinder is increased, the area of its end is increased in the ratio of the *square* of the diameter. The *divellent* force is, therefore, augmented in this ratio. But the retaining force does not, as in the other direction, remain the same, since the *circumference* of a circle increases in the same ratio as the diameter. The *quiescent* force will, consequently, be augmented in the simple ratio of the diameter, without any additional thickness of metal. So that, on the whole, the total tendency to rupture in this direction will increase only in the *simple* ratio of the diameter.

5. Since we have seen that the tendency to rupture, in both directions, increases in the simple direct ratio of the increase of diameter, it is obvious that any position of the sheets which is right for one diameter, must be right for all. Hence there can never be a condition, in regard to mere magnitude, which will require the sheets to be reversed.

6. The foregoing considerations being once admitted, we may proceed to ascertain what is the true direction of the greatest tenacity in the sheet, if any difference exist, and what that difference might amount to, consistently with equal safety of the boiler in both directions.

7. Let $x =$ the diameter of the cylinder.

$f =$ the force or pressure per unit of surface (pounds per square inch, for example).

$T =$ the tenacity of metal which, with the diameter x, and the

force f, will be required in the lineal unit of the circumference, in order to hold on the head.

Then will the whole *quiescent* force be $3.1416\,x\,T$, while the *divellant* will be $.7854\,x^2f$; consequently, $.7854\,x^2f = 3.1416x$ as above stated.

Dividing by $.7854\,x$, we have $xf = 4T$; and we derive immediately—

$$x = \frac{4T}{f},$$

$$f = \frac{4T}{x},$$

$$\text{and } T = \frac{xf}{4}.$$

That is, the tenacity of the *longitudinal bar of the assumed unit in width, will be one-fourth of the product of the diameter into the pressure,* measuring the tenacity by the same standard as the pressure, whether in pounds or kilograms.

8. Now assuming the tenacity required in the *circular band* of the same width to be t, we shall, agreeably to what has already been said, have the *divellant* force expressed by xf, and the *quiescent* by $2t$, so that $xf = 2t$ and $t = \frac{xf}{2}$. Also $f = \frac{2t}{x}$; and $x = \frac{2t}{f}$.

Having thus obtained two expressions for each of the quantities x and f, we may, by comparing them, readily discover the relative values of T and t,—thus:

$$\left.\begin{array}{l} x = \dfrac{4T}{f} \\ x = \dfrac{2t}{f} \end{array}\right\} \text{hence } \frac{4T}{f} = \frac{2t}{f} \text{ whence } 4T = 2t, \text{ or } t = 2T. \text{ From which}$$

it follows, that *under a known diameter, and with a given force or pressure, the tenacity of metal in a cylindrical boiler of uniform thickness, ought to be twice as great in the direction of the curve as in that of the length of the cylinder, and that if this could be the case the boiler would still have equal safety in both directions.*

In whatever direction, therefore, the rolling of metal gives the greatest tenacity, in the same direction must the sheet always be bent in forming the convexity of the cylinder. It follows that if we suppose the tenacity precisely equal in both directions, the liability to rupture by a mere internal pressure *ought to be twice as great along the longitudinal direction as at the juncture of the head.* This supposes the strain regular, and the riveting not to weaken the sheet.

9. To know how large we may safely make a cylindrical boiler, having the absolute tenacity of the metal, in the *strongest direction,* and with a known thickness, we have only to revert to the formula $x = \frac{2t}{f}$. That is, *the diameter will be found by dividing twice the*

tenacity by the greatest force per unit of surface, which the boiler is ever to sustain.

10. When knowing the absolute tenacity of a metal, or other material reckoned in weight, to the bar of a given area in its cross section, we would determine the *thickness* of that metal which ought to be employed in a boiler of given diameter, and to sustain a certain force, we may use the formula $t = \frac{xf}{2}$, and dividing the latter number of this equation by the *strength* of the square bar, which we may call s, we obtain the thickness demanded in the direction of the curve, which we may denominate p; so that $p = \frac{xf}{2s}$; this will give the thickness of the boiler plate either in whole numbers or decimals. Thus, suppose the diameter of a cylindrical boiler is to be thirty-six inches—that it is to be formed of iron which will bear 55,000 lbs. to the square inch, and is to sustain 750 lbs. to the square inch—what ought to be the thickness of the metal?

$$\text{Here } x = 36$$
$$f = 750$$
$$2s = 110000, \text{ consequently,}$$

$p = \frac{36 \times 750}{110000} = .2454$, or a little less than one-quarter of an inch.

It must, however, be evident that the *minimum* tenacity of any particular description of metal, is that on which all the calculations ought to be made when there is any probability that the actual pressure will, in practice, ever reach the limit assigned as the value of f in the calculation.

If we had plates of different metals, or of different known degrees of tenacity in the same kind of metal, and were desirous of ascertaining how strong a kind we must employ under a limited *thickness*, *diameter*, and *pressure*, we should decide the point by transforming the formula $p = \frac{xf}{2s}$, into $ps = \frac{xf}{2}$, and then into $s = \frac{xf}{2p}$. In other terms, in order to know the strength of the metal required, or the direct strain which an inch square bar of the same ought to be capable of sustaining, we must *multiply the diameter of the boiler in inches by the pressure per square inch in pounds, and divide the product by twice the intended thickness in parts of an inch.*

Thus, how strong a metal ought to be employed to sustain a pressure of 1000 lbs. to the square inch, in a boiler thirty inches in diameter, and one-fourth of an inch thick?

Here $s = \frac{30 \times 1000}{2 \times .25} = 60.000$. Hence we see that the metal must be capable of sustaining *sixty thousand pounds* to the inch bar, or in that proportion for any other size. This formula enables us to determine whether among the metals of known tenacity, *any* one can be found to fulfil the conditions under the thickness assigned.]

DIVISION II.

APPLICATION OF MECHANICS TO MACHINERY.

INTRODUCTION.

§ 38. *Machines.*—Machines are artificial arrangements, by which forces are applied to produce mechanical effect. *Tools* or *instruments* differ from machines chiefly in their being applied *immediately* to the work to be done, whilst machines are *intermediate.*

In every machine we have to distinguish between the *power* and the *resistance*. *Power* is the cause of the motion of the machine, and *resistance* is that which opposes the motion, and which it is the object of the machine to overcome. The *powers* applied to machines are modifications of those supplied by nature in the expansive force of heat, the action of gravity, the physical force of men and animals, &c. (Vol. I. § 60). The *resistances* to be overcome are the *transport*, and *the change of form* and *texture of materials.*

There are in every machine three principal parts. One which *receives* the power, a second *transmitting*, *communicating*, or *modifying* the power, and a third *applying* it. In the common flour mill, considered as a machine, a water *wheel* receives the power of a water *fall;* the spur wheel and pinion, or a *train of gear*, communicates the motion of the water wheel to a pair of stones revolving in a different *plane*, and at quite *different speed* it may be, from that of the water wheel, and these stones grind the corn, or *do the work* desired.

Remark. This sub-division is not always manifest; for there are machines, in which the power is transmitted so directly to the work to be done, that the communicators above mentioned are not apparent. The sub-division is, however, convenient, though it would, perhaps, be equally so to apply to recipients of power, the generic term *engine* or machine; to the communicators of the motion, the general term *mechanism;* and to the parts doing the work, the general term of *operators*, and in this manner to consider each separately, as they are, in fact, perfectly distinct. On this subject, there are excellent observations in Willis' "*Principles of Mechanism*, 1840," and in Ampère's "*Philosophie des Sciences.*"—TR.

§ 39. *Mechanical Effect.*—The mechanical effect produced by a machine, is measured by the *work* done in a given time, or by the

product of the force exerted, and the distance gone through in a unit of time in the direction of that force. If P be the force exerted, and s the distance passed through in a second, then is Ps a true measure of the effect of the machine $L = Ps$ ft. lbs.

It is very usual to assume a somewhat arbitrarily chosen, but now pretty generally adopted measure, termed *horse power*, as the unit of mechanical effect of engines or machines. The *horse power* is in England 33,000 lbs. avoird. raised 1 foot high in a minute. This is the *cheval vapeur* of the French, and which in French measures is 75 *kilogrammes* raised 1 *metre* high in a *second*. It is the *Pferdekraft* of the Germans, or 510 lbs. Prussian, raised 1 foot high in a second.

We have to distinguish the *useful effect*, the lost effect, and the total effect of machines. The useful effect is the work done, the lost effect is that consumed in overcoming the friction of the parts of the machine lost in shocks, &c., and the total effect is the sum of these—the effect inherent in the *power*, or the effect taken out of it. An engine or machine is so much the more perfect, the smaller the lost effect compared with the total effect, or the less loss there arises in adapting and transmitting the power. The ratio of the useful effect, produced to the total effect, has been termed the *efficiency* of the machine. If $L =$ the total effect $L_1 =$ the useful effect, and $L_2 =$ the lost effect, the efficiency $\eta = \frac{L_1}{L} = \frac{L - L_2}{L}$. Thus, the more perfect the machine, the more nearly its efficiency approaches to unity; but as there is always friction, and other resistances and losses, that degree of perfection cannot be attained.

Example. An ore stamping mill consists of 20 *stampers*, each of which weighs 250 lbs., and each is raised 40 times per minute, 1 foot high. The machine driving these is a water wheel, taking on 260 cubic feet per minute, and the *fall* is 20 feet high—required the efficiency of this machine. The useful effect is:

$20 \cdot \frac{250 \cdot 40 \cdot 1}{60} = 3333\frac{1}{3}$ ft. pounds per second $= 6$ horse power; the total effect, however, is: $\frac{260 \times 62{,}25}{60}$ 270 pounds water through 20 feet per second $= 5400$ feet lbs. per second $= 9{,}8$ horse power; the *lost effect* $= 5400 - 3333\frac{1}{3} = 2066\frac{2}{3}$ feet lbs. $= 3{,}75$ horse power; and the efficiency of the whole arrangement $= \frac{3333\frac{1}{3}}{5400} = 0{,}62$.

§ 40. *Useful and prejudicial Resistance.*—The resistance to be overcome by machines may be subdivided, in like manner, into useful and prejudicial resistance, but as the power is applied to the useful and prejudicial resistances at different points, we cannot directly set the power equal to the sum of the useful and prejudicial resistance, but there must be a preliminary *reduction*. This reduction is made by means of the spaces simultaneously passed through by the different points of resistance of the machine. If the power P be exerted for a space s, and the useful resistance P_1 for a space s_1, and the prejudicial resistance P_2 for a space s_2, we have

$$Ps = P_1 s_1 + P_2 s_2, \text{ hence } P = \frac{s_1}{s} P_1 + \frac{s_2}{s} P_2.$$

The point in the machine or system at which P is applied, is termed the point of application of the power, and the points at which P_1 and P_2 act, are the points of application of the resistances; we have in $\frac{s_1}{s} P_1$ the useful resistance reduced to the point of application of power, and in $\frac{s_2}{s} P_2$, the prejudicial resistance *reduced* to the same point. *The power is, therefore, equal to the sum of the useful and prejudicial resistances, reduced to the point of application of the power.* Again $P_1 = \frac{s}{s_1} P - \frac{s_2}{s_1} P_1$, or the useful resistance is equal to the difference of the power reduced to the point of application of that resistance, and the prejudicial resistance reduced to the same point. Hence the efficiency of a machine: $\mu = \frac{P_1 s_1}{P s} = \frac{s_1}{s} P_1 : P = P_1 : \frac{s}{s_1} P$, that is, the quotient of the useful resistance reduced to the *power-point* and the power, or the quotient of the useful resistance, and the power reduced to the point of application of the useful resistance.

Very many machines are adaptations of the wheel and axle (Vol. I. § 152), and hence the *reductions* may often be accomplished as for a lever. If in the wheel and axle ABC, Fig. 90, the radius of the wheel $CA = a$, the drum's radius $CB = b$, then the statical moment of the power P, $= Pa$, and that of the useful resistance $P_1 = P_1 b$, and therefore the useful resistance reduced to the power-point $A = \frac{b}{a} P_1$, and the power reduced to the point of application b of the resistance $= \frac{a}{b} P$. If the prejudicial resistance P_2, consist in the axle friction $f(P + P_1 + G)$, and if $r =$ the radius DC of the axle, the moment of it is $= P_2 r$, and therefore the prejudicial resistance reduced to the application of power $= \frac{P_2 r}{a} = \frac{fr}{a}(P + P_1 + G)$, the prejudicial resistance reduced to the point of application of the resistance $= \frac{P_2 r}{b} = \frac{fr}{b}(P + P_1 + G)$.

Fig. 90.

Hence $P = \frac{b}{a} P_1 = \frac{fr}{a}(P + P_1 + G)$, also $P_1 = \frac{a}{b} P - \frac{fr}{b}(P + P_1 + G)$, lastly, $\eta = \frac{b}{a} P_1 : P = P_1 : \frac{a}{b} P = \frac{P_1 b}{Pa}$.

Example. For a *wheel* and *drum* weighing 250 lbs, the wheel being 30 inches radius,

and the drum 6 inches radius, the axle $\frac{1}{2}$ inch radius—the useful resistance being 500 lbs., the co-efficient of axle friction $\frac{1}{10}$, then the useful resistance reduced to the point of application of the power $= \frac{b}{a} P_1 = \frac{6}{30} 500 = 100$ lbs., and the prejudicial resistance reduced to the same point:

$$= \frac{fr}{a}(P + P_1 + G) = \frac{1}{10} \cdot \frac{1}{2 \cdot 30}(750 + P) = \frac{5}{4} + \frac{P}{600},$$

and hence we have to put the power:

$$P = 100 + \frac{5}{4} + \frac{P}{600}, \text{ i. e. } P = 101{,}25 \cdot \frac{600}{599} = 101{,}42 \text{ lbs.},$$

and the *efficiency* of the machine: $\eta = \frac{100}{101{,}42} = 0{,}986$.

ON THE RIGIDITY OF CORDAGE.

Amontons, and, after him, Coulomb, experimented on the rigidity of hemp ropes and cords: and Weisbach, adopting Coulomb's method, has recently experimented on the rigidity of hemp and wire rope, such as are used in the drawing-shafts of mines.

Coulomb deduced from his experiments, that the law of this *resistance to winding* may be represented by a formula composed of two terms; the one, *a constant for each* drum or pulley, which we may designate by a, and which the distinguished experimenter termed "*natural rigidity*," because it depends on the mode of manufacture of the rope, and on the degree of twist given to the threads and strands; the other, proportional to the tension T on the rope, and expressed by the product βT, in which β is a constant for any rope or drum. Thus the resistance to winding, $R = a + \beta T$.

Coulomb also deduced from his experiments that the resistance to winding *varies inversely as the diameter* d *of the drum or pulley;* so that $R = a + \frac{\beta T}{d}$.

Naviet, in using Coulomb's experiments to construct a formula, assumed that the co-efficients, a and β, are proportional to a certain power of the diameter, depending on the state of *wear* of the rope; but this assumption is not true. For it would lead to this, that a *worn* rope of 1 foot diameter has the same rigidity as a new one, which is evidently not true: and besides, the comparison of the values of a and β prove that the power to which the diameter has to be raised cannot be the same for the two terms of the resistance.

Coulomb's experiments, however, show that the *rigidity is proportional to the number* n *of threads in the rope*, for ropes of a given manufacture.

For new white ropes, the formula:

$$R = n[.0002 + .000171\, n + .000243\, Q] \text{ lbs.}$$

for drums or pulleys of 1 foot in diameter, and

$$R = \frac{n}{d}[.0002 + .000171\, n + .000243\, Q] \text{ lbs.}$$

for a drum of diameter d in feet, accords well with experiments.*

* For the complete discussion of this subject, see Morin, "Leçons de Mécanique pratique," lère partie.

For tarred ropes: $R = \frac{n}{d}[.001 + .000232\, n + .00028\, Q]$ lbs.

Whence it appears that *tarred ropes are rather more rigid* than white ropes.

Weisbach has deduced from his experiments on wire rope, (4 wires round a core in each strand, and 4 strands round a core in the rope,) weighing 3 lbs. to the fathom, the formula:

$$R = 0{,}72 + 0{,}0262 \frac{Q}{d} \text{ lbs.},$$

in which Q is the strain on the rope in cwts., and d the diameter of the pulley. Whereas, for the hemp ropes, *fit for the same uses*, or of the same strength, $R = 3{,}02 + 0{,}086 \frac{Q}{d}$: or the rigidity is considerably *greater*.

Wire ropes, newly tarred or greased, have about 40 per cent. less rigidity than untarred ropes.*

§ 41. *Working Condition.*—When a machine is set in motion, it soon comes to its *working condition*, that is, there recur at regular periods the same relative position of the parts, the periodic motion becomes uniformly so. In this condition we assume machines to be in applying our principles, but their working condition may, according to circumstances, be either uniform or variable. The causes inducing irregularity are variations in the power or in the resistance, as also the proportions of, or construction of the machine, in reference to variations in the spaces described in a given time by the power and resistance, and the state of motion of inert masses.

In a steam-engine, the power is variable when the engine "works expansively," that is, when the steam is cut off during the progressive motion of the piston. In a mill for rolling iron, the power and resistance are continually varying, because the forge hammer is *out of gear* when *falling* on the blooms, and, therefore, the working condition of the machines is *irregular*. If the engine work expansively, then there would arise from the combination of the engine, and hammer, and rollers, three causes of irregularity. When a weight G, Fig. 91, is raised by a steam-engine with uniform pressure by means of a wheel CA_0, and crank CB_0, the machine has a variable working condition, because equal spaces A_0A_1, A_1A_2, A_2A_3, A_3A_4, of the resistance correspond to very unequal distances described by the power, and, therefore, the ratio during a half revolution is variable, but for *periods* of a half revolution it is uniform.

Fig. 91.

* Weisbach's paper on this subject is contained in the first number of a journal published at Freyberg, under the title "Der Ingenieur Zeitschrift für das gesammte Ingenieurwesen," 1846.

In the case of uniform working condition, the inert masses on a machine are without influence, because it is only at first, when the machine is still accelerating in motion, that they absorb mechanical effect, but later, when uniform motion has established itself, there is neither loss nor gain of mechanical effect (Vol. I. § 52). But if, on the other hand, a machine be subject to irregular working conditions, the inert masses of the parts have an essential influence on the motion of the machine, because they absorb mechanical effect at every acceleration of speed, and this they again give off at each retardation. If M be the sum of all the masses reduced to the power or resistance-point of a machine, v_1 and v_2, the minimum and maximum velocities of the power, or resistance-points, we have the mechanical effect which the inert masses absorb during their transition from the velocity v_1 to v_2, and which they again give out in passing from v_2 to $v_1 = \left(\frac{v_2^2 - v_1^2}{2}\right) M$. Thus, in each period, the inertia of the masses increases and diminishes the lost effect by the above amount, and, therefore, the total effect for the whole period, or the mean effect is the same as if these inert masses were not there. Hence, as a general formula, $Ps = P_1 s_1 + P_2 s_2$ holds good for a variable working condition, if by s, s_1, s_2, we understand the spaces described in a complete period, or if for P, P_1, P_2, we substitute the mean values of the power, and useful and prejudicial resistance, for a given period. For the case of accelerating motion: $Ps = P_1 s_1 + P_2 s_2 + \left(\frac{v_2^2 - v_1^2}{2}\right) M$, hence $v_2 - v_1 \frac{Ps - (P_1 s_1 + P_2 s_2)}{\left(\frac{v_2 + v_1}{2}\right) M}$. This formula shows that the variations of velocity of a machine are not only less, the less the difference between the effects of the power and the sum of the effects of the resistances, but also the greater the masses of the parts of the machine, and the greater their velocity.

Remark. It does not follow that because the mass of the parts do not affect the efficiency of a machine, but only its working condition, that it is a matter of indifference, whether the parts of a machine have more or less mass. Weight increases friction, gives rise to shocks, &c., which are prejudicial. But of this in the sequel.

SECTION II.

OF MOVING POWERS AND THEIR RECIPIENT MACHINES.

CHAPTER I.

OF THE MEASURE OF POWERS AND THEIR EFFECTS.

§ 42. *Dynamometer.*—In order to determine the mechanical effect produced by powers and machines, in terms of the *horse power* unit, three elements are necessary, viz: The *magnitude of the power or effort*, the *distance passed through by it*, and the *time during which the power has acted.*

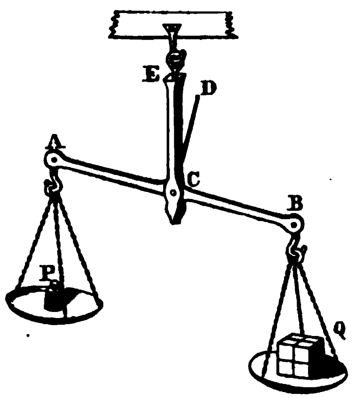
To enable us to represent the effect of forces, we must, therefore, have measures of the force applied, the distance, and the time. *Dynamometers* serve to measure the force applied, the *chain* is generally used for measuring space, and *clocks* or *watches* measure time. If P be the magnitude of the force indicated by the dynamometer, and s the distance throughout which it has acted during the time t, then, the *work* or *mechanical effect* produced in this time is $= Ps$, and the work per second $L = \frac{Ps}{t}$.

Of dynamometers, there are various forms. The common balance is a dynamometer, and is used to measure the force of gravity or *weight*. Modifications of spring balances, and the friction brake are the dynamometers applied to measure forces producing mechanical effect. The friction brake is applied to measuring the mechanical effect given off by revolving axles.

Balances are simple or compound levers, on which the force or weight to be measured is set in equilibrium with standard weights. Balances are either equal or unequal-armed levers, and the latter are variously combined, according to the purposes to which they are applied.

§ 43. *Common Balance.*—The common balance is a lever with equal arms, Fig. 92, on which the weight Q to be measured is equilibriated by an equal weight P. AB is the beam with its points of suspension, (Fr. *fléau*, Ger. *Waagebalken*,) CD the index or

Fig. 92.

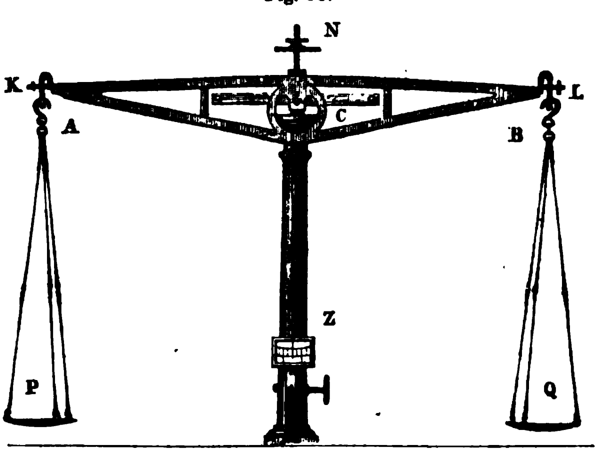


point, (Fr. *aiguille*, Ger. *Zunge*,) CE the support or fork, (Fr. *support*, Ger. *Scheere*,) C is the knife-edge or fulcrum, a three-sided prism of hard steel.

The requisites of a balance are: 1. That it shall take a horizontal position when the weights in the two scales are equal, and only then. 2. The balance must have *sensibility* and *stability*, that is, it must *play* with a very slight difference of weight in either of the scales, and must readily recover its horizontal position, when the weights are again made equal.

That a balance with equal weights in the two scales may be in adjustment, the arms must be perfectly equal. If a be the length of the one, and b be that of the other arm; P the weight in the one scale, and Q that in the other. Then when the beam is horizontal $Pa = Qb$. If, however, we transpose the weights P and Q, we have again $Pb = Qa$, if the beam retain its horizontal position. From the two equations we have $P^2 ab = Q^2 ab$, therefore, $P = Q$, and likewise $a = b$. When, therefore, on transposing the weights, the equilibrium is not disturbed, it is a test of the truth of the balance. A balance may also be tested in the following manner. If we put one after the other two weights P and P into equilibrium with a third Q in the opposite scale, the two weights P and P are equal to each other though not necessarily equal to Q. If, then, we lay the two equal weights in the opposite scales, removing Q, we should have in case of equilibrium $Pa = Pb$, and hence $a = b$. Thus the hori-

Fig. 93.



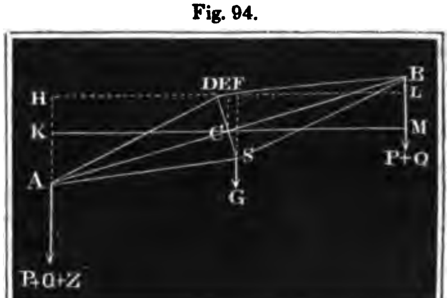
zontality of the balance when two equal weights are laid upon it, is a direct proof of its *truth* or *justness*. Small inaccuracies may be adjusted by means of the screws K, L, as shown on the balance (Fig. 93), which serves to *press out* or *pull in* the *points of suspension.*

If a balance indicates weights P and Q for the same body, according as it has been weighed in the one scale or the other, we have for the true weight X of that body: $Xa = Pb$ and $Xb = Qa$, hence $X^2 \cdot ab = PQ \cdot ab$, or, $X^2 = PQ$, and $X = \sqrt{PQ}$, or the *geometric mean between the two values is the true weight of the body.*

We may also put $X = \sqrt{P(P + Q - P)} = P\sqrt{1 + \frac{Q-P}{P}}$, and approximately $= P\left(1 + \frac{Q-P}{2P}\right) = \frac{P+Q}{2}$, when, as is usual, the difference $Q - P$ is small; we may, therefore, take the *arithmetical mean of the two weighings as the true weight.*

§ 44. *Sensibility of Balances.*—That the balance may move as freely as possible, and particularly that it may not be retarded by friction at the fulcrum, this is formed into a three-sided prism or *knife-edge* of steel, and it rests on hardened steel plates, or on agate, or other stone. In order, further, that the direction of the resultant of the loaded or empty scale may pass through the point of suspension uninfluenced by friction, in order, in short, that the leverage of the scale may remain constant, it is necessary to hang the scales by knife-edges. In whatever manner such a balance is loaded, we may always assume that the weights act at the points of suspension, and that the points of application of the resultant of these two forces is in the line joining the points of suspension. As, according to Vol. I. § 122, a suspended body is only in equilibrium when its centre of gravity is under the point of suspension, it is evident that the fulcrum D of the balance, Fig. 94, should be above the centre of gravity S of the empty beam, and also not below the line AB drawn through the points of suspension. In what follows, we shall assume that the fulcrum D is above AB and above S.

Fig. 94.



The deviation of a balance from horizontality is the measure of its sensibility, and we have to investigate the dependence of this on the difference of weight in the scales. If, for this, we put the length of the arms CA and $CB = l$, the distance CD of the fulcrum from the line passing through the points of suspension $= a$, the distance SD of the centre of gravity from the fulcrum $= s$, if further we put the

angle of deviation from the horizontal $= \phi$, the weight of the empty beam $= G$, the weight on the one side $= P$, and that on the other $= P + Z$, or the difference $= Z$, and, lastly, the weight of each scale, and its apurtenances $= Q$, we have the statical moment on the one side of the balance: $(P + Q + Z) \,.\, DH = (P + Q + Z)(CK - DE) = (P + Q + Z)(l \cos.\phi - a \sin.\phi)$ and on the other side: $(P + Q) \,.\, DL + G \,.\, DF = (P + Q)(CM + DE) + G \,.\, DF = (P + Q)(l \cos.\phi + a \sin.\phi) + G\, s \sin.\phi$; therefore, equilibrium: $(P + Q + Z)\, l \cos.\phi - a \sin.\phi) = (P + Q)(l \cos.\phi + a \sin.\phi) + G\, s \sin.\phi$, or, if we introduce $tang.\phi$, and transform:
$[[2(P + Q) + Z]\, a + G\, s]\, tang.\phi = Z\, l$, therefore,

$$tang.\phi = \frac{Z\, l}{[2(P + Q) + Z]\, a + G\, s}.$$

This expression informs us that the deviation, and, therefore, the *sensibility* of the balance, increases with the length of the beam, and decreases as the distances a and s increase. Again, a heavy balance is, *cæteris paribus*, less sensible than a light one, and the sensibility decreases continually, the greater the weights put upon the scales. In order to increase the sensibility of a balance, the line AB joining the points of suspension and the centre of gravity of the balance, must be brought nearer to each other.

If a and s were equal to 0, or if the points D and S were in the line AB, we should have $tang.\phi = \frac{Zl}{0} = \infty$, therefore $\phi = 90^\circ$; and therefore the slightest difference of weights would make the beam *kick* or deflect 90°. In this case for $Z = 0$, we should have: $tang.\phi = \frac{0}{0}$, *i. e.* the beam would be at rest in any position, if the weights were equal in each scale, and the balance would therefore be useless. If we make only $a = 0$, or put the fulcrum in the line AB, then $tang.\phi = \frac{Zl}{Gs}$, or the sensibility is independent of the amount weighed by the balance. By means of a counterweight N with a screw adjustment, Fig. 98, the sensibility may be regulated.

§ 45. *Stability and Motion of Balances.*—The stability, or statical moment, with which a balance with equal weights returns to the position of equilibrium, when it has inclined by an angle ϕ, is determined by the formula:

$$S = 2(P + Q) \,.\, DE + G \,.\, DF = [2(P + Q)\, a + G\, s] \sin.\phi.$$

Hence, the measure of stability increases with the weights P, Q, and G, and with the distances a and s, but is *independent of the length of the beam.*

A balance vibrating may be compared with a pendulum, and the time of its vibrations may be determined by the theory of the pendulum. $2(P + Q)\, a$ is the statical moment, and $2(P + Q) \,.\, AD^2 = 2(P + Q)(l^2 + a^2)$ is the moment of inertia of the loaded scales, and $G\, s$ is the statical moment of the empty beam. If we put the moment of inertia of the beam $= G y^2$, we have for the length of

the pendulum which would be isochronous with the balance (Vol. I. § 250):

$$r = \frac{2(P+Q)(l^2+a^2)+Gy^2}{2(P+Q)a+Gs},$$

and hence the time of vibration of the balance:

$$t = \pi\sqrt{\frac{2(P+Q)(l^2+a^2)+Gy^2}{g[2(P+Q)]a+Gs}};$$

for which, when a is very small or 0, we may put:

$$t = \pi\sqrt{\frac{2(P+Q)l^2+Gy^2}{gGs}}.$$

It is evident from this that the time of a vibration increases as P, Q, l increase, and as a and s diminish. Therefore, with equal weights, a balance vibrates the more slowly, the more sensible it is, and therefore weighing by a sensible balance, is a slower process than with a less sensible one. On this account it is useful to furnish sensible balances with divided scales (as Z, Fig. 93). In order to judge of the indication of such a scale, let us put Z the additional weight = 0 in the denominator of the formula:

$$tang.\ \phi = \frac{Zl}{[2(P+Q)+Z]a+Gs},$$

and write ϕ instead of $tang.\ \phi$, we then get:

$$\phi = \frac{Zl}{2(P+Q)a+Gs}.$$

If we then put Z for Z_1, and ϕ for ϕ_1, we get:

$\phi_1 = \dfrac{Z_1 l}{2(P+Q)a+Gs}$, hence $\phi : \phi_1 = Z : Z_1$; or, for small differences of weight, the angle of deviation is proportional to that difference. Hence, again $\phi : \phi_1 - \phi = Z : Z_1 - Z$; and therefore $Z = \dfrac{\phi}{\phi_1 - \phi}(Z_1 - Z)$. We can, therefore, find the difference of weights corresponding to a deviation ϕ, by trying by how much the deviation is increased, when the difference of weights is increased by a given small quantity, and then multiplying this increase $(Z_1 - Z)$ by the ratio of the first deviation to the greater deviation obtained.

Remark. Balances such as we have been considering, are used of all dimensions, and of all degrees of delicacy and perfection. Fig. 92 is the usual form of this balance used in trade, and Fig. 93 represents the balances used in assaying, analysis, and in physical researches. Such balances as Fig. 93, are adapted to weigh not more than 1 lb.; but they will turn with $\frac{1}{50}$ of a grain, or with $\frac{1}{350000}$ of a pound. The finest balances that have been made, render $\frac{1}{1{,}000{,}000}$ part of the weight appreciable, but such balances are only for extremely delicate work. Even large balances may be constructed with a very high degree of sensibility. For minute details on this subject, the student is referred to Lardner's and Kater's Mechanics, in "Lardner's Cyclopædia."

§ 46. *Unequal-armed Balances.*—The balance with unequal arms, termed *statira*, or Roman balance and *steelyard*, presents itself in

three different forms, viz: steelyard with movable weight, steelyard with proportional weights, and steelyard with fixed weight. The steelyard with running weight is a lever with unequal arms AB, Fig. 95, on the shorter arm of which CA, a scale is suspended, and on the longer divided arm of which there is a running weight, which can be brought into equilibrium with the body to be weighed Q. If l_0 be the leverage CO of the running weight G, when it balances the empty scale, we have for the statical moment with which the empty scale acts $X_0 = Gl_0$, but if $l_n =$ the leverage CG, with which the running weight balances a certain weight Q, we have for its statical moment: $X_n = Gl_n$; and hence, by subtraction, the moment of the weight Q, $= X_n - X_0 = G(l_n - l_0) = G \,.\, \overline{OG}$. If again $a = CA$, the leverage of the weight, and if b be the distance OG of the running weight from the point O, at which it balances the empty scale, we have $Qa = Gb$, and therefore the weight $Q = \frac{G}{a} b$. Hence the weight Q of the body to be weighed is proportional to b the distance of the running weight from the point O. $2\,b$ corresponds to $2\,Q$, $3\,b$ to $3\,Q$, &c. And, therefore, the scale OB is to be divided into equal parts, starting from O. The unit of division is obtained by trying what weight Q_n must be put on the scale to balance G, placed at the end B. Then Q_n is the number of division, and therefore $\frac{OB}{Q_n}$ the scale or unit of division of OB. If, for example, the running weight is at B, when $Q = 100$ lbs., then OB must be divided into 100 equal parts, and, therefore, the unit of the scale $= \frac{OB}{100}$. If for another weight Q, the weight G has to be placed at a distance $b = 80$, to adjust the balance, then $Q = 80$ lbs.; and so on.

Fig. 95.

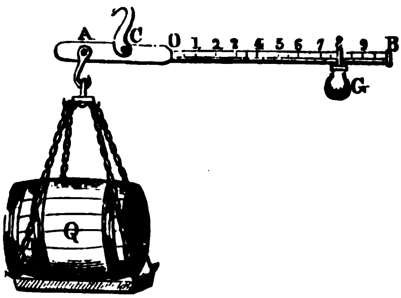
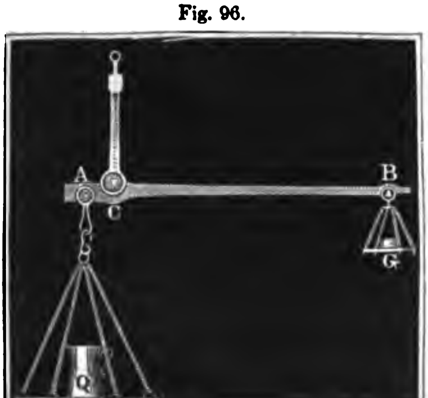


Fig. 96.

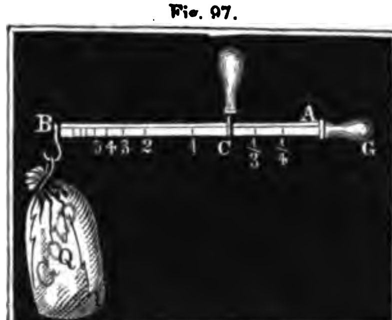


In the steelyard, Fig. 96, with proportional weights, the body to be weighed hangs on the shorter, and the standard weights are put on the longer arm. The ratio $\frac{CB}{CA}$

$=\frac{b}{a}$ of the arms is generally simple, as $\frac{10}{1}$, in which case the balance becomes a *decimal balance.* If the balance has been brought to adjustment or horizontality by a standard weight, then for the weight Q of the body in the scale, we have $Qa = Gb$; hence $Q = \frac{b}{a} G$, and therefore the weight of the merchandise is found by multiplying the small weight G by a constant number, for instance, 10 in the decimal balance, or if the latter be assumed $\frac{b}{a}$ times as heavy as it really is.

The steelyard with *fixed weight*, Fig. 97, called the Danish balance, has a movable fulcrum C, (or it is movable on its fulcrum,) which can be placed at any point in the length of the lever, so as to balance the weights Q hung on one end, by the constant weight G, fixed at the other.

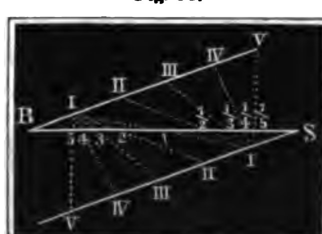
Fig. 97.



The divisions in this case are unequal, as will appear in the following remark:—

Remark. In order to divide the Danish steelyard, Fig. 98, draw through its centre of gravity S and its point for suspension B two parallel lines, and set off on these from S and B equal divisions, and draw from the first point of division of the one, lines to the points of division I, II, III, IV of the other parallel straight line. These lines cut the axis BS of the beam in the points of division required. The point of intersection (1) of the line I, I, bisects SB, and, by placing the fulcrum there, the weight of the merchandise is equal to the total weight of the steelyard, if it be horizontal, or in a state of equilibrium. The point of intersection (2) in the line I, II, is as far again from S as from B; and, therefore, when this point is supported $Q = 2\,G$, when equilibrium is established similarly, the point of division (3) in the line I, III, is 3 times as far from S as from B; and hence for $Q = 3\,G$, the fulcrum must be moved to this point. It is also evident that by supporting the points of division $\frac{1}{2}$, $\frac{1}{3}$, &c., the weight $Q = \frac{1}{2}\,G$, $\frac{1}{3}\,G$, and so on, when the beam is in a state of equilibrium. We see from this that the points of division lie nearer together for heavy weights, and further apart for light weights, and that, therefore, the sensibility of this balance is very variable.

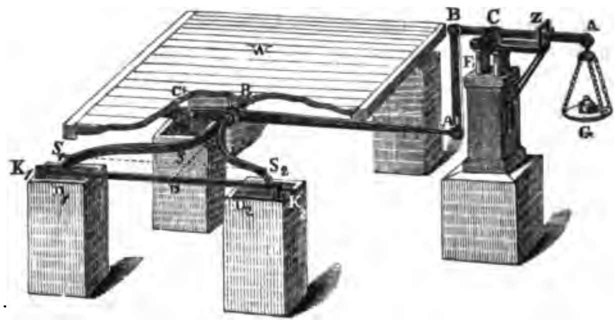
Fig. 98.



§ 47. *Weigh-bridges.*—Compound balances consist of two, three, or more simple levers, and are chiefly used as *weighing-tables* or *weigh-bridges* for carts, wagons, animals, &c. Being used for weighing great weights, they are generally *proportional* balances. The *scale* of the ordinary steelyard is replaced here by a *floor*, which should be so supported, and connected with the levers, that the receiving and removing of the body to be weighed may be as conveniently done as possible, and that the indication of the balance may be independent of the position of the body on the floor.

Fig. 99 represents a very good kind of weigh-bridge by Schwilgue in Strasbourg (*balance à bascule*). This weigh-bridge consists of a

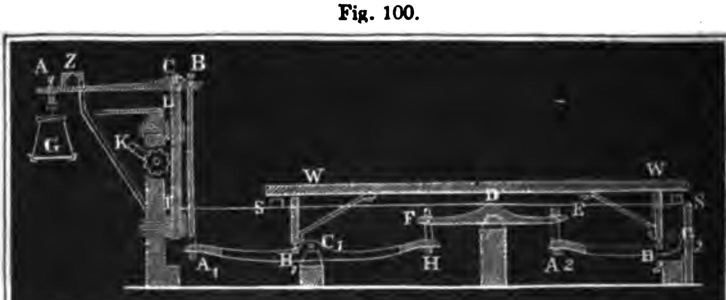
Fig. 99.



two-armed lever ACB, of a simple single-armed lever $A_1C_1B_1$, and of two fork-like single-armed levers B_1S_1, DS_2, &c. The fulcrum of these levers are C, C_1 and D_1D_2. The bridge or floor W is only partially shown, and only one of the fork-formed levers is visible. The bridge usually rests on four bolts K_1, K_2, &c., but during the weighing of any body, it is supported on the four knife-edges S_1, S_2, &c., attached to the fork-shaped levers. In order to do this, the support E of the balance AB is movable up and down by means of a pinion and rack (not visible in the drawing). The business of weighing consists in raising the support EC, when the wagon has been brought on to the floor, in adjusting the weight G in the scale, and finally in lowering the bridge on to its bearings K_1, K_2, &c. The usual proportions of the levers are: $\frac{CA}{CB} = 2$, $\frac{CA_1}{CB_1} = 5$, and the arms $\frac{DB_1}{BS_1} = 10$. If, therefore, the empty balance has been adjusted, the force at B or $A_1 = 2\ G$, the force at $B_1 = 5$ times the force in $A = 10\ G$, and lastly, the force in $S = 10$ times that in $B_1 = 100\ G$. And, therefore, when equilibrium is established, the weight on the floor is 100 times that laid on the scale at G, and this makes a centesimal scale.

Another form of weigh-bridge such as is constructed at Angers, is shown in Fig. 100. The bridge W of this balance, rests by means of

Fig. 100.

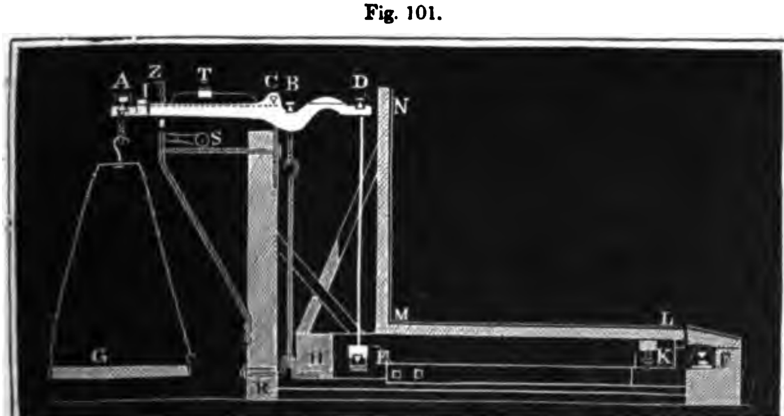


four supports at B_1, B_2, &c., on the fork-shaped single-armed levers $A_1B_1C_1$, $A_2B_2C_2$, of which the latter is connected with the prolongation C_1H of the former, by a lever DEF with equal arms. Until the bridge is to be used, it rests on beams S, S, but when the load is brought on, the support LL of the balance AB is raised (and with it the whole system of levers), by means of a pinion and rack-work, and then so much weight is laid in the scale at G, as is necessary to produce equilibrium.

In whatever manner the weight Q is set upon the bridge W, the sum of the forces at B_1, B_2, &c., is always equal to that weight. But the ratio $\frac{C_2A_2}{C_2B_2}$ is equal to the ratio $\frac{C_1A_1}{C_1B_1} = \frac{a_1}{b_1}$ of the length of the arms, and the length of the arm DE = length of arm DF, as also $C_1H = C_1A_1$. It is, therefore, the same in effect, whether a part of the weight Q is taken up on B_2, or directly on B_1; or the conditions of equilibrium of the lever $C_1B_1A_1$ are the same, whether the whole weight Q act directly on B_1, or only a part of it in B_1, and the rest in B_2, and only transferred by the levers $C_2B_2A_2$, EDF and C_1H to $C_1B_1A_1$. If, further, $\frac{a}{b}$ be the ratio of the arms $\frac{CA}{CB}$ of the upper balance, the force on the connecting rod $BA_1 = \frac{a}{b} \cdot G$, and hence the weight on the floor supposed previously adjusted, is: $Q = \frac{a_1}{b_1} \cdot \frac{a}{b} G$. Generally $\frac{a}{b} = \frac{a_1}{b_1} = \frac{10}{1}$, hence, $\frac{Q}{B} = \frac{100}{1}$, and the balance is centesimal.

§ 48. *Portable Weigh-bridge.* — In factories and warehouses, various forms and dimensions of weighing-tables, after the design of that of Quintenz, are used. This balance, which is represented in Fig. 101, consists of three levers ACD, EF, and HK. On the first lever hangs the scale-pan G, for the weights, and two rods DE and BH.

Fig. 101.



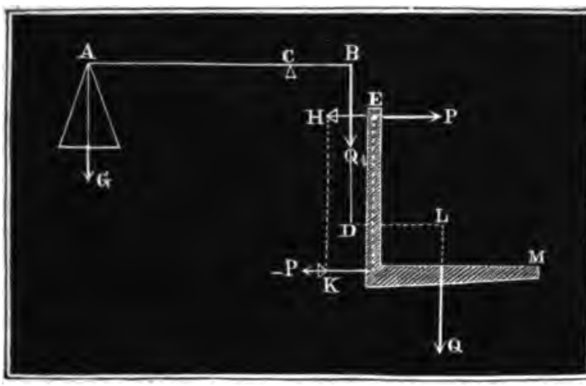
The rod DE carries the lever turning on the fixed point F, and the second rod BH carries the lever HK, the fulcrum of which rests upon the lever EF. In order to provide a safe position for the two latter levers, they are made fork-shaped, and the axes F and K on which they turn, are formed by the two knife-edges. On the lever HK, the trapezoidal floor ML is placed to receive the loads to be weighed, and it is provided with a back-board MN, which protects the more delicate parts of the balance from injury. Before and after the act of weighing, the lever formed by the border of the floor rests on three points, of which only one, R, is visible in our section; and the balance beam AD is supported by a lever-formed *catch* S, provided with a handle. When the merchandise is laid on the table, the catch is put down, and weights are laid on at G, till the balance AD is in adjustment. The catch is again raised, so that HK comes again to bear on the three points, and the merchandise can be removed without injury to the balance. The balance AD is known to be horizontal by an index Z, and the empty balance is adjusted by a small movable weight T, or by a special adjusting weight laid on the scale at G.

In this, as in other *weigh-bridges*, it is necessary that its indications be independent of the manner in which the goods are placed upon the table or floor. That this condition may be satisfied, it is necessary that the ratio $\frac{EF}{KF}$ of the arms of the lever EKF, be equal to the ratio $\frac{CD}{CB}$ of the arms of the balance beam AD. A part X of the weight Q on the floor is transferred by the connecting rod BH to the balance beam AH, and acts on this with the statical moment, $\overline{CB} \cdot X$; another part Y, goes through K to the lever EF, and acts at E with the force $\frac{KF}{EF} \cdot Y$. But this force goes by means of the rod DE to D to act on the balance beam. The part Y, therefore, acts with the statical moment, $\overline{CD} \cdot \frac{KF}{EF} \cdot Y$, and at B with the force $\frac{CD}{CB} \cdot \frac{KF}{EF} \cdot Y$ on the balance beam AD. That the equilibrium of the balance beam may not depend on either X or Y alone, but on the sum of them $Q = X + Y$, it is requisite that Y should act on the point B, exactly as if it were applied there directly, or that: $\frac{CD}{CB} \cdot \frac{KF}{EF} \cdot Y = Y$, i. e. $\frac{CD}{CB} \cdot \frac{KF}{EF} = 1$, therefore, $\frac{CD}{CB} = \frac{EF}{KF}$. If we denote the arms CA and CB by a and b, we have here as in the simple balance $Ga = (X + Y)\, b = Q\, b$, and, therefore, the weight required $Q = \frac{a}{b}\, G$, for example, $= 10\, G$, if the length CB is $= \frac{1}{10}$ of CA. Such a balance is tested by laying a certain weight on different parts of the floor, particularly the ends, to ascertain whether it

everywhere equilibrates the weight $G \frac{a}{b}$ times smaller than itself, placed on the scale.

Remark 1. Messrs. George, at Paris, manufacture weighing-tables of a peculiar construction described in the "Bulletin de la Société d'Encouragement, April, 1844." This balance has only one suspending rod BD, Fig. 102; but to provide against the floor FM

Fig. 102.



turning, there are two knife-edge axes on the back, which are united with two pair of knife-edges H and K, by four parallel rods EH and FK. According to the theory of couples, the tension Q, on the rod BD is equal to a weight Q laid on the floor; but besides this, the floor itself acts outwards with a force P in E, and with an opposite force $-P$ in F inwards. If d be the distance DL of the weight Q from the rod BD, and e the distance of the knife edges E and F, then $e\,P = d\,Q$, and, therefore, each horizontal force $P = \frac{d}{e} Q$. These forces do not influence the lever, and therefore the weight $Q = \frac{a}{b} G$, if, as hitherto, a and b denote the lever arms CA and CB, and G the weight in the scale.

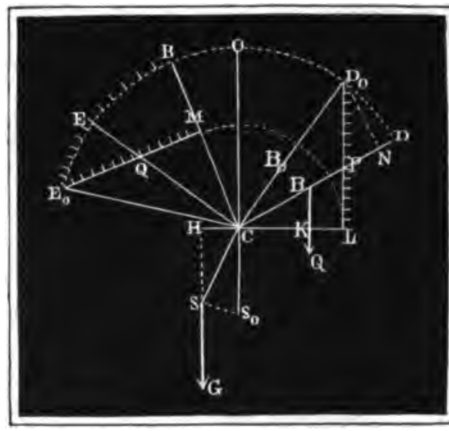
Remark 2. Weigh bridges are treated of in detail in the "Allgemeinen Maschinen Encyclopädie, Bd. 2, Art. Brückenwaagen," under the art. "Weighing Machine, in the Encyclo. Britannica Edinensis."

§ 49. ***Index Balances***, or ***Bent Lever Balances***.—The bent lever balance is an unequal-armed lever ACB, Fig. 103, which shows the weight Q of a body hung on to it at B, by an index CA moving over a scale DE, the weight G of the index head being constant. To determine the theory of this balance, let us first take the simple case, in which the axis of the pointer passes through the point of suspension B of the scale, Fig. 104. When the empty balance is in equilibrium, *i. e.* its centre of gravity S_0 vertically under the centre of motion C, let the index stand in the position CD_0, and let the point of suspension be in B_0. If now we add a weight Q, then B_0 comes to B, and D_0 to D, and S_0 to S, and thus the weight Q acts with the leverage CK, and the weight G of the empty balance, with the leverage CH. Therefore, for the new state of equilibrium $Q \,.\, CK = G \,.\, CH$. If now D_0N falls perpendicularly on CD, we have CD_0N and SCH, two similar triangles, and, therefore, $\frac{CH}{CS} = \frac{D_0N}{CD_0}$, and as besides,

the triangles D_0PN and CBK are similar, we have also $\frac{CK}{CB} = \frac{D_0N}{D_0P}$,

Fig. 103.

Fig. 104.



and hence:

$$Q \cdot \frac{CB \cdot D_0N}{D_0P} = G \cdot \frac{CS \cdot D_0N}{CD_0}, \text{ i. e } Q = \frac{CS}{CB} \cdot \frac{D_0P}{CD_0}; \text{ or, if:}$$

$CS = a$, $CB = b$, $CD_0 = CD = d$, and $D_0P = x$, $Q = \frac{a}{b} \cdot \frac{x}{d}$. Therefore the weight Q increases with the portion $D_0P = x$, cut off by the index from the vertical D_0L, and therefore D_0L may be divided as a scale of equal parts. If a point P on the scale has been found for a known weight put on the balance, other points of division are got by dividing D_0P into equal parts. If the index centre line of the CD_0 does not pass through the point of suspension, but has another direction CE_0, the scale of equal parts corresponding to it, is found, if we place the right-angled triangle CD_0L as CE_0M or CE_0, and lastly, in order to get the circular scale E_0R, we have to draw radii from the centre C through the points of division of the line E_0M to the periphery of the circle passed over by the point of the index.

Remark. There are other index balances described in Lardner's and Kater's Mechanics. Such balances are chiefly used for weighing letters and paper, thread, and such like manufactures, where samples have frequently to be weighed.

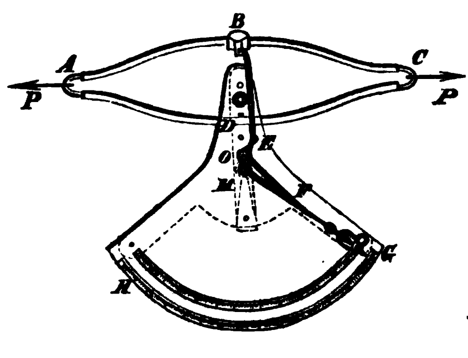
§ 50. *Spring-balances* or *Dynamometers.*—Spring-balances are made of hardened steel springs, upon which the weights or forces act, and are furnished with pointers, indicating on a scale the force applied in deflecting the spring. The springs must be perfectly tempered, or they must resume their original form on removal of the force applied. Thus spring-balances should never be strained beyond a certain point proportional to their strength; for if we surpass the limits of their elasticity in any application of them, they are

afterwards useless as accurate measures of weight. The springs applied for such balances are of many different forms. Sometimes they are wound spirally on cylinders, and enclosed in a cylindrical case, so as to indicate the forces applied in the direction of the axes of the cylinder by the compression or extension of the spiral. In other balances the spring forms an open ring *ABDEC*, Fig. 105, and the index is attached by a hinge to the end *C*, and passed through an opening in the end *A*. If the ring *B* be held fast, and a force *P* applied at *E*, the ends *A* and *C* separate in the direction of the force applied, and the index *CZ* rises to a certain position on the scale fastened at *D* to the spring. If the scale has been previously divided by the application of standard weights, the magnitude of any force *P* applied, though previously unknown, is indicated by the pointer.

Fig. 105.

Fig. 106 is a representation of Regnier's dynamometer. *ABCD* is a steel spring forming a closed ring, which may either be drawn out by forces *P* and *P*, or pressed together by *OB* and *D*; *DEGH* is a sector connected with the spring, on which there are two scales; *MG* is a double index turning on a centre at *M*, and *EOF* is a bent lever, turning on *O*, and which is acted upon by a rod *BE*, when the parts *B* and *D* of the spring approach each other in consequence of the application of weights or other forces as above mentioned. That the index may remain where the force has put it, for more convenient reading, a friction leather is put on the under side.

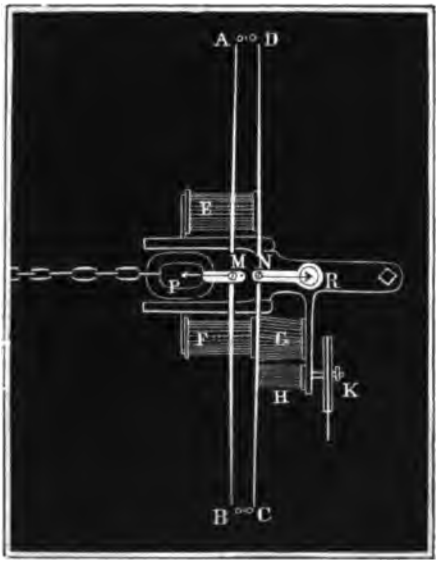
Fig. 106.



The most perfect, and most easily applied dynamometer for mechanical purposes, is that described by Morin, in his Treatise, "Description des Appareils chronométriques à style, et des Appareils dynamométriques, Metz, 1838," and used by him in his various researches on friction and other important mechanical inquiries. Morin's dynamometer consists of two equal steel springs *AB* and *CD*, Fig. 107, of from 10 to 20 inches in length, and the force applied is measured by the separation which it produces between the two spring plates at *M*. In order to determine the force, for example, the force of traction of horses on a carriage, the spring plate *N* is connected by a bolt with the carriage, and the horses are attached to the chain

M in any convenient manner. There is a pointer on *M* which indicates on a scale attached to *N*, the separation of the plates produced by the force *P* applied. If the springs be plates of uniform breadth and thickness, and be $l =$ the length, $b =$ the breadth, and $h =$ the thickness; according to Vol. I. § 190, we have for the deflexion corresponding to a force P:

Fig. 107.



$$a = \tfrac{1}{48}\frac{Pl^3}{WE} = \tfrac{1}{4}\frac{Pl^3}{Ebh^3},$$ or the deflexion increases as the force applied, and, therefore, a scale of equal parts should answer in this dynamometer.

As the deflexion s of two springs is called into action, the amount is double of that of one of them, or it is $= \frac{1}{2}\frac{Pl^3}{Ebh^3}$, and, therefore, generally $s = \frac{\phi\, l^3}{b\,h^3} P$, if ϕ be a number determined by direct experiment. If, before application of such an instrument, a known weight or force be put upon it, and the deflexion s ascertained, the ratio between force and deflexion may be calculated, and a scale prepared. It has been proved by experience that when the best steel is used, the deflexion may amount to $\frac{1}{10}$ the length, without surpassing the limits for which proportionality between force and deflexion subsists. The springs that have been employed by Morin and others are made into the form of beams of equal *resistance* throughout their length (Vol. I. § 204), and have, therefore, a parabolic form, or thicker in the centre than at the ends.

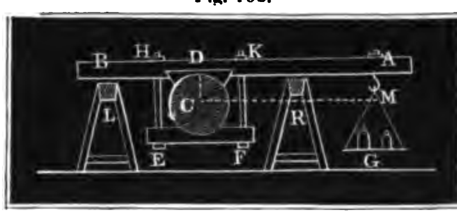
Remark. Forces do not generally act uniformly, but are continually changing, and therefore the usual object is to ascertain the *mean effort*. The usual index dynamometers only give the force as it has acted at some particular instant, or only the *maximum effort*. There is, therefore, extreme uncertainty in the indication of such dynamometers, modified as they have been by M'Neill and others, when applied to measuring the effort of horses applied to ploughing, canal traction, &c. Morin has completely provided against this defect, by attaching to his dynamometer, a self-registering apparatus, first suggested by Poncelet, (see Morin's work above quoted,) by which, in one case, the force for each point of a distance passed over is registered in the form of a curved line, drawn on paper, and in the other case, the force as applied at each instant is summed up or integrated by a machine. Both apparatuses give the product of the force into the distance described, and, therefore, the *mean effort* may be produced when the *mechanical effect* is divided by distance passed over—by a canal boat, for example.

In the dynamometer with pencil and continuous scroll of paper, the measure of the force is marked by a pencil passing through *M*, till its point touches a scroll of paper

passing under it. This scroll is wound from the roller E, (Fig. 107,) to the roller F, which is set in motion by bands or wheel-work, by the wheels of the carriage itself: When no force acts on the springs, the pencil would mark a straight line on the paper, supposing it set in motion; but by application of a force P, the springs are deflected, and therefore a line more or less tortuous is drawn by the pencil at a variable distance from the above alluded to zero line, but on the whole parallel to it. The area of the space between the two lines is the measure of the mechanical effect developed by the force; for the basis of it is a line proportional to the distance passed over, and the height is itself proportional to the force that has acted to bend the spring.

§ 51. *Friction Brake.*—The dynamometrical brake (Fr. *frein dynamométrique de M. Prony*), is used to measure the power applied to, and mechanical effect produced by a revolving shaft, or other revolving part of a machine. In its simplest form, this instrument consists of a beam AB, Fig. 108, with a balance scale AG; and of two wooden segments D and EF, which can be tightened on the revolving axis C, by means of screw-bolts EH and FK. To measure, by means of this arrangement, the power of the axis C for a given number of revolutions, weights are laid in the scale, and the screw-bolts drawn up until the shaft makes the given number of revolutions, and the beam maintains a *horizontal* position, without support or check from the blocks L or R. In these circumstances the whole mechanical effect expended is consumed in overcoming the friction between the shaft and the wooden segments, and this mechanical effect is *equal* to the work or useful effect of the revolving shaft. As, again, the beam hangs freely, it is only the friction F acting in the direction of the revolution that counterbalances the weight at G, and this friction may be deduced from the weights. If we put the lever CM of the weight G referred to the axis of the shaft $= a$, the statical moment of the weight, and therefore also the *moment of the friction*, or the friction itself acting with the lever equal to unity $= G a$, then, if ϵ represent the angular velocity of the shaft, the mechanical effect produced $L = Pv = Ga \cdot \epsilon = \epsilon\, a\, G$ per second.

Fig. 108.



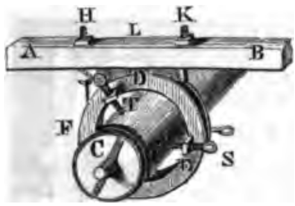
If, again, $u =$ the number of revolutions of the shaft per minute, then $\epsilon = \frac{2\pi u}{60} = \frac{\pi u}{30}$, and, therefore, the work required $L = \frac{\pi u a}{30} G$.

The weight G must of course include, not only the weight in the scales, but the weight of the apparatus reduced to the point of suspension. To do this, the apparatus is placed upon a knife-edge at D, and a cord from A attached to a balance would give the weight required.

The friction brake as represented in Fig. 109, with a cast iron friction ring DEF is a convenient form of this instrument. This ring is fastened by three pairs of screws on any sized shaft that will

pass through the ring. For the wooden segment an iron band is substituted, embracing half the circumference of the iron ring. The band ends in two bolts passing through the beam AB, and may be tightened at will by means of screw nuts at H and K.

Fig. 109.



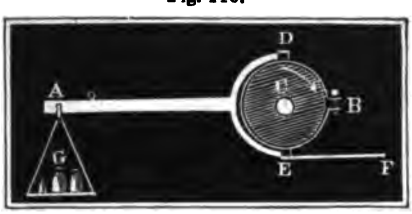
To hinder the firing of the wood, or excessive heating of the iron, water is continually supplied through a small hole L. This apparatus is known in Germany as "Egen's Friction Brake."

Example. To determine the mechanical effect produced by a water wheel, a friction brake was placed on the shaft, and when the water let on had been perfectly regulated for six revolutions per minute, the weight G including the reduced weight of the instrument was 530 lbs., the leverage of this weight was $a = 10{,}5$ feet. From these quantities we deduce the effect given off by the water wheel to have been:

$$L = \frac{\pi \cdot 6 \cdot 10{,}5}{30} \cdot 530 = 3497 \text{ ft. lbs.} = 6{,}3 \text{ horse power.}$$

§ 52. In more recent cases, various forms of friction brake have been adopted, some of them very complicated. The simplest we know of is that of Armstrong, shown in Fig. 110. This consists of an iron ring, which is tightened round the shaft by a screw at B, and of a lever ADE with a scale for weight G on one side, and a fork-shaped piece at the other, which fits into snuggs projecting from the ring. There is a prolongation of one prong of the fork, by which the weight of the instrument itself can be counterbalanced, and which is otherwise convenient in the application of the instrument.

Fig. 110.



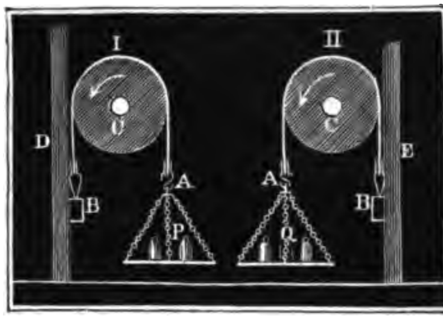
Navier proposed a mode of determining the effect given off at the circumference of a shaft by laying an iron band round the shaft, attaching the one end of this to a spring balance, the other end being weighted, so that the friction on the wheel causes a resistance, in overcoming which only the required number of revolutions take place. The difference of this weight Q and that indicated by the spring balance P, is of course = the friction F between the shaft and the band. If then p be the circumference of the shaft, and u the number of revolutions per minute, the effect produced

$$= L = F\frac{up}{60} = \frac{up}{60}(Q - P).$$

When a spring balance cannot be obtained, a simple band, as shown in Fig. 111, is sufficient for the purpose, if the experiment be made twice, and the end B be fastened to an upright or other fixture, first on the one side and then on the other of the shaft. In this way one experiment gives us $Q = P + F$, and in the other $Q = P$,

because in the one case, the friction F acting in the direction of the revolution of the shaft, counteracts the weight hanging in the scale on the end A, and in the other it acts with this friction. For this arrangement, used by the author in many experiments, the mode of calculation already explained applies precisely. As the power has only a small leverage in this arrangement, it is only suitable for cases in which the effort exerted is small. The strain on the band may be multiplied by means of an unequal-armed lever attached at A, instead of the direct application of the weight in the scale. The author has successfully applied a leverage of 10 to 1 in this way. In order to avoid the objectionable increase of friction of the axle or gudgeons induced by this apparatus, the band may be made to pass round the shaft, carrying the one end upwards and the other down.

Fig. 111.



Remark. Egen treats of the different forms of dynamometers in his work, entitled "Untersuchungen über den Effect einiger Wasserwerke, &c.," and Hülsse, in article Bremsdynamometer in the "Allgemeinen Maschinenencyclopädie." James White, of Manchester, invented the friction brake in 1808. See Hachette "Traité elémentaire des Machines, p. 460." Prony's original paper is in the "Annales des Mines, 1826. There are remarks on its use in the writings of Poncelet and Morin, worthy attention. W. G. Armstrong's paper is in the "Mechanic's Magazine," vol. xxxii. p. 531. Weisbach's papers on the friction band, in the "Polytechnishes Central Blatt," 1844.

CHAPTER II.

OF ANIMAL POWER, AND ITS RECIPIENT MACHINES.

§ 53. *The Power of Animals.*—The working power of animals is, of course, not only different for individuals of different species, but for animals of the same species. The work done by animals of the same species depends on their race, age, temper, and management, as well as on the food they get, and their keeping generally, and also on the nature of the work to which they are applied, or the *manner* of putting them to their work, &c. We cannot discuss these different points here, but for each kind of animal employed by man, we shall assume as fair an average specimen as possible,—that the animal is judiciously applied to work it has been used to perform, and that its food is suitable. But the working capabilities of animals depend also on the effort they exert, and on their speed, and

on the time during which they continue to work. There is a certain *mean effort*, *speed*, and length of *shift* for which the *work done* is a maximum. The greater the effort an animal has to exert, the less the velocity with which it can move, and *vice versâ*. And there is a maximum effort when the speed is reduced to nothing, and when, therefore, the work done is nothing. It is evident, therefore, that animal powers should work with only a certain velocity, exerting a certain mean effort, in order to get the maximum effect; and it also appears that a certain mean length of shift, or of *day's work*, is necessary to the same end. Small deviations from the circumstances corresponding to the maximum effect produced, are proved by long experience to be of little consequence. It is also a matter of fact that animals produce a greater effect, when they work with variable efforts and velocities, than when these are constant for the day. Also pauses in the work, for breathing times, makes the accomplishment of the same amount of work less *fatiguing*, or the more the work actually done in a unit of time differs from the mean amount of work, the less is the fatigue.

The main point to be attended to in respect to animal powers is the "*day's work.*" If this be compared with the daily cost of maintaining the animals, and interest on capital invested, we have a measure of the value of different animal powers.

§ 54. The manner and means of employing the power of men and animals is very different. Animal powers produce their effects either with or without the intervention of machines. For the different means of employing labor, the degree of fatigue induced is not proportional to the work done. Many operations fatigue more than others; or what amounts to the same thing, the mechanical effect produced is much smaller in some modes of applying labor than in others. Again, all labor cannot be measured by the same standard as is involved in our definition of mechanical effect. The work done in the transport of burdens on a horizontal road cannot be referred to the same standard as the raising of a weight is referred to. According to the notions we have acquired hitherto, the mechanical effect produced in the transport of burdens on a horizontal road is nothing, because there is no space described *in the direction* of the force (Vol. I. § 80) exerted, that is, at right angles to the road; whilst in drawing or lifting up a weight, the work done, or mechanical effect produced, is determined by the product of the weight into the distance through which it has been raised. It is true, that walking or carrying fatigues as much as lifting does, *i. e.* the "day's work" is consumed by the one as by the other kind of labor; and, therefore, a certain day's work is attributable to the one as there is to the other, although they are essentially different in their nature. According to experience, a man can walk, unburdened, for ten hours a day at $4\frac{3}{4}$ feet per second (something under $3\frac{1}{4}$ miles per hour). If we assume his weight at 140 lbs. we get as the day's labor 140 . 4,75 . 10 . 60 . 60 = 23,940000 ft. lbs.

If a man carry a load of $85\frac{1}{2}$ lbs. on his shoulders, he can walk

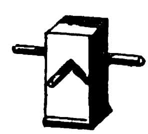
for 7 hours daily with a speed of 2,4 feet per second, and, therefore, produces daily the quantity of work = 85,5 . 2,4 . 7 . 60 . 60 = 5′,171000 ft. lbs., neglecting his *own* weight.

A horse will carry 256 lbs. for 10 hours daily, walking 3½ feet per second, so that its day's work amounts to 256 . 3,5 . 10 . 60 . 60 = 32′256000 ft. lbs., or more than 6 times as much as a man doing the same kind of work. If the horse carries only 171 lbs. on his back, he will trot at 7 feet per second for 7 hours daily, and the work done in this case is only 171 . 7 . 7 . 60 . 60 = 30′164400 ft. lbs. daily.

The amounts of work done in raising burdens is much smaller, for in this case mechanical effect, according to our definition, is produced, or *the space is described in the direction of the effort exerted.*

If a man, unburdened, ascend a flight of steps, then, for a day's work of 8 hours, the velocity measured in the vertical direction is 0,48 feet per second; therefore, the amount of work done daily = 140 . 0,48 . 8 . 60 . 60 = 1′935000 ft. lbs. It thus appears that a man can go over 12½ times the space horizontally that he can vertically.

Fig. 112.



In constructing a reservoir dam, the author observed that 4 practised men, raised a dolly, Fig. 112, weighing 120 lbs., 4 feet high 34 times per minute, and after a *spell* of 260 seconds, rested 260 seconds; so that, on the whole, there were only 5 hours work in the day. From this it appears that the day's work of a man $= \frac{120}{4} . 4 . 34 . 5 . 60 = 1'224000$ feet lbs.

Remark 1. In the "Ingenieur," there is detailed information on the work done by animal power. In the sequel, the effect produced by animals by aid of machines is given for each machine respectively.

Remark 2. The effect produced by men and animals is far from being accurately ascertained. The effect produced by men working under disadvantageous circumstances, or by unpractised laborers, is not one-half of that produced by well-trained hands. Coulomb, in his "Théorie des Machines simples," first entered on investigations of the effect of animal powers. Desaguiliers ("Cours de Physique expérimentale,") and Schulze ("Abhandlungen der Berliner Akademie,") had previously occupied themselves with the subject. Many experiments have been made and recorded in more recent times. See Hachette, "Traité élémentaire, &c.," Morin, "Aide Mémoire," Mr. Field in the "Transactions of the Institution of Civil Engineers, London," Sim's "Practical Tunneling," and Gerstner's "Mechanik," Band 1.

§ 55. *Formulas.*—Effort and velocity have a very close dependence in the application of animal power; but the law of their dependence is by no means known, and is still less deducible *à priori.* The following empirical formulas, given by Euler and Bouguer, are only to be considered as approximations. If K_1 be the maximum effort which an animal can exert without velocity, and c_1 the greatest velocity it can give itself when unimpeded by the necessity of extraneous effort, we have for any other velocity and effort:

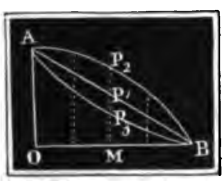
$$\text{according to Bouguer : } P = \left(1 - \frac{v}{c_1}\right) K_1,$$

$$\text{according to Euler}: P = \left(1 - \frac{v^2}{c_1{}^2}\right) K_1,$$

$$\text{“} \qquad \text{Euler}: P = \left(1 - \frac{v}{c_1}\right)^2 K_1.$$

The first of these is the most simple, and that which, according to Gerstner, corresponds best with observation. According to Schulze's observations, on the other hand, the last formula appears to be most consistent with experiment. If we draw v as abscissa, and P as ordinates to a curve, the first formula corresponds to a straight line AB, Fig. 113, the second with a concave parabolic curve AP_2B, and the third with a convex parabolic curve AP_3B, and the ordinates MP_1 of the straight line always lie between the ordinates MP_2 and MP_3 of the two parabolic curves. The abscissa OM, for example, $= v = \frac{1}{2} c_1$ corresponds to the ordinates $MP_1 = \frac{1}{2} K = \frac{1}{2} OA$, also $MP_2 = \frac{3}{4} K = \frac{3}{4} OA$, and $MP_3 = \frac{1}{4} K = \frac{1}{4} OA$. The formula of Bouguer, therefore, gives values of the *effort* which lie between the values given by the two formulas of Euler; and we may, therefore, make use of Bouguer's formula until some special reason for adopting Euler's formula be adduced. If we introduce into Bouguer's formula, instead of the maximum values K_1 and c_1, the halves of these, or their mean values $K = \frac{1}{2} K_1$, and $c = \frac{1}{2} c_1$, we get a formula first applied by Gerstner:

Fig. 113.



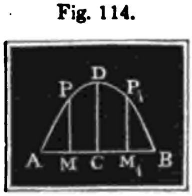
$$P = \left(1 - \frac{v}{2c}\right) 2\, K, \text{ or } P = \left(2 - \frac{v}{c}\right) K,$$

and inversely, $v = \left(2 - \frac{P}{K}\right) c$. Although this formula can be but little depended upon as accurate for extreme values of v and P, yet it may be presumed, that for values not very different from the mean, they are sufficiently near for practical uses. The mechanical effect produced per second would follow from this:

$$Pv = \left(2 - \frac{v}{c}\right) v\, K. \quad \text{As} \left(2 - \frac{v}{c}\right) v\, K = (2\, c - v)\, v \frac{K}{c},$$

the mechanical effect is a maximum, as in Vol. I. § 386, when $v = c$, or when $P = K$, or when the velocity and effort are mean values, *i. e.*, $Pv = Kc$. If we try to get a greater or less velocity, or a greater or less effort, we get an effect $L = Pv$ less than Kc. If we set off the velocities as abscisses, and the amounts of mechanical effect produced as ordinates, we get as the projected curve a parabola ADB, Fig. 114; and it is evident that not only for abscissa $AM < AC$, but also for $AM_1 > BC$, the ordinates MP, M_1P_1, are less than for the abscissa $AC = c$. For $v = \frac{c}{2}$, as also for $v = \frac{3}{2}$ c, it follows from the above that $L = \frac{3}{4} Kc = \frac{3}{4} CD$.

Fig. 114.



According to Gerstner, the following table represents the *draught* of animals applied properly to draw by *traces*.

Animals.	Weight.	Mean effort K in lbs.	Mean speed c in feet per second.	Mean period of day's work. Hours.	Effect produced p. sec. in ft. lbs.	Daily effect feet lbs.
Man .	150	30	2,5	8	75	2'160000
Horse .	600	120	4	8	480	13'824000
Ox . .	600	120	2,5	8	300	8'640000
Ass . .	360	72	2,5	8	180	5'184000
Mule .	500	100	3,5	8	350	10'080000

Example 1. According to the above table, a man working with an effort of 30 lbs., and mean velocity of $2\frac{1}{2}$ feet per second, produces in a day an amount of mechanical effect represented by 2'160000 feet lbs. If he be urged to work at 3 feet per second, the effort will be reduced to $P = \left(2 - \frac{3}{2,5}\right) 30 = 24$ lbs., and his daily effect would only be $24 . 3 . 8 . 60 . 60 = 2'073600$ feet lbs.

Example 2. If a horse be obliged to draw with an effort of 150 lbs., it can only be done with a velocity $v = \left(2 - \frac{150}{120}\right) 4 = 3$ feet per second, and thus his effective work is reduced to only $3 . 150 = 450$ feet lbs. per second.

Remark. Fourier, in the *Annales des Ponts et Chaussées*, 1836, gives a complicated formula for the effect produced by horses. See also Crelle's *Journal der Baukunst*, Band xii. 1838.

§ 56. *Work done by aid of Machines.*—If we follow Gerstner's notion, that the period or time of each *shift*, or day's work, has the same influence on the amount of work done as the velocity, we must then put for the effort:

$$P = \left(2 - \frac{v}{c}\right)\left(2 - \frac{z}{t}\right)K,$$

and from this we get the daily effect produced:

$$L = \left(2 - \frac{v}{c}\right)\left(2 - \frac{z}{t}\right)K v z.$$

There can be no doubt that the effect produced is a maximum, that is $= K c t$, when the animal is made to work, not only with a mean velocity and effort, but also when the time of work is kept within the mean for this. It is to be kept in mind, however, that this formula only applies when the values of v, z, and P do not differ widely from c, t, and K.

M. Maschek, of Prague, recommends the expression:

$$P = \left(3 - \frac{v}{c} - \frac{z}{t}\right)K,$$

which is certainly more convenient for calculation.*

Eight to ten hours per day is a good average day's work, and, therefore, the factor $\left(2 - \frac{z}{t}\right)$ may generally be neglected, or the day's effect may be written $L = \left(2 - \frac{v}{c}\right) K v z$. If, however, an

* Neue Theorie der menschlichen und thierischen Kräfte, &c., von F. J. Maschek, Prag.

animal be applied to a machine, its effort P would be divided into an effort P_1 for doing the work, and an effort P_2 for overcoming prejudicial resistances, or $P = P_1 + P_2$, both resistances being reduced to the point of application of the effort. It is also usual, as we shall learn in the sequel, to find the prejudicial resistances P_2 composed of a constant part R, independent of the strain on the machine, and a part $\delta \,.\, P_1$, proportional, or nearly so, to the useful effect produced or work done, where δ is co-efficient derived from experiment, thus $P_2 = R + \delta \,.\, P_1$; and, therefore,

$$P = (1 + \delta)\, P_1 + R \text{; and again } \left(2 - \frac{v}{c}\right) K = (1 + \delta)\, P_1 + R.$$

The total effect produced per second is, therefore,

$$Pv = \left(2 - \frac{v}{c}\right) Kv = (1 + \delta)\, P_1 v + Rv.$$

and, therefore, the useful effect produced:

$$P_1 v = \frac{(2K - R)\, v - \frac{Kv^2}{c}}{1 + \delta} = \left[\left(2 - \frac{R}{K}\right) c - v\right] v \,.\, \frac{K}{(1 + \delta)\, c}.$$

That this effect may be the greatest possible (see previous paragraph), we must have $v = \frac{1}{2}\left(2 - \frac{R}{K}\right) c = \left(1 - \frac{R}{2K}\right) c$, or the velocity less than the mean velocity; and so much the less, the greater the constant part of the prejudicial resistance is. The effort corresponding would be, according to this:

$$P = \left(1 + \frac{R}{2K}\right) K = K + \frac{R}{2},$$

or greater than the mean effort. The useful resistance, on the other hand, is $P_1 = \dfrac{K - \frac{R}{2}}{1 + \delta}$. The total effect produced is:

$Pv = \left[1 - \left(\frac{R}{2K}\right)^2\right] K\, c$, and the useful effect produced is:

$P_1 v = \left(1 - \frac{R}{2K}\right)^2 \frac{Kc}{1 + \delta}$, and the efficiency of the machine:

$$\eta = \frac{\left(1 - \frac{R}{2K}\right)^2}{1 + \delta}.$$

Example. If in a machine turned by two horses, the *constant* prejudical resistance reduced to the point of application of the horses' effort $= 60$ lbs., the velocity at which the horses should work when $K = 2 \,.\, 120 = 240$ lbs., and $c = 4$ feet, is reduced to $v = \left(1 - \frac{60}{480}\right) c = \frac{7}{8} \,.\, 4 = 3{,}5$ feet. Further, the effort of the horses $= 240 + \frac{60}{2} = 270$ lbs., and, therefore, that of one horse $= 135$ lbs. If, now, the constant part of prejudicial resistance be 15 per cent. of the useful resistance, then $\delta = 0{,}15$, and, therefore, the resistance to be put on the machine $P_1 = \frac{240 - 30}{1{,}15} = 182{,}5$ lbs., and the efficiency of the machine would be $\eta = (\frac{7}{8})^2 \div 1{,}15 = 0{,}67$.

Remark. Gerstner reduces the calculation of the effect of animal power to motion on an inclined plane. If G be the weight of the animal, P the effort exerted, and α the angle of inclination of the inclined plane, upon which the moving power ascends with its load, then the effort is $P + G \sin. \alpha$ (see *Theory of Inclined Plane*, Vol. I. § 134), and hence $\left(2 - \frac{v}{c}\right) K = P + G \sin. \alpha$. Hence, we have the load with which an animal can ascend an inclined plane, and, conversely, the inclination corresponding to a given load, viz.: $\sin. \alpha = \frac{\left(2 - \frac{v}{c}\right) K - P}{G}$, thus when $P = 0$, and $v = c$, or when the animal has no resistance to overcome, and goes with the mean velocity $\sin. \alpha = \frac{K}{G}$. But, the weight of an animal is almost always five times as great as the mean effort it can exert, therefore $\sin. \alpha = \frac{1}{5}$ and $\alpha = 11\frac{1}{2}°$ is the angle of inclination of a plane which an animal can ascend with the mean amount of exertion and fatigue. This corresponds to a rise of one foot in five feet, or nearly so.

§ 57. *The Lever.*—Animal powers are applied to work by means of the lever or the wheel and axle. The latter are either horizontal or vertical. We shall first speak of the lever as a machine for receiving (and transmitting) animal power. The general theory of this machine is known from Vol. I. §§ 126, 127, and 170. The lever is either single as ACB, Fig. 115, or double, as $ACBA_1$, Fig. 116; the one has only one arm for the application of the power CA, whilst the other has two arms CA and CA_1. The lever produces an oscillating circular motion, and is, therefore, chiefly applied, when a reciprocating *up* and *down* motion is desired, as in

Fig. 115.

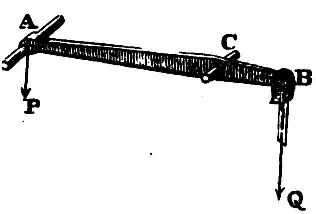
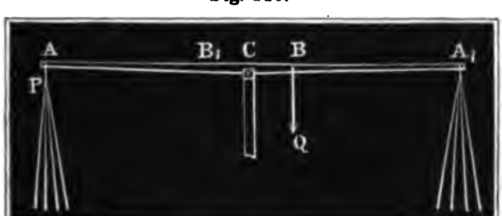


Fig. 116.



pumping. Handles, suited to the number of hands to be applied, are affixed to the lever. As the strength can be better exerted in pulling downwards than in lifting upwards, it is usual to make the *down-stroke* the *working-stroke*, and counter-balances are attached so as to aid the workmen in the *up-stroke*, or the double lever is used, on which the workers alternately pull downwards. When the down-stroke is the effective stroke, ropes, hanging from the end of the lever, are frequently substituted for handles. Levers are sometimes moved by the tread of the feet.

That there may not be too great a change of direction during a stroke, the lever's motion is confined to an arc of not more than

60°, and, in order to facilitate the exertions of the power, the space passed through at each stroke is kept proportional to the length of arm of the workers, or at from $2\frac{1}{2}$ to $3\frac{1}{2}$ feet. Again, the handles should not come within from 3 to $3\frac{1}{2}$ feet from the floor. According to experience, men work 8 hours per day, exerting an effort of $k = 10{,}7$ lbs. on the end of a lever, with a velocity $c = 3{,}5$ feet. Therefore, the mechanical effect produced by a man applied to a lever, as in pumping, is per second: $L = 10{,}7 \,.\, 3{,}5 = 37{,}45$ ft. lbs., and, therefore, the *day's work*

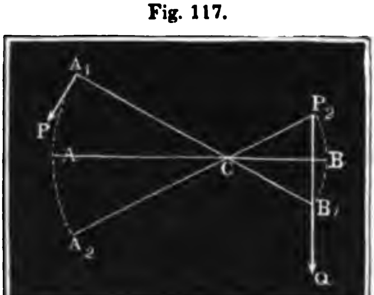
$$= Kct = 37{,}45 \,.\, 8 \,.\, 3600 = 1'078560 \text{ ft. lbs.}$$

In putting up a lever, it is necessary to take care that the workmen shall be applied so as to exert the ascertained mean effort with the mean velocity; or rather, that the effective effort shall exceed the mean effort by only one-half of the constant prejudicial resistance.

The lever itself is subject to only one prejudicial resistance, viz.: the friction of the fulcrum. If R be the pressure on the fulcrum arising from the weight of the lever and from the effort and resistance, r the radius of the fulcrum, f the co-efficient of friction, and a the leverage CA of the power, then the axle friction reduced to the point of application of the power $F = \frac{fr}{a} R$; as, however, f, and also $\frac{r}{a}$ are generally small fractions, F is so small that it may be neglected in most cases, compared with the other resistances.

If we suppose a useful resistance Q and a prejudicial resistance $\delta\, Q + W$ acting at the point B, and if we put the leverage CB of these resistances $= b$, the moment of the effort becomes: $Pa = [(1 + \delta)\, Q + W]\, b$, and, therefore, the effort itself: $P = \frac{b}{a}\, [(1 + \delta)\, Q + W]$. But, that the power of men may be most advantageously applied: $P = K + \frac{b}{a} \,.\, \frac{W}{2}$ and, therefore, $\frac{a}{b} K = (1 + \delta)\, Q + \frac{W}{2}$, and, therefore, the ratio of the lever arms $\frac{a}{b} = \frac{(1 + \delta)\, Q + \frac{1}{2} W}{K}$ is to be employed.

Fig. 117.



Remark. The arms of the lever are variable to a certain extent during the stroke, and, therefore, it may be well to determine the amount of this variation.

If the arm CB, Fig. 117, be horizontal at the *half-stroke*, and if the angle B_1CB_2 passed through in a stroke $= \beta^\circ$, the height through which the resistance is overcome $s = \overline{B_1B_2} = 2\, b\, sin. \frac{\beta}{2}$, and, therefore, the mechanical effect produced or expended in one stroke $= 2\, b\, sin. \frac{\beta}{2} \,.\, Q$. If, however, the resistance were constant during the stroke at a

leverage $CB = b$, the space passed over at each stroke would be $=$ arc $B_1BB_2 = \beta\, b$, and, therefore, the resistance would be $= \frac{2\, b\, sin. \frac{\beta}{2}}{\beta\, b}\, Q = \frac{2\, sin. \frac{1}{2}\beta}{\beta}\,.\, Q$, and the statical moment $= \frac{2\, sin. \frac{1}{2}\beta}{\beta}\, Q\, b$. Conversely, we may assume that the resistance Q acts during a stroke on the mean length of arm $\frac{2\, b\, sin. \frac{1}{2}\beta}{\beta}$. For $\beta^0 = 60^0$ this lever $= \frac{b}{arc.\ 60^0}$ $= \frac{b}{1,0472} = 0,955\, b$, or not quite 5 per cent less than b, and for smaller arcs of oscillation, the difference is still much less.

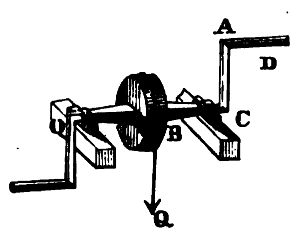
Example. What proportion of arms should be chosen for a lever, that for a useful resistance of 160 lbs. and a prejudicial resistance $Q_1 = 0,15\ Q + 55 = 0,15\ .\ 160 + 55 = 79$ lbs., four men may work to the best advantage? $K = 4\ .\ 10,7 = 42,8$ lbs. therefore $\frac{a}{b} = \frac{1,15\ .\ 160 + \frac{1}{2}\ 55}{42,8} = \frac{2115}{428} = 4,9$. If the resistance passes through 1 foot for each stroke, the power must at the same time pass through 4,9 feet, and if we take the angle of oscillation $\beta = 50^0$, we get for the suitable length of lever $b = \frac{s}{2\, sin. \frac{\beta}{2}}$ $= \frac{0,5}{sin.\ 25^0} = 1,183$ feet, and the length of arm $a = 4,9\ .\ b = 4,9\ .\ 1,183 = 5,80$ ft. The *effort* necessary is $P = \frac{160 + 79}{4,9} = 48,78$ lbs., therefore, the effort of each man $=$ 12,195 lbs., and the efficiency

$$\eta = \frac{\left(1 - \frac{55}{2\ .\ 4,9\ .\ 42,8}\right)^2}{1,15} = \frac{(1 - 0,1311)^2}{1,15} = 0,657.$$

We see, therefore, that four men capable of a day's work each $=$ 1'075860 ft. lbs., or 4'303440 ft. lbs. in all, would only produce 0,657 . 4'303440 $=$ 2'800000 feet lbs. *useful effect* with this machine.

§ 58. ***Windlass.***—The best means of applying the power of men, is the windlass (Fr. *treuil, tour*; Ger. *Haspel*). This machine consists of a horizontal axle, at the circumference of which the resistance acts, and of a crank, handle, or *winch*, Fig. 118, or series of handles on a wheel, Fig. 120, or of fixed or movable levers (hand-spikes), Fig. 119. With the winch, the laborers have a continuous hold throughout the revolution, whilst with the wheel or hand-spike, the action is hand over hand, or otherwise at short intervals. The winch is the form used for general purposes. The wheel is applied principally in working the tiller on board-ship, and the movable levers are chiefly used for weighing anchor by means of the capstan.

Fig. 118.



That a laborer may produce the best effect by means of the crank-handled windlass, the length of the lever must not be more than from 16 to 18 inches, corresponding to the length of arm of the laborer, and the axis of the barrel must not be more than 36 to 39 inches above the floor on which the laborer stands, for men of average height. The handle of the windlass is adapted for one, two, or more men, according to circumstances. As a man can work with less

fatigue while pushing and pressing, than while lifting and pulling, the effort required at each point of a revolution of the handle is not

Fig. 119.

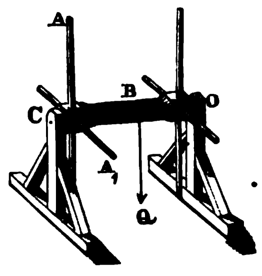
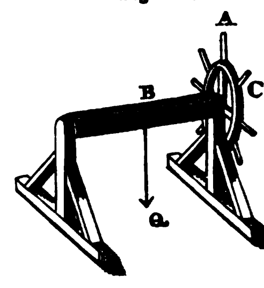


Fig. 120.

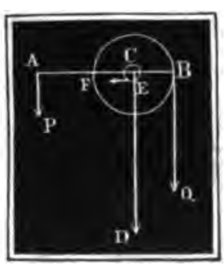


equal, and, therefore, it is well in double-handled windlasses to set the handles 180° apart, and, when more handles are applied, to distribute them equally.

The day's work of a man working a windlass has been found to be 1'175040 feet lbs. with a mean effort $K = 17$ lbs., and mean velocity $c = 2{,}4$ feet, and length of day 8 hours. The calculations for the windlass are the same as for the wheel and axle.

If the resistance Q, Fig. 121, act with the lever $CB = b$, and the power P on the lever $CA = a$, then $Pa = Qb$; and, therefore, the power corresponding to a given resistance is $P = \frac{b}{a} Q$. If, again, D be the pressure on the *journals* or *gudgeons* and r the radius of the gudgeons CE, then $Pa = Qb + f\,Dr$, and hence $P = \frac{b}{a} Q + \frac{r}{a} . f D$. If the resistance Q, together with the friction $\frac{r}{b}$ $f\,D$, consist of the useful resistance Q_1, the constant prejudicial resistance W, and the variable prejudicial resistance $\delta\, Q$, or, $Q = (1 + \delta)\, Q_1 + W$, then $P = \frac{b}{a} [(1 + \delta)\, Q_1 + W] = K + \frac{b}{a} . \frac{W}{2}$ and, therefore, the proportion of the winch and barrel radius should be: $\frac{a}{b} = \frac{(1 + \delta)\, Q_1 + \frac{1}{2} W}{K}$. But as the winch has a prescribed height of 16 to 18 inches, the leverage of the resistance, or radius of the barrel is to be determined by this, viz:

Fig. 121.



$b = \frac{K\,a}{(1 + \delta)\, Q_1 + \frac{1}{2}\, W}$, in order that the laborer may work to the greatest advantage.

Example. On a two-handled windlass the resistance is 200 lbs., viz.: 150 lbs. of useful resistance, and 30 lbs. constant, and 20 lbs. variable prejudicial resistance. The leverage of the resistance is 4 inches, that of the power 18 inches, the radius of the

journal ½ inch, the co-efficient of friction $f = 0{,}1$, and the weight of the barrel, &c., 80 lbs.; required the useful effect of such a machine. The whole power required, if the pressure on the journals be taken $200 + 80 = 280$ lbs., is $P = \frac{4}{18} \cdot 200 + 0{,}1 \frac{1}{2 \cdot 18} \cdot 280$ $= 44{,}44 + 0{,}5 = 44{,}94$ lbs., and, therefore, the effort of each laborer must be 22,47 lbs., and, according to Gerstner's formula, the velocity of the power, or of the handle of the windlass:

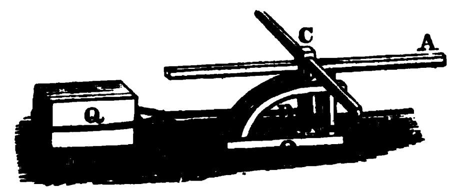
$v = \left(2 - \frac{P}{K}\right) c = \left(2 - \frac{22{,}47}{17}\right) \cdot 2{,}4 = 1{,}628$ feet, and that of the resistance:

$w = \frac{a}{b}\, v = \frac{2}{9} \cdot 1{,}628 = 0{,}362$ feet, and the useful effect per second:

$Q_1 w = 0{,}362 \cdot 150 = 54{,}3$ feet lbs., and daily $= 1{,}563840$ lbs., and the efficiency of such an application of the power of two laborers, the day's work of each of whom is assumed to be $1{,}175040$: $\eta = \frac{1{,}563840}{2{,}350080} = 0{,}665$.

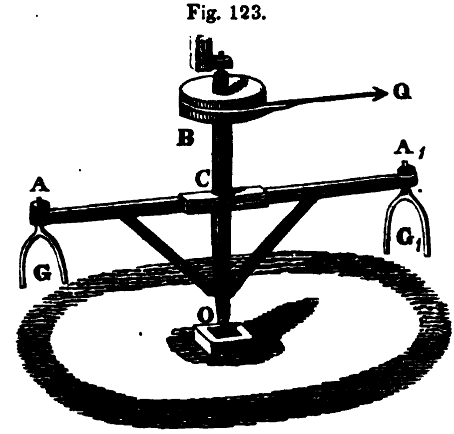
§ 59. *Vertical Capstan.*—When the axis or barrel of the windlass is vertical, it is termed a capstan, Fig. 122. It is chiefly used on land for moving great weights a short distance, or for removing great weights, as blocks of stone from a quarry, or for the erection of obelisks, &c. Its use on board-ship is well known.

Fig. 122.



The *horse-capstan*, in its different applications as the prime mover of mill work, or as a *whim-gin*, as it is termed, by miners, is a modification of the windlass easily comprehended. The cattle employed in working the vertical windlass or *gin*, go round in a given path, pushing or pulling at the arm of the machine. Fig. 123 shows the usual construction of the whim-gin (Fr. *baritel à chevaux manège;* Ger. *Pferdegöpel, Handgöpel*). BO is the axis, having a *pivot* at O resting in a *footstep*, ACA_1 is the double arm or lever, with fork-shaped *shafts* G, G_1. These cross the backs of the horses, and the harness is attached to them. The resistance Q acts at the circumference of a barrel or drum, or toothed wheel B, either directly or indirectly. The length of lever is made as great as conveniently can be done, that the animals may have the largest possible circle to move in. The radius should not be less than from 20 to 30 feet. The *line of traction* must be as nearly horizontal as possible, and, therefore, the height of the lever should be fixed, according to the height of the animals working on it. By the ar-

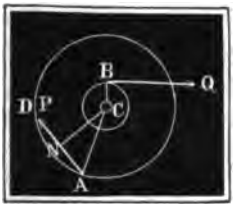
Fig. 123.



rangement shown in Fig. 123, the horses or other cattle work very nearly at right angles to the beam or lever; but if the horses be attached by traces to a cross bar and hook, the direction of traction makes a certain angle with the beam, becoming, in fact, a *chord* of the circular path.

From the length of beam $CA = a$, Fig. 124, and the length of traces $AD = d$, the length of levers of the horses is:

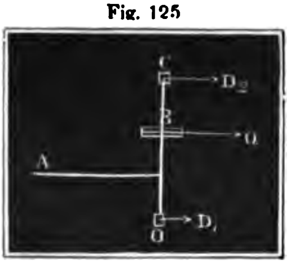
Fig. 124.



$$CN = a_1 = \sqrt{a^2 - \frac{d^2}{4}},$$

or, approximately, $= a - \frac{d^2}{8a}$. It is a result of experiment, that a man can work eight hours daily on the beam of capstan or gin, exerting an effort of 25½ lbs. at the rate of 1,9 feet per second, and can, therefore, produce a day's work $= 25.5 \,.\, 1{,}9 \,.\, 28800 = 1'395360$ feet lbs.; that, on the other hand, a horse working on a gin for 8 hours daily, with a speed of 2,9 feet per second (a walk) can exert an effort of 95 lbs., or produce a day's work $= 95 \,.\, 2{,}9 \,.\, 28800 = 7'934400$ feet lbs. The *power* is to the resistance, on the capstan or gin, as for any wheel and axle, or $P = Q \frac{b}{a}$, when b and a are the arms or leverages of the resistance Q, and power P respectively. The frictions at the footstep and at the periphery of the pivots at top and bottom have to be considered; for these require an increase of the power. If G be the weight of the gin or capstan complete, and r_1 the radius of the pivot, the statical moment of the friction on the footstep $= \frac{2}{3} f\, G\, r_1$, (Vol. I. § 171.) The point of application of the resistance B, Fig. 125, generally lies nearer the one pivot, than the other, and thus the pressure on the two is different, and their dimensions are, of course, proportional to the strain.

Fig. 125



If the point of application of the resistance be at the distance $BO = l_1$, from the pivot O, and $CB = l_2$ from the pivot C, and if the whole length of the upright shaft $CO = l = l_1 + l_2$, then the pressure on the lower pivot $D_1 = \frac{l_2}{l} Q$, and the pressure on the upper pivot $D_2 = \frac{l_1}{l} Q$, as is manifest if we first consider C and then O, as the fulcrums of the lever CBO. Thus, the sum of the statical moments of the lateral friction on the pivots $= f\, D_1 r_1 + f\, D_2 r_2 = \frac{r_1 l_2 + r_2 l_1}{l} \,.\, f\, Q$, and the equation of equilibrium for the gin, is

$$Pa = Qb + \tfrac{2}{3} f\, G r_1 + f\, Q \,.\, \frac{r_1 l_2 + r_2 l_1}{l}.$$

Remark 1. The application of the *whim gin*, for drawing from mines, is treated of in the third section.

Remark 2. French authors assert that a horse, going at a trot, can work daily $4\frac{1}{2}$ hours, exerting an effort of 30 kilog. = 66 lbs. at a speed of 2 metres = 6,6 feet, and, therefore, can produce a day's work of 7'055000 feet lbs. If we apply Gerstner's formula, and put $K = 120$ lbs., $c = 4$ feet, $v = 6,6$ feet, $t = 8$ hours, and $z = 4\frac{1}{2}$ hours, we get the power $P = \left(2 - \frac{6,6}{4}\right)\left(2 - \frac{4,5}{8}\right) . 120 = 60$ lbs., and, therefore, the day's work $= 60 . 6,6 . 4,5 . 3600 = 6'415200$ feet lbs., or pretty nearly the result alluded to. If, however, we take the velocity 2,9 feet of a walk as the basis, we get by Gerstner's formula a much greater effort, viz: $\left(2 - \frac{2,9}{4}\right) . 120 = 153$ lbs., and, therefore, the day's work (8 hours) = 12'778560 lbs.

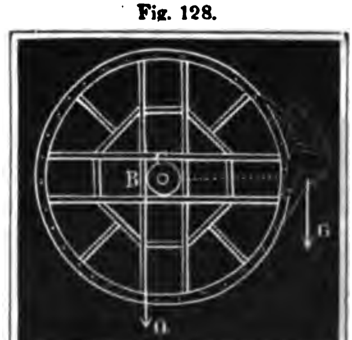
§ 60. *Tread-wheels*, or *Tread-mills.*—The weight of men and cattle is sometimes used as the moving power of machines, the effort being exerted by *climbing* on the periphery of the wheel. The wheels consist of two *crowns*, connected with an axle by arms, and with each other by a *flooring*. The laborer treads either at the internal or on the external circumference, cross pieces, as steps, being provided for his steadier support at intervals of $1\frac{1}{2}$ feet. Figs.

Fig. 126.

Fig. 127.

126 and 127 represent the more usual construction of tread-mills. Fig. 128 is a construction of wheel analogous to an endless ladder, but is not much used. On it, the laborer is placed at the level of the axis, so that his weight acts entirely, and with the radius $CA = a$ surpassing that of the wheel itself. In tread-mills the laborer is placed at an acute angle $ACK = \alpha$ from the summit, or the bottom of the wheel, and, therefore, the leverage of his weight G, is less than the radius of the wheel $CA = a$, viz: $CN = a_1 = CA \sin. CAN = a \sin. \alpha$. But then the fatigue of the laborer on the endless ladder is greater than on the tread-mill. In

Fig. 128.



the former case, it is the effort necessary to mount a vertical ladder; in the other, it is that for going up an inclination given by the tangent AT, making the angle $TAH = CAN = \alpha$. The effort P in the case of the ladder is, therefore, G, while in the tread-mill it is G $sin. \alpha$. If the resistance Q act with the leverage $CB = b$, then for the ladder-wheel $Ga = Qb$, while for the tread-mill $Ga \; sin. \alpha = Qb$, by substituting the power or effort, as in the wheel and axle; $Pa = Qb$. Mathematically considered, therefore, the tread-mill gives no advantage over the windlass or capstan; but the laborer can produce a much greater day's work by the one than by the other, and, therefore, they are often advantageously employed. The application of four-footed animals on these wheels is inconvenient, and not advantageous in any point of view.

It has been deduced from experiment that a man can work near the centre of the wheel, *i. e.*, near the level of the axis for 8 hours daily, exerting an effort of 128 lbs., and going at 0,48 feet per second, while he can work for the same time, exerting an effort of $25\frac{3}{8}$ lbs., and going at $2\frac{1}{4}$ feet per second, when his position is 24° from the vertical. In the one case, the day's work amounts to 1'769000 feet lbs., and in the other 1'663000 feet lbs. Horses and other cattle produce less effect on such machines than by means of a gin. A part of the advantage arising from the use of tread-wheels is lost in the increased friction of their axles beyond that of windlasses or capstan. If $n\,G$ be the weight of the laborers, G_1 the weight of the machine, and if the resistance Q act vertically downwards, the pressure on the journals $D = n\,G + G_1 + Q$, and if r be the radius of the journal, the moment of friction $= f\,(n\,G + G_1 + Q)\,r$, and the ratio of power to resistance is: $n\,G\,a \; sin. \alpha = Qb + f\,(n\,G + G_1 + Q)\,r$.

If the resistance be given, the angle of ascent may be determined, viz.:

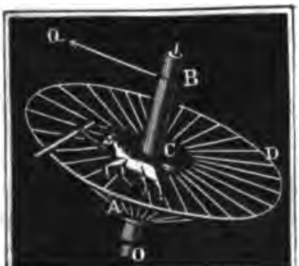
$$sin\, \alpha = \frac{Q\,b + f\,(n\,G + G_1 + Q)\,r}{n\,G\,a},$$

or the number of laborers

$$n = \frac{Q\,b + f\,(G_1 + Q)\,r}{G\,(a\; sin. \alpha - fr)}.$$

Men work to the greatest advantage when their effort: $n\,P = n\,G \; sin. \alpha = n\,K + \frac{b}{a} \cdot \frac{W}{2}$, or when $sin. \alpha = \left(K + \frac{b}{a} \cdot \frac{W}{2n}\right) + G.$

Fig. 129.



§ 61. *Movable Inclined Planes.*—For farming purposes, in breweries, &c., the arrangement sketched in Fig. 129, is sometimes applied. The horse or ox works on such an inclined plane for short spells. The machine has this advantage, that the animal may be left without a

driver. The action of the animals is in every respect the same as in tread-mills, when they work near the horizontal radius. The machine consists of a shaft BO, the axis of which is inclined 20° to 25° from the vertical, and of a plane, from 20 to 25 feet in radius, set at right angles to the shaft, and, therefore, having an inclination of 20° to 25° to the horizon. If the animal moving the machine work at a distance $CA = a$ from the axis of the shaft, and if the angle of inclination of the plane, or the inclination upon which the animal may be supposed to be moving $= \alpha$, then the power $P = G \sin. \alpha$, and, therefore, the moment of rotation $= Pa = Ga \sin. \alpha$.

If the resistance be applied with a leverage b, its moment is $Q\,b$; and if G_1 be the weight of the machine when in work, and r be the radius of the pivot, the moment of friction on the footstep $= \frac{2}{3} f (G + G_1) \cos. \alpha \,.\, r$, and the moment of friction on the periphery of the pivots $= f\,(G + G_1 - Q) \sin. \alpha \,.\, r$; because the weight $G + G_1$ resolves itself into the components $(G + G_1) \cos. \alpha$, in the direction of the axis, and $(G + G_1) \sin. \alpha$, in the direction of the inclination of the plane, whilst Q acts in the opposite direction to this latter. Whence follows

$$G\,a \sin. \alpha = Q\,(b - f\,r \sin. \alpha) + f(G + G_1)\,(\tfrac{2}{3} \cos. \alpha + \sin. \alpha)\,.\,r.$$

Example. How many men are required to be put upon a tread-mill of 20 feet diameter, in order to raise a weight of 900 lbs., acting with a leverage of 0,8 feet? If we estimate the weight of the wheel, and its load at 5000 lbs., and taking the radius of the pivot at $2\frac{1}{2}$ inches, and the co-efficient of friction at 0,075, then the statical moment of the resistance $= 0{,}8 \,.\, 900 + 0{,}075 \,.\, \frac{5}{24} \,.\, 5000 = 720 + 78 = 798$ feet lbs., and, therefore, the power at the circumference of the wheel $= \frac{798}{10} = 79{,}8$ lbs. A laborer placed 24° back from the summit of the wheel, exerts an effort of $25\frac{3}{4}$ lbs., and, therefore, the number of men required is $\frac{79{,}8}{25\frac{3}{4}} = 3$. These men could produce $3 \,.\, 1663000 = 4989000$ feet lbs. per day of 8 hours, and, therefore, they could raise the weight Q daily through $\frac{4989000}{900} = 5890$ feet high; or, supposing the load had to be raised only 200 feet high, the three men could raise $\frac{5980}{200} = 30$ times 900 lbs. to the height of 200 feet.

CHAPTER III.

ON COLLECTING AND LEADING WATER THAT IS TO SERVE AS POWER.

§ 62. *Water-conduits.*—Water that is to serve as power (Fr. *l'eau motrice;* Ger. *Aufschlagewasser*), to be applied to machines, is collected from streams and rivers, or from springs. In most cases the machines have to be erected at some distance from the point at which the water can be collected, and must be led to the machine in what is termed the *lead* or *lete*, or *water-conduit*, or *water-course*.

The lead may either be an open *channel* or canal (Fr. *canales,*

rigoles), or it may be a close pipe (Fr. *tuyaux de conduite;* Ger. *Röhrenleitungen*). Pipes are best adapted for smaller quantities of water. They have this great advantage, that they may be led in any way within the hydraulic range of variation of level, whilst canals as letes, must have a continuous fall. Valleys and hills may often be passed by pipes without trouble or expense, while the open channel requires the cutting of *drifts* or *tunnels*, and the erection of *aqueducts*.

§ 63. *Dams.*—The *vis viva* of running waters, of brooks and rivers having velocities of from 1 to 6 feet per second, is seldom sufficient to allow of their direct application as power to drive machines. To increase the *vis viva*, or to bring the weight of the water into action, it is necessary to *dam* it up to create a *head* or *fall* (Fr. *chute;* Ger. *Gefälle*). Water is dammed up by *weirs, dams*, or *bars* (Fr. *barrages;* Ger. *Wehre*).

Weirs are either *overfall weirs*, or they are *sluice weirs*. Whilst in the former the water flows freely over the *saddle-beam cill*, or highest edge of the weir, in the latter movable sluice-boards dam the water above the summit of a weir, which may be either natural or artificial. The overfall weir is usually laid down with the view of constraining a portion at least of the water of a river or stream to enter a side canal above it, or a lete by which it is conducted to the machine by which the power of the water is to be applied; and the sluice weir, is used when the object is to get an increased *vis viva* to the water, which is then directly applied to a machine immediately below the weir.

In large rivers, dams are frequently built to occupy only a part of the width of the stream. These dams are termed *incomplete* wiers, in contradistinction to *complete* weirs, which are laid from side to side of the stream. The piers of bridges are examples of incomplete weirs (Fr. *barrage discontinus;* Ger. *Lichte Wehre*), contracting the passage for the stream to a certain extent.

Overfall weirs, too, may either be complete or imperfect. The summit of the complete overfall rises above the surface of the water in the part of the stream below it, whilst the top of the incomplete weir lies below that level, so that a part of the water flowing over undergoes a resistance from the water *below-weir*.

§ 64. *Swell*, or *Back-water.*—Any of the constructions we have above alluded to, *dam back* the water, produce a *swell* above the weir, an elevation of the water's surface, and, therefore, a decrease of velocity. The height and amplitude or extent backwards to which this rise of the water surface extends, is a matter important to be determined with reference to the dimensions of the weir.

A knowledge of this relation between the weir, and its effects on the river above it, is not only necessary because by damming up the water too high, we should involve the district above in *floods* to which they had not been previously subjected, but we may interfere with other establishments, robbing them of a part of their fall, by throwing *back-water* upon them. The level of the summit of weirs is often

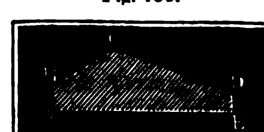
fixed by law or prescription according to a standard peg, or mark; any alteration of which is an offence liable to penalties (see, in reference to the English law on this subject, "Fonblanque on Equity"). The peg generally has a *scale* attached to it, by which the supply of water may be read off at a glance.

The water flowing over an overfall, or through an incomplete weir, acquires a waving eddying motion, the action of which is very severe on the bed of the river immediately below the weir, so that particular arrangements have to be made in the erection of weirs to withstand this action.

The quantity of water contained in or flowing through streams or rivers, is different at different times, so that we have the expressions *full*, *average*, and *dry*, applied to the *state* of rivers, corresponding in Britain, to winter, autumn, and summer, though not very definitely fixed as to the particular period of the seasons. It is evidently necessary to have accurate information as to the mean supply of water yielded by a brook or stream, proposed to be applied as water power. The state of the stream in autumn and spring may be taken as the *mean state*, but for any important undertaking of this nature a series of hydrometrical observations should be instituted, that the question of the supply of water may be accurately determined. Any one of the methods discussed in Vol. I. § 376, &c., may be adopted for this purpose.

§ 65. *Construction of Weirs.*—For obtaining water power, the *overfall* weir is the most important means. Weirs are built either square across the stream, or inclined to the axis of it. They are often built in two parts inclined to each other, the angle, which is laid up-stream, being rounded or not; they are formed as polygons also, and as segments of a circle, the convexity being always turned *to the* stream. Weirs are built of wood, or of stone, or of both combined. They have frequently to be founded on piles, from the difficulty of getting a sound foundation. The cross section of wooden, or other dams, is more or less of the form of a five-sided figure *ABCDE*, Fig 130, in which *AB* is termed the *breast*, *BC* the *front slope*, *CD the apron*, *DE* the *back*, and *EA* the sole, and *C* the saddle or cill. The cross section of stone weirs is generally composed of curved lines, as regards the apron and back, the object being to get the rush of water smoothly away from the foot of the apron, so as to prevent corrosion in time of floods.

Fig. 130.

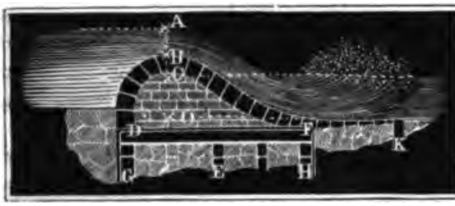


An overfall weir, such as is represented in Fig. 131, consists of a row of piles *D*, going across the stream, and

Fig. 131.

a walling-piece, or *saddle-beam* C on the top—of walling E in front of the piles—a second row of piles F further down-stream and parallel to the first—of a casing of hard laid pavement G, between the two, and which is continued onwards with the same curvature, forming an apron (which should be continued so that it turns slightly *upwards*). The weir in Fig. 132, shows the manner of founding on piles, the intervals between the piles being cleared out as far as possible, and rammed with concrete, and upon this the superstructure is raised.

Fig. 132.



The construction of wooden wiers is sketched in Fig. 133. AB is a wall of beams, lying tight, one on the other, on the top of which comes the saddle-beam A. These beams are confined by a double row of piles CD and C_1D_1, and the piles EF and GH, driven as breast and back of the dam, form resting points for the planking of the dam. The interior of the dam is filled with stone, clay, concrete, or such material. The apron K of the dam is continued onwards as substantially as possible, in the manner shown in the sketch. This latter is a point of great importance. At L the sluice of the lete is visible. A submerged weir is shown in Fig. 134. A is the saddle-beam, AB are the guide-columns, in grooves, in which the sluice works. The arrangements for raising or *opening* and lowering, or *shutting* the sluice are various. A capstan-like arrangement is shown in the figure, the

Fig. 133.

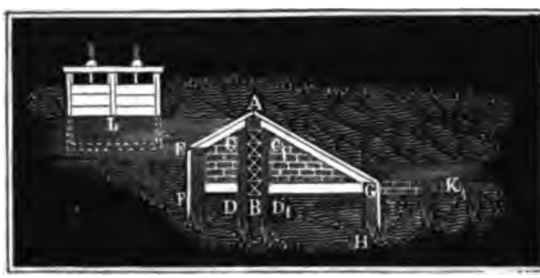


Fig. 134.

sluice-board hanging by chains. The piles in such a construction must be cleared for some depth, and the interstices well rammed with *puddle* or concrete, to prevent leakage.

§ 66. *Height of Swell.*—By aid of the hydraulic formulas we have investigated (Vol. I.), the height and amplitude of the *back-water* for any given dam may be easily determined. If, in the case of a dam represented in Fig. 135, h be the head AB, and if b, be the breadth, and k, the height due to the velocity c of the water as it flows up to the weir, or:

Fig. 135

$k = \frac{c^2}{2g}$, then the quantity of water discharged by the weir is (Vol. I. § 321) $Q = \frac{2}{3}\,\mu\, b\,\sqrt{2g}\,[(h + k)^{\frac{3}{2}} - k^{\frac{3}{2}}]$. If, on the other hand, the quantity discharged be known, the *head* corresponding to it upon the saddle-beam: $h = \left(\frac{\frac{3}{2}\,Q}{\mu b\,\sqrt{2g}} + k^{\frac{3}{2}}\right)^{\frac{2}{3}} - k$. In order, therefore, to give the height $BO = x$ of a weir to produce a given *head*, or rise of the water surface at the weir $= h_1$, we put $AC + CO = AB + BO$, or, if the original depth of the water *down-stream* CO be put $= a$, then $h_1 + a = h + x$, and hence $x = a + h_1 - h$.

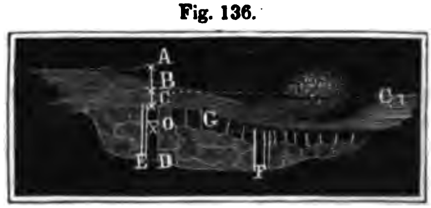
When the back-water or head raised is considerable, say $x =$ at least 2 feet; the velocity of the water, as it comes to the weir k, may be neglected, and, therefore, we may put:

$$x = a + h_1 - \left(\frac{\frac{3}{2}\,Q}{\mu\, b\,\sqrt{2g}}\right)^{\frac{2}{3}},$$

and according to experiments of the author, the co-efficient μ may be taken $= 0{,}80$ for this case.

In the case of the submerged weir, Fig. 136, the calculation is somewhat more complicated, because in this case two different discharges are combined. The height $AC = h$ of the water above the saddle-beam is greater in this case than the height $AB = h_1$, to which the water is raised by the dam, and, therefore, only the water above the level B flows away freely, whilst the water under B flows away under the head or pressure $AB = h_1$. The discharge through

Fig. 136.



$$AB = Q_1 = \tfrac{2}{3}\,\mu\, b\sqrt{2g}\,[(h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}],$$

and that through $BC = h - h_1$, is:

$$Q_2 = \mu b (h - h_1) \sqrt{2g} (h_1 + k)^{\frac{1}{2}},$$

and consequently the whole quantity, or,

$$Q_1 + Q_2 = Q = \mu b \sqrt{2g} \left[\tfrac{2}{3} [(h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}] + (h - h_1)(h_1 + k)^{\frac{1}{2}}\right].$$

From the quantity of water Q, and the height h_1 to which the water is raised, we have the height of water above the saddle:

$$h = h_1 + \frac{Q}{\mu b \sqrt{2g (h_1 + k)}} - \tfrac{2}{3} \cdot \frac{(h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}}{(h_1 + k)^{\frac{1}{2}}},$$

from which the height of weir $CO = x = a + h_1 - h$ may be deduced. It is evident that $h > h_1$ or the weir, is a submerged or imperfect weir, when

$$Q > \tfrac{2}{3} \mu b \sqrt{2g} [(h_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}].$$

Example. A stream of 30 feet width, and 3 feet in depth, discharges 310 cubic feet of water per second. It is required to raise it $4\frac{1}{2}$ feet by means of a weir. What height of weir is necessary? As the height of the water to be raised is considerable in this case, we may confidently use the simpler formula:

$x = a + h_1 - \left(\frac{3Q}{2 \mu b \sqrt{2g}}\right)^{\frac{2}{3}}$. In this formula $a = 3$, $h_1 = 4{,}5$, $Q = 310$, $b = 30$, $\mu = 0{,}80$, and $\sqrt{2g} = 8{,}02$ for the case in question. Hence:

$x = 3 + 4{,}5 - \left(\frac{3 \cdot 10}{2 \cdot 0{,}8 \cdot 30 \cdot 8{,}02}\right)^{\frac{2}{3}} = 5{,}7$ feet; and, therefore, the overfall is a *perfect* weir, as was presumed. If it were required to raise the water up only 2 feet, x would be 3,2 feet, or the weir would still be *perfect*. If $1\frac{1}{2}$ feet only were required, the dam would not require to rise above the level of the water down-stream, or the natural level of the water in the stream; and would be a submerged weir. Applying the complete formulas to this case, and putting

$$k = \frac{c^2}{2g} = 0{,}0155 \left(\frac{Q}{(h + h_1) b}\right)^2 = 0{,}0155 \left(\frac{310}{4{,}5 \cdot 30}\right)^2$$

$= 0{,}0155 \cdot 5{,}27 = 0{,}084$ feet, and taking μ again $= 0{,}80$ we get:

$$h - h_1 = \frac{310}{0{,}8 \cdot 30 \cdot 8{,}02 \sqrt{1{,}584}} - \tfrac{2}{3} \cdot \frac{(1{,}584)^{\frac{3}{2}} - (0{,}084)^{\frac{3}{2}}}{1{,}584^{\frac{1}{2}}}$$

$$= 1{,}28 - 1{,}06 + 0.01 = 0{,}23 \text{ feet.}$$

The saddle overfall must, therefore, be about $\frac{1}{4}$ foot, or 3 inches under the surface of the water on the lower side of the weir, and, therefore, the height of the weir itself $x = a + h - h_1 = 3{,}25$ feet.

§ 67. The height and amplitude of the *back-water* in the case of sluice weirs may be determined according to the theory of the discharge by sluices. Three cases may occur. Either the water flows away unimpeded, or it flows under a counter pressure of water, or it flows partly unimpeded, partly under water. In the case of a free discharge, as in Fig. 184, the velocity of discharge depends upon h above, measured from the centre of the opening to the water's surface. If, then, a be the height of opening, and b the breadth, then $Q = \mu a b \sqrt{2gh}$, and, therefore, inversely

$h = \frac{1}{2g}\left(\frac{Q}{\mu ab}\right)^2$, or, taking into consideration the velocity k with which the water comes up to the sluice,

$h = \frac{1}{2g}\left(\frac{Q}{\mu ab}\right)^2 - k$. For the height of opening, we have the formula:

$a = \frac{Q}{\mu b \sqrt{2gh}}$, or, if h_1 = the height to which the water is raised by the dam above the sill, be given:

$a = \frac{Q}{\mu b \sqrt{2g\left(h_1 - \frac{a}{2}\right)}}$. According to the author's experiments μ is here = .60.

If the under-water lie back to the sluice, as in Fig. 137, then the difference of level $AB = h$, is the *head* to be introduced as pressure in the above formula. In this case, therefore, the opening corresponding to a given head h is : $a = \frac{Q}{\mu b \sqrt{2gh}}$.

When the level of the under-water is within the range of the

Fig. 137.

Fig. 138.

sluice's opening, as shown in Fig. 138, one part flows away unimpeded, whilst the other flows under water. If the height of the water is raised, or the difference of level AB, Fig. 138 = h the height BC of the part of orifice of discharge above the surface of the water $= a_1$, and BD the height of the part under this surface $= a_2$, then the quantity of water for the former part:

$Q_1 = \mu\, a_1\, b \sqrt{2g\left(h - \frac{a_1}{2}\right)}$, and for the other:

$Q_2 = \mu\, a_2\, b \sqrt{2gh}$, therefore, the whole quantity:

$$Q = Q_1 + Q_2 = \mu b \sqrt{2g}\left(a_1 \sqrt{h - \frac{a_1}{2}} + a_2 \sqrt{h}\right).$$

From the quantity of water discharged Q, the height to which the water is raised h, and the depth a_2 of the sill or saddle of the weir under the under-water surface, we deduce the distance of the sluice-board from this surface:

$$a_1 = \left(\frac{Q}{\mu b \sqrt{2g}} - a_2 \sqrt{h}\right) : \sqrt{h - \frac{a_1}{2}}.$$

Example 1. How high must the boards of the sluice weir, Fig. 134, be raised, which has to let off 250 cubic feet of water per second, the breadth b being = 24 feet, and the height h_1, to which the water is dammed above the sill = 5 feet? In the case of unimpeded discharge:

$$a = \frac{250}{0{,}6 \,.\, 24 \,.\, 8{,}02 \sqrt{5 - \frac{a}{2}}} = \frac{2{,}16}{\sqrt{5 - \frac{a}{2}}},$$

approximately: $a = 1$, hence: $\sqrt{5 - \frac{a}{2}} = \sqrt{4,5} = 2,12$, therefore, the height of opening required: $a = \frac{2,16}{2,12} = 1,02$ feet $= 12,24$ inches.

Example 2. What amount must the sluice, Fig. 137, be drawn up, in order that it may discharge 120 cubic feet of water per second, under a head of 1,5 feet, the width of opening being 30 feet? This is a case of discharge under water, therefore,

$$a = \frac{120}{0,6 \cdot 30 \cdot 8,02\sqrt{1,5}} = 0,678 \text{ feet} = 8,14 \text{ inches.}$$

Example 3. It is required to determine the quantity of water which flows through a sluice opening (Fig. 138) of breadth $b = 18$ feet, height $CD = a_1 + a_2 = 1,2$ feet, when the head $AB = 2$ feet $= h$, and the height of water above the sill, $a_2 = 0,5$ feet. In this case $\mu b \sqrt{2g} = 0,6 \cdot 18 \cdot 8,02 = 86,6$. Further $a_2 \sqrt{h} = 0,5\sqrt{2} = 0,707$, and $a_1 \sqrt{h - \frac{a_1}{2}} = 0,7\sqrt{1,65} = 0,899$, therefore, the quantity of water required $Q = 86,6\,(0,707 + 0,899) = 86,6 \cdot 1,606 = 139,07$ cubic feet.

§ 68. *Discontinuous Weirs.*—The height of the back-water in the case of incomplete or discontinuous weirs, such as piers of bridges, jetties, &c., may be calculated in very much the same way as that for overfalls. For the jetty BE, Fig. 139, there results a damming back of the waters, because the stream is contracted from the width AC to AB. If, therefore, the *lead* be closed, which it is well to

Fig. 139.

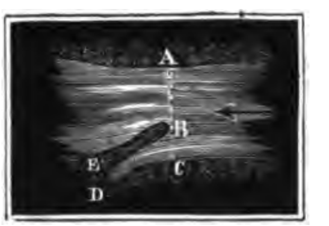


Fig. 140.

assume, the whole of the water of the stream Q must pass through the contracted passage AB. If we put the width $AB = b$, the height of dammed water $= AB_1 = h$, Fig. 140, and the depth B_1C_1 of the under-water $= a$, then the quantity flowing freely above the under-water is $Q_1 = \frac{2}{3}\mu b \sqrt{2gh^3}$, and the quantity flowing away as under-water $= Q_2 = \mu b a \sqrt{2gh}$. Therefore, the whole quantity going away: $Q = \mu b \sqrt{2gh}\,(\frac{2}{3}h + a)$. Hence, inversely, the breadth of weir corresponding to given height h of dammed water, is $b = \frac{Q}{\mu(\frac{2}{3}h + a)\sqrt{2gh}}$. If the height of back-water h, be small, or the velocity of the water great, the velocity of the water as it comes up to the jetty, must be taken into consideration. If k be again taken to represent the height due to the velocity of the water as it comes to the weir, we have:

$$Q_1 = \frac{2}{3}\mu b \sqrt{2g}\left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right], \text{ and } Q_2 = \mu b a \sqrt{2g(h+k)},$$

and, therefore:

$$Q = \mu b \sqrt{2g}\left[\tfrac{2}{3}\left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right] + a(h+k)^{\frac{1}{2}}\right],$$

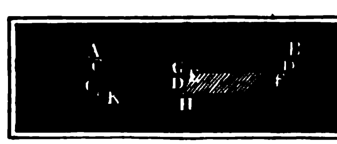
and inversely:

$$b = \frac{Q}{\mu \sqrt{2g}\left[\tfrac{2}{3}\left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right] + a(h+\ )^{\frac{1}{2}}\right]}.$$

Whilst in the unimpeded motion of water in river channels, the velocity is greatest at the surface, and decreases gradually as we go downwards in the vertical depth, the case is different when the water is dammed up by any obstruction in the stream. Then the velocity *increases* from the surface of the upper-water down to that of the under-water, and diminishes very little from thence downwards to the bottom. There is, therefore, a change of velocity as represented by the arrows in Fig. 140. This must necessarily be the case, because the water *above* the under-water surface flows away under a pressure or head increasing from O to h, and the water *under* it, flows away under the constant pressure h, whilst for unimpeded motion, the pressure or head at all depths $= 0$. This formula is likewise applicable in the case of bridge piers, if b be put to represent the sum of the openings between the piers. In order to prevent as much as possible injurious effects from the eddying motion of the water behind and in front of the piers, the *starlings* are added, presenting a rounded or angled *prow* to the water. If the starling of the piers be round, or form a very obtuse angle, then μ is to be taken $= 90$, if the angle be acute $\mu = .95$, and if the acute angle be formed by the meeting of two elliptical or circular arcs, as in Fig. 141, μ becomes even .97, or very nearly 1.

Fig. 141.

Fig. 142.



Remark. If a jetty, or other building contracting a stream, does not reach above the surface, the whole quantity of water Q may be considered as composed of 3 parts. If the top of the construction be beneath the under-water surface CD, Fig. 142, then the quantity of water flowing away through the section $ABDC$, is:

$$Q_1 = \tfrac{2}{3}\mu b \sqrt{2g}\left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right].$$

h being the height of the back-water, and b the breadth AB.

Secondly, the remaining part above the top of the building, and under the constant head h_1 or $Q_2 = \mu b_1 (a - a_1)\sqrt{2g(h+k)}$, where $a = GH$ the depth of under-water, $b_1 =$ the breadth EF of the building, and $a_1 =$ its height EH.

Lastly, the part flowing away at the end of the building under the constant head h, is $Q_3 = \mu b_2 a \sqrt{2g(h+k)}$, b_2 being the free width CD. Thus:

$Q = \tfrac{2}{3}\mu b \sqrt{2g}\left[(h+k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right] + \mu\left[b a - b_1 a_1\right]\sqrt{2g(h+k)}$, and, therefore, we can calculate the length and height of building necessary to produce a given amount of dam. If, on the other hand, $C_1 D_1$ be the under-water surface, or if the construction reach above the surface,

$$Q = \tfrac{2}{3} \mu b_1 \sqrt{2g} \left[(a + h - a_1 + k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right]$$
$$+ \tfrac{2}{3} \mu b_2 \sqrt{2g} \left[(h + k)^{\frac{3}{2}} - k^{\frac{3}{2}}\right] + \mu\, a\, b_2 \sqrt{2g\,(h + k)}.$$

Example. What width BC must be given to the dam BE, Fig. 139, in order that the river, which is 550 feet wide, and 8 feet deep, and delivers 14000 cubic feet of water per second, may be dammed up 0,75 feet?

$$k = 0{,}0155 \left(\frac{14000}{550\,.\,8}\right)^2 = 0{,}0155\,.\,3{,}18^2 = 0{,}156,$$

and if $\mu = 0{,}9$, then the width of the *contracted stream*:

$$b = \frac{14000}{09\,.\,8{,}02\left[\tfrac{2}{3}\left(0{,}906^{\frac{3}{2}} - 156^{\frac{3}{2}}\right) + 8\,.\,0{,}906^{\frac{1}{2}}\right]},$$

$$= \frac{14000}{7{,}218\,(0{,}522 + 7{,}608,)} = \frac{14000}{7{,}218\,.\,8{,}13} = 238{,}5 \text{ feet},$$

and, therefore, the length or projection of the dam $= 550 - 238{,}5 = 311{,}5$ feet.

§ 69. ***Amplitude of the Back-water.***—We have now to resolve the other important question. According to what law does the height of the dammed water diminish in stretching back, up stream? Without having resort to any peculiar theory, this problem can be solved by the theory of the variable motion of water in river channels, explained Vol. I. § 869, § 370.

Let us suppose the length of river on which back-water from the dam ABK, Fig. 143, is perceptible, divided into separate lengths,

Fig. 143.

and let us submit each length separately to calculation. If a_0 be the depth of water AB at the weir, a_1 the depth DE at the upper end of such a length; $ABDE$, F_0 the section of the flowing water at the weir, F_1 the section at DE, Q the quantity of water, p the mean circumference of the section for this length, and a the angle of inclination of the river's bed, then, from (Vol. I. § 370) the length of the first division, (a_0 and a_1, and F_0 and F_1 being substituted for each other) is:

$$l = \frac{a_0 - a_1 - \left(\frac{1}{F_1^2} - \frac{1}{F_0^2}\right)\frac{Q^2}{2g}}{\sin.\, a - \zeta\,.\,\frac{p}{F_0 + F_1}\left(\frac{1}{F_0^2} + \frac{1}{F_1^2}\right)\frac{Q^2}{2g}}.$$

If a_2 be the depth of water GH at the upper end of a second length $DEGH$, F_2 its section, and p_1 the mean perimeter of the water section of this part, then its length

$$DH = l_1 = \frac{a_1 - a_2 - \left(\frac{1}{F_2^2} - \frac{1}{F_1^2}\right)\frac{Q^2}{2g}}{\sin.\, a = \zeta\frac{p_1}{F_1 + F_2}\left(\frac{1}{F_1^2} + \frac{1}{F_2^2}\right)\frac{Q^2}{2g}}.$$

Continuing in this manner, namely, assuming arbitrary decreases of depth $a_0 - a_1$, $a_1 - a_2$, $a_2 - a_3$, &c., and calculating from this the sections F_1, F_2, F_3, &c., and the mean perimeters, we get by the formula, the distances l, l_1, l_2, corresponding, or the distances l, $l + l_1$, $l + l_1 + l_2$, &c., from the weir.

To find the depth y corresponding to a given distance x, we may either apply the method of interpolation to the values l, $l + l_1$, $l + l_1 + l_2$, &c., just found, or we may make use of this other formula, likewise given, Vol. I. § 370, viz.:

$$a_0 - a_1 = \frac{\left(\sin.\, \alpha - \zeta \,.\, \frac{p_0}{a_0 b_0} \,.\, \frac{v_0^2}{2g}\right)}{1 - \frac{2}{a_0} \,.\, \frac{v_0^2}{2g}} l.$$

If we put in this instead of b_0, the breadth, and instead of p_0, the perimeter, and for v_0 the velocity at the weir, this formula gives the decrease $(a_0 - a_1)$ of the height of back-water on the first short length l, and for a next following short length l_1 this decrease is:

$$a_1 - a_2 = \frac{\left(\sin.\, \alpha - \zeta \,.\, \frac{p_1}{a_1 b_1} \,.\, \frac{v_1^2}{2g}\right)}{1 - \frac{2}{a_1} \,.\, \frac{v_1^2}{2g}} l_1, \text{ \&c.,}$$

and, lastly, for a given distance $l + l_1 + l_2 + \ldots$ the depth: $a_0 - (a_0 - a_1) - (a_1 - a_2) - \ldots$ may be calculated.

Example 1. A weir is to be built in a river 80 feet wide, 4 feet deep, and discharging 1400 cubic feet per second, in order to dam up the water 3 feet high. Required the relative amount of damming, at distances back from the weir. Without the dam, the velocity of the water $c = \frac{1400}{8\,.\,04} = 4{,}375$ feet, and, therefore, according to the table, Vol. I. p. 447, the co-efficient of resistance $\zeta = 0{,}00747$, and the inclination of the channel $\sin.\, \alpha = 0{,}00747 \,.\, \frac{p}{F} \,.\, \frac{c^2}{2g}$. If, therefore, $p = 84$, $F = 80\,.\,4 = 320$, $c = 4{,}375$, and $\frac{1}{2g} = 0{,}0155$, then the inclination:

$$\sin.\, \alpha = 0{,}00747 \frac{84}{320} \,.\, 0{,}0155\, (4{,}375)^2 = .0{,}0005818, \text{ or, } .00058,$$

near enough. The depth of water close to the weir is $4 + 3 = 7$ feet, and we shall now determine the distances at which the depths $6\frac{1}{2}$, 6, $5\frac{1}{2}$, and 5 feet occur. If we first introduce into the formula:

$$l = \frac{a_0 - a_1 - \left(\frac{1}{F_1^2} - \frac{1}{F_0^2}\right)\frac{Q^2}{2g}}{\sin.\, \alpha - \zeta . \frac{p}{F_0 + F_1}\left(\frac{1}{F_0^3} + \frac{1}{F_1^3}\right)\frac{Q^2}{2g}},\ a_0 - a_1 = 0{,}5,\ F_0 = 80\,.\,7 = 560,$$

$F_1 = 80\,.\,6{,}5 = 520$, $Q = 1400$, $\sin.\, \alpha = .00058$, $p = 86$, and then for the mean velocity $= \frac{2\,Q}{F + F_1} = \frac{2800}{1080} = 2{,}59$ feet, $\zeta = .0075$, the value of

$$l = \frac{0{,}5 - (0{,}0000036982 - 0{,}0000031888)\,.\,30434}{0{,}00058 - 0{,}0075 \frac{86}{1080}\,(0{,}0000036982 + 0{,}0000031888)\,.\,30434}$$

$$= \frac{0{,}5 - 0{,}0155}{0{,}00058 - 0{,}000128} = \frac{0{,}4845}{0{,}000452} = 1071 \text{ feet.}$$

To find the distance back at which a depression of 1 foot in the water's surface occurs, we must again put $a_0 - a_1 = 0{,}5$, but $F_0 = 520$, $F_1 = 80\,.\,6 = 480$, $p = 85{,}5$ and the

mean velocity $\frac{2800}{1000} = 2,80$ gives $\zeta = 0,00749$. Hence, by means of the same formula as above, we get for the distance in which the surface lowers, so that the depth becomes 6 feet instead of 6,5,

$$l = \frac{0.5 - 0,0000006421 \,.\, 30434}{.00058 - .00749 \frac{85,5}{1000} .0,0000080385 \,.\, 30434} = \frac{0,4845}{.000424} = 1142 \text{ feet.}$$

The water at a distance $1071 + 1142 = 2213$ feet, is, therefore, only 6 feet deep, or the height of the back-water is 2 feet. If, again, we put $a_0 - a_1 = 0,5$, and $F_0 = 480$, $F_1 = 440$, $p = 85,1$, and $\zeta = 0,00749$, then $l = 1205$ feet, and for a further depression of 0,5 feet, $l = 1413$ feet, so that at 2213 feet + 1203 feet + 1413 feet = 4829 feet back from the weir there is still a rise of 1 foot, occasioned by it. For the $4\frac{1}{2}$ feet deep length $l = 1922$ feet, for $4\frac{1}{4}$ feet, $l = 1584$ feet, and for 4,1 feet, $l = 1850$ feet, so that there is still a difference of $\frac{1}{10}$th of a foot at a distance $4829 + 1922 + 1584 + 1850 = 10185$ feet back from the weir, and diminishes upwards; but for 4 feet, or *complete cessation of back-water*, $l = \infty$ by our formula.

Example 2. Required the height of the back-water at the distance 2,500 feet back from the weir of the last example. According to the calculations above, there is a rise of 2 feet at 2122 feet above the weir, and the question, therefore, is, how does the rise diminish in the distance $2500 - 2122 = 378$ feet? The distance back from the 6 feet depth at which a further reduction of 0,5 feet takes place, has been found above to be 1205 feet. Therefore, for each foot a depression of $\frac{0,5}{1205}$ feet, so that for 377 feet, we should have $\frac{0,5 \,.\, 378}{1205} = 0,157$ feet, and, therefore, the rise of the back-water at 2,500 feet back from the weir is $2 - 0,157 = 1,843$ feet, and, therefore, the depth of water $= 5,843$ feet. If we calculate according to the second formula:

$$a_0 - a_0 = \frac{\left(sin.\, a - \zeta \,.\, \frac{p_0}{a_0 b_0} \,.\, \frac{v_0^2}{2g}\right)}{1 - \frac{2}{a_0} \,.\, \frac{v_0^2}{2g}} \, l,$$ and if we put into this $l = 800$, $p_0 = 86$, $a_0 = 7$, $a_0 b_0 = 560$, $v_0 - \frac{1400}{560} = 2,5$, and $\zeta = 0075$, we get the depression corresponding $= 0,399$ feet, and if we again put $l = 800$, $p_0 = 85,8$, $a_0 = 7 - 0,399 = 6,601$, $a_0 b_0 = 528$, $v_0 = \frac{1400}{528} = 2,652$, and $\zeta = .0075$, the depression is found to be 0,383 feet. Continuing in this manner, but setting l this time $= 900$, $p_0 = 85,5$, $a_0 = 6,601 - 0,383 = 6,218$, $a_0 b_0 = 497,44$, $v_0 = \frac{1400}{497,44} = 2,88$, and $\zeta = 00749$, we get the depression $a_0 - a_1 = 0,403$ feet, so that for $800 + 800 + 900 = 2,500$ feet back from weir, the depth of water is $6,218 - 0,403 = 5,818$ feet, and the height of the back-water here is 1,815 feet. The first method gave 1,843, so that the difference in the results of the two methods is not quite $\frac{1}{4}$ of an inch.

§ 70. ***Back-water Swell.***—If we consider somewhat closely the equation of the *curve* of the back-water, that is, of its longitudinal section, viz:

$$a_0 - a_1 = \left(\frac{sin.\, a - \zeta \,.\, \frac{p}{F} \,.\, \frac{v^2}{2g}}{1 - \frac{2}{a} \,.\, \frac{v^2}{2g}}\right) l,$$

we discover several interesting circumstances in reference to the back-water. In the fraction:

$$\frac{sin.\, a - \zeta \,.\, \frac{p}{F} \,.\, \frac{v^2}{2g}}{1 - \frac{2}{a} \,.\, \frac{v^2}{2g}}$$

the numerator and denominator become more nearly equal to 0, the greater the velocity v, and according as the one or the other first becomes 0, we have:

$$l = \frac{(a_0 - a_1)\left(1 - \frac{2}{a} \cdot \frac{v^2}{2g}\right)}{0} = \infty \text{ or } l = \frac{(a_0 - a_1) \cdot 0}{sin.\ \alpha - \zeta \cdot \frac{p}{F} \cdot \frac{v^2}{2g}} = 0.$$

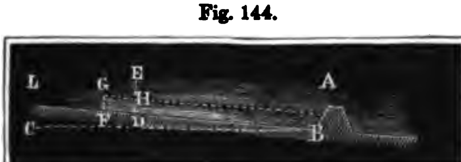
We perceive from this that when the numerator becomes $= 0$, the division l, or the limit of the back-water becomes infinitely distant, and in the case of the denominator becoming $= 0$, the length $l = 0$, or there is no back-water. Now the numerator becomes $= 0$, when $\zeta \cdot \frac{p}{F} \cdot \frac{v^2}{2g} = sin.\ \alpha$, or, when the velocity of the dammed water differs in an infinitely small degree from the velocity $v = \sqrt{\frac{2g\ F\ sin.\ \alpha}{\zeta\ p}}$ of the uniformly flowing water of the stream, and the denominator becomes $= 0$, when:

$$\frac{2}{a} \cdot \frac{v^2}{2g} = 1, \text{ or, } \frac{v^2}{2g} = \frac{a}{2},$$

that is, when the height due to the velocity = half the depth of the stream.

When the height due to the velocity of the water before the introduction of a weir, is less than half the depth of the undammed water, the back-water takes the form shown in Fig. 143, *and if the height due to the velocity be greater than half the depth, the back-water has the form* Fig. 144, there being a rise or swelling at the point EG.

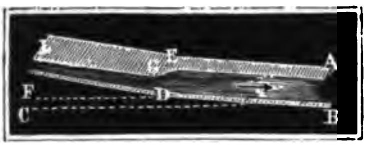
Fig. 144.



If in the equation $sin.\ \alpha = \zeta \frac{p}{F} \cdot \frac{v^2}{2g}$, we put $\frac{v^2}{2g} = \frac{a}{2}$ $F = ab$, and p (though it be only approximately) $= b$, we have: $sin.\ \alpha = \frac{1}{2}\ \zeta$. Thus the circumstances represented in Fig. 144, are likely to occur when the fall or inclination of the stream α, is greater than $\frac{1}{2}$ the co-efficient of resistance $\zeta = .0075$, that is when $\alpha > .00375$, or $\alpha > \frac{3}{800}$, or 1 in 266. As rivers and water-courses have generally a less inclination than this, the sudden depression EG, Fig. 144, is eldom observable in them.

Remark 1. This sudden depression of the back-water was first observed by Bidone, in a 12 inch wide trough, in which α was $= 0{,}033$. The same appearance is manifested when the inclination of the channel changes, as shown in the Fig. 145. If the degree of inclination of the upper part be greater than $\frac{1}{2}\ \zeta$, and inclination of the lower part

Fig. 145.



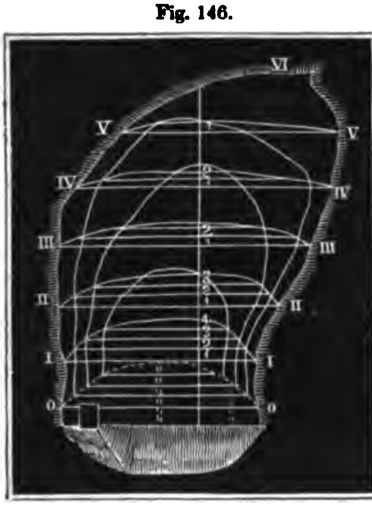
less, there is formed at the point of change a *swell*, or sudden rise where the less depth corresponding to the greater inclination, passes into the greater depth corresponding to the less inclination.

Remark 2. Saint-Guilhem has given an empirical equation for the curve of the back-water, but the author has given one more simple and accurate in the "*Allgemeinen Maschinen Encyclopädie*," article "Bewegung des Wassers."

§ 71. *Reservoirs.*—In districts where the supply of water is small, but where powerful machines are nevertheless required, as in mining districts generally, the construction of reservoirs (Fr. *étangs ;* Ger. *Teichen*), or large artificial ponds, that fill during seasons of rain, and supply the demands of drier seasons, is a matter of practical importance. The site to be chosen for a reservoir is regulated by a variety of circumstances. The main question is that of the relative level of the machines to which the water is to be applied. This being satisfied, they are most advantageously placed in a deep *dean*, or part of the valley where they can collect, not only the rain-water, but the streamlets and springs of as large a surrounding district as possible. In such a situation a single dyke or dam going square across the valley is sufficient to *enclose* the reservoir. The shorter the dyke, and the less the superficial area of a reservoir for a given cubical contents, the better. The steeper the banks, therefore, the more economically a reservoir is formed. The lower the level of the reservoir compared to the surrounding district, the greater supply of water may be led into it, or will flow to it naturally.

In selecting the site for a reservoir, great attention must be paid to the nature of the bottom, that is, its impermeability must be thoroughly ascertained; also its fitness for bearing the weight of the dyke or dam. Artificial puddling is, of course, a resource available in many cases; but for very extensive reservoirs, it is a precarious and expensive remedy for want of natural impermeability. Fissures in rocks, deposits of sand and gravel, morasses or bogs are to be avoided by all means.

Fig. 146.



Remark. On this subject, see Smeaton's "Reports," Sganzin, "*Cours de Construction*," and Hagen "*Wasserbaukunst*."

The value of a reservoir depends chiefly on its superficial and cubical contents. For ascertaining these, an accurate survey is necessary. The points I, II, III, &c., of Fig. 146, are laid down from a

survey with the chain or theodolite, and *cross sections* are then taken by leveling (and sounding, when there exists a natural reservoir), on equi-distant parallel lines 0—0, I—I, &c.

If $b_0, b_1, b_2 \ldots b_n$, be the widths 0—0, I—I, II—II, &c., and if the distance between the parallels be a, the area of the dam is:

$$G = \left[b_0 + b_n + 4(b_1 + b_3 + \ldots + b_{n-1}) + 2(b_2 + b_4 + \ldots + b_{n-2})\right] \cdot \frac{a}{3},$$

and if, in like manner, F_0, F_1, F_2, &c., be the area of the cross sections corresponding to the widths b_0, b_1, b_2, &c., respectively, the volume of the dam:

$$V = \left[F_0 + F_n + 4(F_1 + F_3 + \ldots + F_{n-1}) + 2(F_2 + F_4 + \ldots + F_{n-2})\right] \cdot \frac{a}{3}.$$

By dividing the cross sections by parallel lines, drawn at equal depths, we get the means of laying down contour lines of equal depth, and so ascertain the contents of the dam for each depth.

Remark. The author's work "*Der Ingenieur*," contains detailed instructions for measuring reservoirs and dykes.

§ 72. *Dykes.*—The dykes or dams of reservoirs are generally of earth-work, seldom of stone. The face inside, or next the reservoir is covered with clay puddle, and with a carefully laid course of gravel. They are carried up of a uniform slope, or with *offsets* or *terraces*. They are carefully rammed at every foot of additional height laid upon them. Especial care must be taken with the foundation, which must be carried down to an impermeable stratum with which the superstructure must be connected, so that the bed of junction may be perfectly water-tight. When a water-tight substance cannot be found, a system of piles must be used to insure this most important point of the reservoir's efficiency. The depth of the foundations depends on the nature of the ground, as above explained, and 5, 10, and 20 feet deep foundations have been executed.

The dyke, in its main features, is shown in Fig. 147, having a trapezoidal section EK or FL. AC, is sometimes termed the *crown* of the dam; it must be well paved, and generally has a parapet wall to prevent the wash of water during high winds from damaging the crown, or washing over to injure the back of the dam NE or ME. The piece KME of the dyke is termed the *middle* or centre piece, and the pieces ANH and BMC are termed the *wings* of the dam. As to the dimensions of dams, the breast is generally made to slope at the rate of 1 to 3, and the back at the rate of 1 to 2. The width on

Fig. 147.

the top is very various. For high dykes it varies from 10 to 20 feet. A common rule is, to make the width at top equal to the height, but this only applies to dams of small height. The dyke should be carried from 3 to 6 feet higher than the highest water intended to be in the reservoir.

Fig. 148.

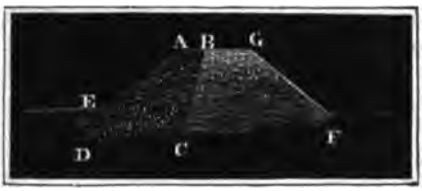


Fig. 148 represents a cross section of a dyke for a reservoir. $ABCE$ is the breastwork of clay carried down to water-tight substratum, $BGFC$ is the backing of earth-work, AE is the paved face, the paving being 4 feet thick at bottom, and 2 feet at top.

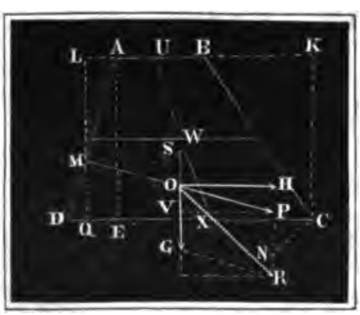
Remark. If l be the length along the top, and l_1 the length along the bottom, b the breadth on top, and b_1 the breadth at bottom, and if h be the height of a dyke such as Fig. 148, the cubic contents of the dam are:

$$V = [l b_1 + l_1 b + 2 (l b + l_1 b_1)] \frac{h}{6}.$$

In applying this formula, it must be borne in mind, that the well-rammed clay does not occupy quite one-half of that of the earth-work that has not been rammed.

§ 73. *Stability of Dykes.*—Dykes are exposed to the pressure, and sometimes, though rarely, to the shock or impetus of water. They must, therefore, be of proportions that will resist either being overturned or shoved forward by the action of the water. The conditions under which they resist being shoved forward have been examined, Vol. I. § 280; and we shall now consider the question of stability in reference to dislocation by rotation. The water acts on the internal slope or breast AD of a dyke, Fig. 149, with a normal pressure $OP = P$, the point of application of which is M is at the distance $LM = \frac{2}{3}$ the depth $CK = \frac{2}{3} h$ from the surface of the water

Fig. 149.



(Vol. I. § 278). For a length of dam $= 1$, $P = AD \cdot \gamma \cdot \frac{h}{2}$, γ being the density of the water, or weight of cubic unit. The horizontal component of this pressure is: $H = h \cdot 1 \cdot \gamma \cdot \frac{h}{2} = \frac{1}{2} h^2 \gamma$, and the vertical component, (if m be the relative batter, or mh the absolute batter DE of the breast,) $V = mh \cdot 1 \cdot \gamma \cdot \frac{h}{2} = \frac{1}{2} m h^2 \gamma$. The weight of the piece of the dyke of length $= 1$, acting at the centre of gravity

S of the trapezoidal section $ABCD$, is $G = \left(b + \frac{m+n}{2} h\right) h \gamma_1$, in which $b =$ the breadth AB, and n relative, or $n\,h$ the absolute batter or slope of the back of the dyke. From P and G, or from H, V, and G, there arises a resultant force $OR = R$, the statical moment of which $CN \,.\, R$, referred to the corner C, represents the stability of the dam. If we suppose P, and also H and V, acting in M, the statical moment of $P =$ statical moment of H minus statical moment of $V = \frac{1}{2} h^2 \gamma \,.\, \overline{MQ} - \frac{1}{2} m h^2 \gamma \,.\, CQ = \frac{1}{2} h^2 \gamma (\overline{MQ} - m \,.\, CQ) = \frac{1}{2} h^2 \gamma \left[\frac{1}{3} h - m (nh + b + \frac{2}{3} m h)\right]$;
hence we have the statical moment of G working in a contrary direction:

$$= \tfrac{1}{2} n h^2 \gamma_1 \,.\, \tfrac{2}{3} n h + b h \gamma_1 \left(n h + \frac{b}{2}\right) + m h^2 \gamma_1 (n h + b + \tfrac{1}{3} m h)$$
$$= h \gamma_1 (\tfrac{1}{3} n^2 h^2 + n b h + \tfrac{1}{2} b^2 + \tfrac{1}{2} m n h^2 + \tfrac{1}{2} m b h + \tfrac{1}{6} m^2 h^2)$$
$$= h \gamma_1 \left[\left(\frac{m^2 + 2 n^2}{3} + m n\right) \frac{h^2}{2} + \left(n + \frac{m}{2}\right) b h + \tfrac{1}{2} b^2\right];$$ and, hence, we have the stability of the dykes:

$$S = h \left(\left[\left(\frac{m^2 + 2 n^2}{3} + m n\right) \frac{h^2}{2} + \left(n + \frac{m}{2}\right) b h + \tfrac{1}{2} b^2\right] \gamma_1 - \left[\tfrac{1}{3} h - m (n h + b + \tfrac{2}{3} m h)\right] \frac{h}{2} \gamma\right).$$ In order now to find the point X, in which the line of resistance UWX cuts the base CD of the dyke, we must determine the distance CX of this point from the edge C, and for this we put: $\frac{CX}{CN} = \frac{OR}{HR} = \frac{R}{V+G}$; and from this

$$CX = a = \frac{CN \,.\, R}{V+G} = \frac{S}{G+V} = \left(\left[\left(\frac{m^2 + 2 n^2}{3} + m n\right) \frac{h^2}{2} + \left(n + \frac{m}{2}\right) b h + \tfrac{1}{2} b^2\right] \gamma_1 + \left[\left(\frac{2 m^2 - 1}{3} + m n\right) h + m b\right] \frac{h}{2} \gamma\right)$$
$$: \left(\left[\left(\frac{m+n}{2}\right) h + b\right] \gamma_1 + \tfrac{1}{2} m h \gamma\right);$$ or,

$$a = \frac{[(m^2 + 2 n^2 + 3 mn) h^2 + (2n + m) \,.\, 3 bh + 3 b^2] \gamma_1 + [(2 m^2 - 1 + 3 mn) h + 3 mb] h \gamma}{3 ([(m+n) h + 2 b] \gamma_1 + m h \gamma)}.$$

By aid of this formula, other points W in the line of resistance may be found, if for h different heights of dyke be introduced, or we may ascertain the stability of any part of the dam bounded by a horizontal plane.

For a dyke with vertical sides, $m = n = 0$, hence
$a = \frac{3 b^2 \gamma_1 - h^2 \gamma}{6 b \gamma_1} = \frac{1}{2} b - \frac{h^2 \gamma}{6 b \gamma_1}$ (Vol. II. § 10). If the inclination of the breast and back be 1 to 1, or 45°, $m = n = 1$, therefore,

$$a = \frac{3 (2 h^2 + 3 bh + b^2) \gamma_1 + (4 h + 3 b) h \gamma}{3 [2 (b + h) \gamma_1 + h \gamma]};$$

and if $b = h$, then $a = \frac{18 \gamma_1 + 7 \gamma}{4 \gamma_1 + \gamma} \,.\, \frac{h}{3}$, and if $\gamma_1 = 2\ \gamma$, then

$a = \frac{43}{27} h = \frac{43}{27} b$, or, as in this case the breadth at the base $b_1 = 3 b$, or $b = \frac{1}{3} b_1$, $a = \frac{43}{81} b_1$. According to Vauban's practice, there is ample security when $a = \frac{5}{9} \cdot \frac{b_1}{2} = \frac{5}{18} b_1$ (Vol. II. § 11), so that for the last case there is an excess of stability. All things considered, it is well in dykes, for great reservoirs, to make a at least $= 0{,}4\ b_1$, or the line of resistance should cut the base at $\frac{4}{10}$ths of the width of the base from the heel of the dyke.

Example. Required the line of resistance of a dyke, the batter of inclination of the breast of which $m = 1$, that of the back $n = \frac{1}{3}$, the breadth on the summit, or crown being $b = 10$ feet. Assuming that the mass of the dyke has a specific gravity $= 2$. We have:

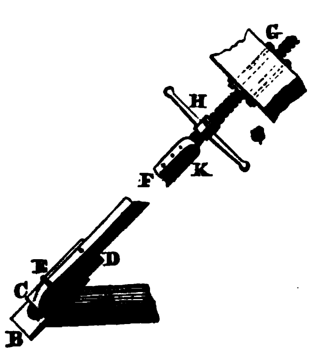
$$a = \frac{2\,(3\,h^2 + 60\,h + 300) + (\frac{5}{3}\,h + 30)\,h}{3\,(3\,h + 40 + h)} = \frac{1200 + 300\,h + 17\,h^2}{24\,(10 + h)};$$

hence for $h = 0$, $a = 5$ feet; for $h = 5$ feet, $a = \frac{3125}{360} = 8{,}68$ feet; for $h = 10$ feet, $a = \frac{5900}{480} = 12{,}29$ feet, for $h = 15$ feet, $a = \frac{9525}{600} = 15{,}87$ feet, for $h = 20$ feet, $a = \frac{14000}{720} = 19{,}44$ feet, &c. If the height of dyke be very great, we may put: $a = \frac{17\,h}{24}$, and $b = \frac{1}{2}\,h$, hence $\frac{a}{b} = \frac{17}{12}$. As $\frac{17}{12}$ is more than 0,4, such a dam would be safe for an infinite height.

Remark. According to the formula $b = \frac{3\,h - a}{2}$ in the example Vol. I. § 280, if we put $a = mh$, then $2\,b = (3 - m)\,h$, hence $h = \frac{2\,b}{3 - m}$, and, therefore, in our last example, in which $m = 1$, $h = b = 10$ feet.

§ 74. *Offlet Sluices of Dykes.*—Offlet sluices and discharge-pipes, or culverts, must be provided in the reservoir dyke. The offlet sluice or regulator, serves for the discharge of any excess of water that would accumulate in times of extraordinary wet. The discharge-pipe or culvert, is for supplying the *lead* or water-course as circumstances require. There may be one or more of each of these accessories in a dyke. For instance, in some dykes an offlet is arranged at the very lowest level, so that the dam may be completely emptied when occasion requires, and above this, a second offlet is laid, by which the water-course is supplied with water to be led to the machine that is to receive it as *power*.

Fig. 150.



The offlet-pipes may be either of wood or iron, or of stone, or may be built culverts. Fig. 150, in the margin, gives a general idea of the arrangement of the drawing-sluice or discharge-sluice of a dyke. *A* is the end of the pipe or culvert, on the face of which is a flat piece of wood or iron *B*, *CD* is a cast iron or wooden *sluice-board*, fitting into

guides, DE is the *sluice-rod*, reaching to the surface or top of the dyke, E is a cross piece by which, in the absence of grooves or guides on the plate on the end of the pipe, the sluice is kept pressed upon its bed, G is a strong beam having a female screw, through which the screw GH passes, and the handle or key, H, being turned, the screw elevates or depresses the sluice-rod, as may be desired, for opening and shutting the sluice.

The discharge-pipe must have a sectional area, such that the discharge when the water, or rather its *head*, is lowest, may be sufficient for the supply of the power required for the machine. If Q be the quantity of water to be discharged per second, h the given least *head*, l the length, and d the diameter of the discharge-pipe, ζ the co-efficient of resistance at entrance, and ζ_1 the co-efficient for internal friction, then, according to Vol. I. § 332,

$$d = \sqrt[5]{\frac{(1+\zeta)\,d + \zeta_1\,l}{2\,g\,h} \cdot \left(\frac{4\,Q}{\pi}\right)^2},$$

or, more simply:

$$d = 0{,}4787 \sqrt[5]{[(1+\zeta)\,d + \zeta_1\,l]\,\frac{Q^2}{h}}.$$

If, therefore, we take ζ from the table in Vol. I. § 325, and ζ_1 from the table in Vol. I. § 331, we can determine by approximation the required width of pipe. As the head is higher, a greater part of the aperture must be closed, so that, according to Vol. I. § 338, there must be introduced a greater co-efficient of resistance for the entrance. If the entrance aperture be very small, the water does not fill the pipe, and, therefore, the calculation is simply referable to the area of the opening $F = \frac{Q}{\mu\sqrt{2\,g\,h}}$, where μ is to be taken from Vol. I. § 325. With table of areas of segments, the calculations are very simple. The prolongation of the discharge-pipe through the dyke must be of very substantial cement-built masonry, and in large dykes should be from 5 to 6 feet high.

Example 1. A discharge-pipe of 100 feet long is required to let off 10 cubic feet per second, when the head is reduced to 1 foot, what must be the diameter? Supposing the inclination of the sluice to be 40° (equal that of the breast of the dyke) then $\zeta = 0{,}87$; and the co-efficient ζ_1 corresponding to a velocity of 5 feet $= 0{,}022$, we have $d = 4787 \sqrt[5]{(1{,}870\,d + 2{,}2)\,.\,100}$, and $d = 1{,}7$ satisfies this equation very nearly. Thus, a discharge pipe of $1{,}7\,.\,12 = 20{,}4$ inches would fulfil the required conditions.

Example 2. In what position must this sluice-board be placed, in order to discharge only 10 cubic feet of water per second, when the head is 16 feet? If we assume that the pipe does not fill in this case, then

$$F = \frac{Q}{\mu\sqrt{2\,g\,h}} = \frac{10}{0{,}731\,.\,802\sqrt{16}} = \frac{5}{11{,}6} = .431 \text{ square feet.}$$

This segment of radius $\frac{1{,}7}{2}$ reduced to radius $1 = 0{,}431\,.\,\frac{4}{2{,}89} = 0{,}598$, and from a table of areas of segments, we find the height of such a segment to be 5 inches.

§ 75. *Water-courses.*—The water of the reservoir is conducted or led to the point at which it is to be applied, *i. e.*, to the machine through which it is to expend its mechanical effect, by *canals*, *water-*

courses, and *mill-leads*. These channels are generally dug out of the natural soil, raised upon embankments and aqueducts over the deeper valleys, and cut as drifts or tunnels through the greater elevations that occur in their course. The bed of the canals are formed of sand or gravel, on a bottom of clay, or are hand-laid stones, or concrete formed with cement, and not unfrequently it consists of a wooden, an iron, or a stone *trough*. The sides of this canal form right lines, or its section is a gently curved trapezium, or it is rectangular when it becomes a *trough*. The section of water-courses is from 1¼ to 3 times as wide as its depth. The slopes of the sides of the course are generally very slight, or none at all in the case of masonry set in cement. An inclination of 1 in 2 is given to dry stone sides, an inclination of 1 in 1 in the case of compact earth or clay, and of 2 to 1 in the case of sand or loose earth. Fig. 151 gives an idea of the construction of a water-course in loose ground,

Fig. 151.

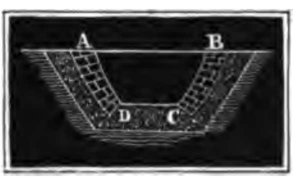


Fig. 152.

not water tight. Fig. 152 represents the manner of forming such a course on the side of a hill, where the earth taken from the cut is made the supporting bank on the under side. Fig. 153 shows the

Fig. 153.

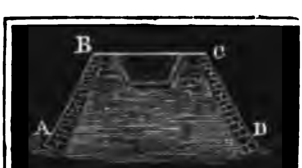


Fig. 154.

manner in which it is sometimes necessary to construct the embankments for aqueducts.

Fig. 154 is a section of a walled drift or tunnel, through ground not considered impermeable to water, and incapable of standing unsupported. The manner of putting troughs together is indicated

in the sketches in Fig. 155 for wood, and Fig. 156 for iron, where the flanges, bolted together, are further made water-tight by what is

Fig. 155.

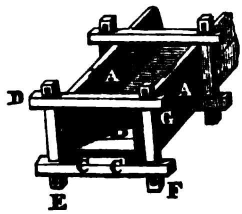
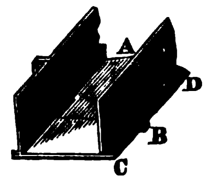


Fig. 156.



termed a *rust-joint* (a cement composed of sal ammoniac and iron filings or turnings).

The junction of a water-course with a river *AA*, Fig. 157, should be gradually widened and rounded off, and the head *D* substantially finished, so that it may not be injured by freshes, or objects carried against it in time of floods. Flood-gates or sluices have to be arranged along the course, if this be of any considerable extent. These sluices should be made self-acting, that no damage may be done to

Fig. 157.

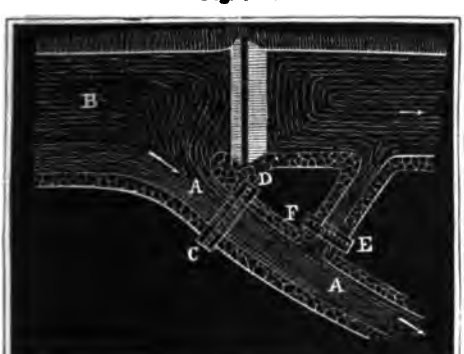
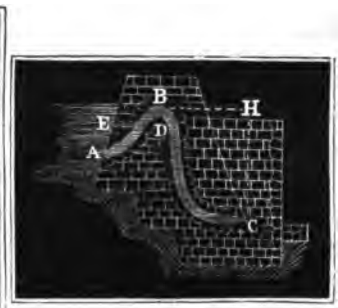


Fig. 158.



the banks by even a momentary overflow (the self-acting sluices on *Shaw's water-works*, in Scotland, is the most notable case of this self-acting arrangement on record). They act generally by a float being raised as the water in the channel rises, which float opens a valve or sluice to discharge the surplus water in convenient localities. Sometimes a case fills as the water rises, overcomes a counter-balance, and in its descent opens a valve or sluice, by which the surplus water is discharged. The syphon, properly adapted, forms a simple contrivance, and is shown in Fig. 158, where *ABC* is the syphon with an air-pipe *DE*. When the water in the water-course rises to the height of the summit of the syphon, which is the highest point for safety, the syphon fills with water, and the water is drawn off and discharged at *C*, the head being *CH*, the depth of *C* under the water surface. When the water has sunk to the level of *DE*,

the air rushes in and stops the action of the syphon. If the water does not fill the section BD of the pipe, the discharge is made under the conditions of a weir.

§ 76. The velocity of the water in a water-course should be neither too slow, for then the course chokes with weeds; nor too fast, for then the bed of the channel may be disturbed; and besides, too much fall must not be lost in the inclination of the course.

A velocity of 7 to 8 inches per second is necessary to prevent deposit of slime and growth of weeds, and $1\frac{1}{2}$ feet per second is necessary to prevent deposit of sand. The *maximum velocity* of water in canals depends on the nature of the channel's bed.

On a	slimy bed	the velocity should	not exceed	$\frac{1}{4}$	foot.
"	clay	"	"	$\frac{1}{2}$	"
"	sandy	"	"	1	"
"	gravelly	"	"	2	"
"	shingle	"	"	4	"
"	conglomerate	"	"	5	"
"	hard stone	"	"	10	"

This applies to the mean velocity.

From the assumed mean velocity c, and the quantity of water to be led through the course Q, we have the section F, and hence the perimeter p of the water section. If we put this in the formula $\delta = \frac{h}{l} = \zeta \cdot \frac{p}{F} \cdot \frac{c^2}{2g}$ (Vol. I. § 367), we get the required inclination δ of the canal, and hence the fall required for the lead, whose length $= l$ is $h = \delta l$.

The inclination may, therefore, be very different according to circumstances. As, however, ζ as a mean is 0,007565, and c generally from 1 to 5 feet, and $\frac{p}{F}$ is something between $\frac{1}{8}$ and 2, the limits of the inclinations for the water-course would be

$$0{,}007565 \cdot \tfrac{1}{8} \cdot 1 \cdot {,}0155 = 0{,}000023, \text{ and}$$
$$0{,}007565 \cdot 2 \cdot 25 \cdot {,}0155 = 0{,}00578,$$

the courses leading from the machine have a greater fall, that the machine may be quite clear of back-water. The course leading from the machine is usually termed the *tail-race*.

Remark 1. The water-courses for the water wheels and general uses of the Freyberg mining districts, have inclinations varying from $\delta = 0{,}00025$ to $\delta = 0{,}0005$, or from 15 inches to 30 inches per mile, the tail-races generally .001 to .002. The Roman aqueduct, at Arcueil, near Paris, has an inclination $\delta = 0{,}000416$, or 2 feet per mile nearly. The New River, which supplies a great part of London, has an inclination $\delta = 0{,}00004735$. [The Croton aqueduct has $\delta = 0{,}000208$, or 1,1 foot; and the Boston aqueduct 0,0000473̇5, or 3 inches per mile, the same as New River.]—Am. Ed.

Remark 2. All sudden changes of sectional area and of direction are to be avoided, because these not only occasion loss of fall, but entail other bad effects in the way of wear and tear and deposits. Bends or curves should have as great a radius as possible, or *the sectional area* should be increased there. If r be the mean width of the course, and R the radius of curvature, the fall lost by a curve may be calculated, according to Vol. I. § 334, by the formula:

$$h_1 = \left[0{,}124 + 3{,}104 \left(\frac{r}{R}\right)^{\frac{7}{2}}\right] \frac{\beta^\circ}{180^\circ} \cdot \frac{c_2}{2g}$$

until we have further experimental data.

Remark 3. The deposit of slime, sand, and the growth of plants, diminishes the section of water-courses, and fall is thereby lost. The water-courses must, therefore, be carefully cleaned out from time to time.

§ 77. *Sluices.*—The entrance of water into a water-course is either free, or regulated by a sluice. If the water enter unimpeded from the weir-dam or reservoir, in which it may be considered to be *still*, the surface of the water sinks where the flow commences, and the depression is proportional to the initial velocity in the water-course, and therefore $= \frac{v^2}{2g}$, which height must be deducted from the total *fall* of the water-course. For moderate velocities of 3 to 4 feet per second, this depression amounts to only $1\frac{1}{2}$ to 3 inches.

If the entrance of water into the lead be regulated by a sluice, two distinct cases may present themselves. Either the water flows freely through the sluice, or it flows into and against the water of the lead. It will generally be found that the depth of the water in the lead, is greater than the height of the sluice-opening, and, therefore, there occurs a sudden rise S at a certain distance from the sluice AC, Fig. 159. The height $BC = x$ of this rise is a function of the velocity v of the water in the lead, and of the velocity v_1 of the water coming up to the sluice, such that

Fig. 159.

$x = \frac{v_1^2}{2g} - \frac{v^2}{2g}$, and if we deduct this height from that due to the velocity v_1, or $AC = h = \frac{v_1^2}{2g}$, then the head causing the initial velocity v is:

$$AB = h_1 = h - x = \frac{v_1^2}{2g} - \left(\frac{v_1^2}{2g} - \frac{v^2}{2g}\right) \frac{v^2}{2g},$$

or exactly the same as if the water were discharging freely. As the sluice-opening is never perfectly smooth, there is, of course, a certain resistance increasing the head required by 10, or even more, per cent.

If we put $G =$ the area of the section of the water flowing in the lead, and $F =$ the area of the sluice-opening CD, then $Gv = Fv_1$, and, therefore, the rise

$$x = a - a_1 = \left[1 - \left(\frac{F}{G}\right)^2\right] \frac{v_1^2}{2g},$$

and substituting for $\frac{v_1^2}{2g}$ the height due to the velocity or the head $AC = h$, $x = \left[1 - \left(\frac{F}{G}\right)^2\right] h$. If the difference $x = a - a_1$ of the depth of water a and a_1 be less than $\left[1 - \left(\frac{F}{G}\right)^2\right] \frac{v_1^2}{2g}$, the rise occurs

further *down* the lead; but, if it be greater, then the rise occurs nearer the sluice, till at last the discharge takes place under back-water, as shown in Fig. 160. In this case, the head $AB = h$ has not only to produce the velocity v in the water of the lead, but also to overcome the resistance arising from the sudden change of the velocity v_1 into the velocity v of the lead. If we put $F =$ the area of the opening, and $G =$ the area of the lead, the loss of head occasioned by this transition is:

Fig. 160.

$$= \frac{(v_1 - v)^2}{2g} = \left(\frac{G}{F} - 1\right)^2 \frac{v^2}{2g},$$

and hence the fall:

$$AB = h = \frac{v^2}{2g} + \left(\frac{G}{F} - 1\right)^2 \frac{v^2}{2g},$$

$$\text{hence } h = \left[1 + \left(\frac{G}{F} - 1\right)^2\right] \frac{v^2}{2g}.$$

It is obvious that the difference of level of the water before and behind the sluice, is so much the greater, the smaller the sluice-opening F in proportion to the section of the water in the lead G.

Example. A lead of 5 feet mean width, and 3 feet depth, supplies 45 cubic feet per second. It is fed through a sluice 4 feet wide, and 1 foot opening. Required how much higher the water will stand, before the sluice than behind it. $G = 5 \times 3 = 15$ square feet. $F = 4 \times 1 = 4$ square feet; $v = \frac{45}{15} = 3$ feet per second, and $v_1 = \frac{3 \,.\, 15}{4} = \frac{45}{4}$ $= 11\frac{1}{4}$ feet.

Now as $\left[1 - \left(\frac{F}{G}\right)^2\right] \frac{v_1^2}{2g} = [1 - (\frac{4}{15})^2]\ 2{,}02 = 1{,}88$ feet is less than $a - a_1 = 3 - 1$ $= 2$ feet, it is evident that there will not be a *free discharge.* The formula $h = \left[1 + \left(\frac{G}{F} - 1\right)^2\right] \frac{v^2}{2g}$ gives the difference of level required
$h = (1 + 2{,}75^2)\ 0{,}139 = 8{,}56 \times 0{,}139 = 1{,}19$ feet, which must, however, be increased 10 per cent. at least, on account of the resistances at the opening.

§ 78. *Pipes, Conduit Pipes.*—Pipes are usually employed when smaller quantities of water are to be brought to supply machines, such as the water-pressure engine, and turbines of very high fall. They have the advantage of much greater *pliability* than open conduits, but their adoption instead of open canals, depends entirely on local circumstances in the question of relative advantage.

Pipes are made of wood, of pottery, of stone, of glass, iron, lead, &c. Wooden and iron pipes are those most usually employed in connection with water-power engines. Wooden pipes are usually formed from large trees, because straight pipes of 12 to 20 feet in length, and from $1\frac{1}{2}$ to 8 inches *bore*, or internal diameter, may be got from this timber. The bore is generally $\frac{1}{3}$ of the diameter of the tree. Wooden pipes are jointed or connected together as shown in Figs. 161 and 162.

Fig. 161 is a conical mortice with a *binding ring* and *packing* of hemp, or linen steeped in tar and oil. Fig. 162 is a connection by

Fig. 161.

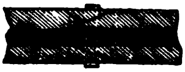


Fig. 162.

means of an iron double spigot going from 1 to 2 inches into the ends of the two pipes.

Iron pipes are the most durable and most universally employed of all pipes. They are cast of any diameter, and have been used as large as 5 feet bore. The length of each pipe rarely exceeds 12 feet, and is less as the diameter is greater. For 3 feet diameter, they are about 9 feet long each, in England. To prevent internal oxidation, they are sometimes boiled in oil, sometimes lined with wood, or with Roman cement. The thickness of metal must be proportional to the pressure they have to bear, and to their diameter, according to Vol. I. § 283. The jointing of iron pipes is effected either by flanges and bolts, as shown in Fig. 163, there being an annular packing between the flanges, or by the *spigot* and *faucet*, as shown in Fig. 164, (which is considered the best and cheapest mode,

Fig. 163.

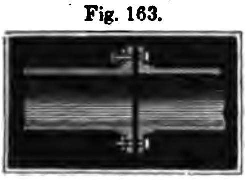
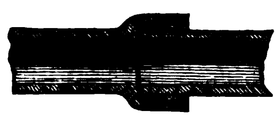


Fig. 164.



when the packing is properly done with small folding wedges of hard wood.) A collar, or ring, as shown in Fig. 165, is sometimes used. The packing is either leather, felt, lead, iron rust, or wood. The more effectually to prevent all leakage, there is sometimes a small internal ring put in (counter-sunk) to cover the joint. A flexible joint, as shown in Fig. 166, is sometimes necessary (as for crossing a river, where it is necessary to let the pipe rest on the original bed of the river). Where the pipes are exposed to changes of temperature, expansion joints, as shown in Fig. 167,

Fig. 165.

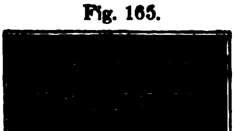


Fig. 166.

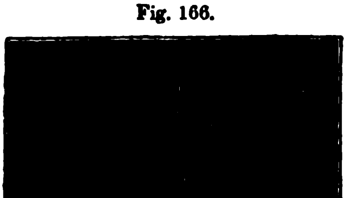
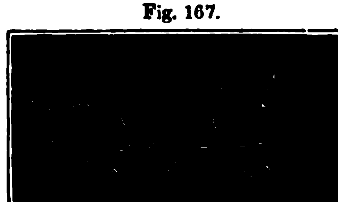


Fig. 167.



must be introduced, that the expansion and contraction of each considerable length may not injure the pipes or joints. The expansion of cast iron is .0000111 of its length, for each degree of centigrade; and, therefore, for a change of temperature of 50°, or from winter frost to summer heat, the expansion would be 0,000553. Therefore, for every 900 feet, there is an expansion and contraction of 6 inches. This is to be *compensated* by an arrangement, such as is shown in our last figure, where the pipe *B* is movable through the water-tight stuffing box *C*. There should be a compensation joint for every length of 300 feet exposed to a change of temperature.

§ 79. Pipes cannot of course be laid so as to maintain a straight line; but rise and fall, and turn from right to left in their course. It is a general maxim to avoid all *sudden* changes of direction in laying pipes. All bends should be effected by curved pipes, of as great radius as possible, or the bore of the pipe should even be increased at bends, to avoid loss of *vis viva* in the water. When a pipe bends over an elevation, as in Fig. 168, there is a disadvantage arises from the collection of air at *L*, as this contracts the section, and would gradually stop the flow of water. To prevent this accumulation of air, vertical pipes *AL*, called *ventilators* or *wind pipes*, are placed on the summit of the pipe, through which air, or other gases given off by the water, can be discharged from time to time, by means of a cock, to be turned by the inspector of the pipes. To make these ventilators self-acting, the arrangement shown in Fig. 169 has been adopted. In this ventilator the discharge valve *V* is connected with a float *S* of tinned iron, which is pressed upwards as long as it is surrounded by water, and thus keeps the valve shut, but falls or sinks downwards when the space about it becomes filled with air, and then the valve is opened to discharge the air. As air collects at the highest points of a conduit pipe, so the sand or slime collects at the lowest points. To remove any deposits of this nature, *waste-cocks* are placed at these points, by which the pipe is scoured, or separate receptacles for the deposits are attached to the pipes, and these are cleared from time to time, as may be found necessary. The deposit is favored by the greater section of these receptacles, and sometimes by the introduction of check or division plates, which still more retard the flow.

Fig. 168.

Fig. 169.

Cocks for *flushing* the pipes are introduced more or less frequently, according to the purity of the water, and the rate of flow through the pipes, and seldom at less intervals than 100 feet. For ascertaining the point in the pipe where any obstruction has occurred, *piezometers* (Vol. I. § 344) are very useful.

For regulating the discharge of water through pipes, cocks and slides, and valves are used. The effect of these has been shown in Vol. I. § 340, &c. In order to moderate the effects of the impulse or shock arising on the sudden closing of a cock, or other valve, it is useful to have a loaded safety valve, so placed that it will open outwards when the pressure exceeds a certain limit.

Remark. The most detailed treatise on the subject of conduit pipes, is Genley's "*Essai sur les Moyens de conduire, d'élever, et de distribuer les eaux.*" Matthew's "*Hydraulia,*" and the "*Civil Engineer and Architect's Journal,*" contain much useful information on this subject. Hagen, "*Wasserbaukunst,*" Vol. I. has a chapter on water pipes.

§ 80. The general conditions of motion in conduit pipes have been already discussed. If h be the fall, and l the length, d the diameter of the pipe, ζ the co-efficient of resistance at entrance, ζ_1 the co-efficient for friction in the pipe, and ζ_2, &c., the co-efficients for resistances in passing bends, cocks, &c., and if v be the velocity of discharge, we have:

$$h = \left(1 + \zeta + \zeta_1 \frac{l}{d} + \zeta_2 + \ldots\right)\frac{v^2}{2g},$$

and if Q be the quantity of water:

$$h = \left(1 + \zeta + \zeta_1 \frac{l}{d} + \zeta_2 + \ldots\right)\left(\frac{4\,Q}{\pi}\right)^2 \cdot \frac{1}{2g\,d^4}.$$

We see from this, that for carrying a certain quantity of water Q, so much less fall is requisite, the greater the width of the lead. If there be two pipes instead of one, the two together having an area equal to the one, and supposing each to take half the whole quantity of water, the fall necessary is:

$$h_1 = \left(1 + \zeta + \zeta_1 \frac{l}{d\sqrt{\frac{1}{2}}} + \zeta_2 + \ldots\right)\left(\frac{2\,Q}{\pi}\right)^2 \cdot \frac{1}{2g\,(d\sqrt{\frac{1}{2}})^4}$$

$$= \left(1 + \zeta + \zeta_1 \cdot \frac{l\sqrt{2}}{d} + \zeta_2 + \ldots\right)\left(\frac{4\,Q}{\pi}\right)^2 \cdot \frac{1}{2g\,d^4};$$

so that in this case the fall is greater, or the head required is greater, so that it is mechanically better to employ one large pipe, than two smaller of equal section when united.

Calculations for whole systems of pipes, where there are numerous subdivisions of branches, become exceedingly complicated. The case in which water is brought from different sources, and the pipes ultimately united, is of the same nature.

The general nature of such calculations is as follows. If the subdivision takes place in a reservoir which has a much greater sectional area than the *main* pipe, the water comes there again to rest, or the whole *vis viva* is destroyed, and has to be acquired again in the branch pipes. The same loss of *vis viva* occurs when several branches come together in a reservoir, from which one main pipe carries off their waters. In this case, the calculation reduces itself to a separate consideration of each branch and pipe, and requires no further elucidation. The collecting reservoir should be, if possible, placed at such levels as will ensure the same mean velocity in all the pipes, in order that the loss of head or of *vis viva* may be the least possible.

In the case of a simple subdivision or *fork*, it is mechanically advantageous to make such arrangements that the water may move in all the pipes with the same velocity. If, besides this, the branches be curved off properly, so that there is no sudden change of direction in the passage of the water from the main into the branches, it may be assumed that there is no loss of head or *vis viva*. In the case sketched in Fig. 170, let h = the head BC, l the length, and

Fig. 170.

d the diameter of the main pipe, and let h_1 = the head or fall D_1E_1, l_1 the length, and d_1 the diameter of the one branch, and $d_2 = D_2E_2$, l_2, and d_2, the fall, length, and diameter of the other branch, and also let c, c_1, c_2, be the velocities of the water in these three branches, and, lastly, let ζ be the co-efficient of resistance for entrance, and ζ_1 the co-efficient for friction of the water. Then, for the length of pipes ACE_1, we may put:

1. $F_1E_1 = BC + D_1E_1 = h + h_1 = \left(\zeta + \zeta_1 \frac{l}{d}\right)\frac{c^2}{2g} + \left(1 + \zeta_1 \frac{l_1}{d_1}\right)\frac{c_1^2}{2g}$ and

for the length of pipes ACE_2:

2. $F_2E_2 = BC + D_2E_2 = h + h_2 = \left(\zeta + \zeta_1 \frac{l}{d}\right)\frac{c^2}{2g} + \left(1 + \zeta_1 \frac{l_2}{d_2}\right)\frac{c_2^2}{2g}$.

But the quantity of water $Q = \frac{\pi d^2}{4} c$ of the main pipe, is equal to the sum of the quantities $Q_1 = \frac{\pi d_1^2}{4} c_1$, and $Q_2 = \frac{\pi d_2^2}{4} c_2$ of the two branches; and hence we may put:

$$3.\ d^2 c = d_1^2 c_1 + d_2^2 c_2.$$

By aid of these three equations, three quantities may be determined. The more usual case is, that of the fall, the length and the quantity of water being given, the necessary diameter of the pipe is required. If, then, we assume a certain velocity c in the main, we get the width of this pipe by the formula:

$d = \sqrt{\frac{4\,Q}{\pi\, c}}$, and we have then only to solve the equations:

$$2\,g\,(h + h_1) - \left(\zeta + \zeta_1 \frac{l}{d}\right)c^2 = \left(1 + \zeta_1 \frac{l_1}{d_1}\right)\left(\frac{4\,Q_1}{\pi\, d_1^2}\right)^2, \text{ and}$$

$$2\,g\,(h + h_2) - \left(\zeta + \zeta_1 \frac{l}{d}\right)c^2 = \left(1 + \zeta_1 \frac{l_2}{d_2}\right)\left(\frac{4\,Q_2}{\pi\, d_2^2}\right)^2.$$

By transformation, we get similar equations for determining d_1 and d_2, as in Vol. I. § 332, viz.:

$$\zeta_1 \frac{l_1}{d_1^5} + \frac{1}{d_1^4} = \left[2\,g\,(h+h_1) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2\right] \left(\frac{\pi}{4\,Q_1}\right)^2, \text{ and}$$

$$\zeta_1 \cdot \frac{l_2}{d_2^5} + \frac{1}{d_2^4} = \left[2\,g\,(h+h_2) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2\right] \left(\frac{\pi}{4\,Q_2}\right)^2;$$

we can, therefore, as in Vol. I. § 332, put:

$$d_1 = \sqrt[5]{\frac{\zeta_1 l_1 + d_1}{2\,g\,(h+h_1) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2} \cdot \left(\frac{4\,Q_1}{\pi}\right)^2}, \text{ and}$$

$$d_2 = \sqrt[5]{\frac{\zeta_1 l_1 + d_1}{2\,g\,(h+h_2) - \left(\zeta + \zeta_1 \frac{l}{d}\right) c^2} \cdot \left(\frac{4\,Q_2}{\pi}\right)^2},$$

and in order to obtain a first approximation to the values of d_1 and d_2, we may omit these from the part under the radical. If c_1 and c_2 come out to be very different from c, attention must be paid to the co-efficient ζ_1, being variable, and its value for each of the pipes introduced, and the determination of d_1 and d_2 repeated.

Example. A system of pipes, to consist of one main and two branches is intended to carry 15 cubic feet of water per minute by one branch, and 24 cubic feet by the other. The levels showed that in a length of 1000 feet of main, the fall was 4 feet, the first branch had a fall of 3 feet in 600 feet length, and the second 1 foot in 200 feet. What must be the diameters of the pipes respectively? If we suppose a velocity of $2\frac{1}{2}$ feet per second in the main, then its diameter

$d = \sqrt{\frac{4\,Q}{\pi\,c}} = \sqrt{\frac{4\,.\,39}{\frac{5}{2}\,.\,60\,\pi}} = \sqrt{\frac{26}{25\,\pi}} = 0{,}5754$ feet $= 6{,}9$ inches. If now (according to Vol. I. § 436), we put the co-efficient of resistance for entrance $\zeta = 0{,}505$, the co efficient of friction (Vol. I. § 435) for velocity $c = 2{,}5$ feet, $\zeta_1 = 0{,}0253$, and as $2\,g = 64{,}4$, and $\left(\frac{4}{\pi}\right)^2 = 1{,}621$, we have for the diameter of the branches

$$d_1 = \sqrt[5]{\frac{0{,}0253\,.\,600 + d_1}{64{,}4\,.\,7 - (0{,}505 + 0{,}0253\,.\,1738)\,.\,\frac{25}{4}} \,.\, 1{,}621\,.\,\left(\tfrac{15}{60}\right)^2}$$

$$= \sqrt[5]{\frac{15{,}18 + d_1}{450{,}8 - 277{,}98} \,.\, 0{,}1013} = \sqrt[5]{\frac{15{,}18 + d_1}{1706}}, \text{ and}$$

$d_2 = \sqrt[5]{\frac{0{,}0253\,.\,200 + d_2}{322{,}0 - 277{,}98}\,.\,1{,}621\,.\,\left(\tfrac{24}{60}\right)^2} = \sqrt[5]{\frac{5{,}06 + d_2}{169{,}7}}$. If we first neglect d_1 and d_2 under the radical, we get the approximate values $d_1 = \sqrt[5]{\frac{15{,}18}{1706}} = 0{,}39$ feet, and $d_2 = \sqrt[5]{\frac{5{,}06}{169{,}7}} = 0{,}495$ feet. If we now introduce the value on the right-hand side of the equation, we get more accurately $d_1 = \sqrt[5]{\frac{15{,}57}{1706}} = 0{,}391$ feet and $d_2 = \sqrt[5]{\frac{5{,}555}{169{,}7}} = 0{,}505$ feet. The diameter $d_1 = 0{,}391$ corresponds to a velocity

$$c_1 = \tfrac{15}{60}\,.\,\frac{4}{\pi\,d_1^2} = \frac{1}{0{,}391^2\,.\,\pi} = 2{,}082 \text{ feet},$$

and the diameter $d_2 = 0{,}505$ corresponds to

$$c_2 = \tfrac{24}{60}\,.\,\frac{4}{\pi\,.\,0{,}505^2} = 1{,}997 \text{ feet},$$

and hence we should have more accurately for the first branch pipe $\zeta_1 = 00263$, and for the other $\zeta_2 = 0{,}0270$, and hence with the best accuracy which the formula admits

$$d_1 = \sqrt[5]{\frac{0{,}0263 \,.\, 600 + 0{,}391}{1706}} = \sqrt[5]{\frac{16{,}171}{1706}} = 0{,}394 \text{ feet} = 4{,}7 \text{ inches, and}$$

$$d_2 = \sqrt[5]{\frac{0{.}0270 \,.\, 200 + 0{,}505}{169{,}7}} = \sqrt[5]{\frac{5{,}905}{169{,}7}} = 0{,}511 \text{ feet} = 6{,}13 \text{ inches.}$$

CHAPTER IV.

OF VERTICAL WATER WHEELS.

§ 81. *Water Power.*—Water acts as a *moving power*, or *moves machines* either by its *weight*, or by its *vis viva*, and in the latter case it may act either by *pressure* or by *impact*. In the action of water by its weight, it is supported on some surface connected with the machine, that sinks under the weight; and in the action by its *vis viva* it comes against a surface yielding to it, in a horizontal direction generally, which is, in like manner, an integral part of the machine. If Q be the quantity of water (or $Q\gamma$ the weight of water) available as power, per second, and h, the *fall*, or the perpendicular height through which the water falls in giving out its mechanical effect, then the mechanical effect produced is: $L = Q\gamma \,.\, h = Q h \gamma$. If, again, c be the velocity with which the water comes upon any machine, the mechanical effect produced by its *vis viva*, is:

$$L = Q\gamma \frac{c^2}{2g} = \frac{c^2}{2g} Q\gamma.$$

That water may pass from rest to the velocity c, a fall, or height due to the velocity $h = \frac{c^2}{2g}$ is necessary, and, therefore, in the second instance we may also put $L = h\,Q\gamma$. So that *the mechanical effect inherent in water is the product of its weight into the height from which it falls*, as in the case of other bodies.

Water sometimes acts by its weight and *vis viva* simultaneously, by combining the effects of an acquired velocity c, with the fall h through which it sinks on the machine. In this case, the mechanical effect produced is again:

$$L = Q\gamma \,.\, h + Q\gamma \frac{c^2}{2g} = \left(h + \frac{c^2}{2g}\right) Q\gamma.$$

The mechanical effect Pv yielded by a machine is of course always less than the above available mechanical effect $Q h \gamma$; because many *losses* occur. In the first place, *all the water* cannot always be brought to work; secondly, a part of the fall is generally lost; thirdly, the water retains a certain amount of *vis viva* after having quitted the machine; and, fourthly, there are the passive

resistances of friction, &c., interfering. The *efficiency* of a water-power machine may be represented by $\mu = \frac{P v}{Q h \gamma}$, and the merits of different machines are proportional to the approximation of this ratio in their case, to unity.

From the general formula $L = Q h \gamma$, it is manifest that fall and quantity of water are convertible terms; so that, by doubling the height of a fall with a given quantity of water, we have the same power as by doubling the quantity of water, and retaining the original height.

Example. There is a fall of 10 feet yielding 12 cubic feet of water per second. The machine uses only 8,5 feet, however, and the water leaves it with a velocity of 9 feet per second, and the friction is ascertained to be 750 feet lbs.; required the efficiency of this machine.

The available mechanical effect $L = 12 \times 10 \times 62,5 = 7500$ feet lbs. (Pruss.), and the effect of the fall used $= 12 \times 8,5 \times 623 = 6375$ feet lbs. The mechanical effect lost from the *vis viva* retained in the water leaving the machine is $0,0165 \times 9^2 \times 12 \times 62,5 = 941,2$ feet lbs.; and the mechanical effect consumed by friction $= 750$ feet lbs.; and, therefore, the useful effect of this machine $Pv = 6375 - (941,2 + 750) = 4683,8$ feet lbs., and the efficiency $= \frac{4683,8}{7500} = ,624$.

§ 82. *Water Wheels.*—The machines used as recipients of water-power, are either wheels, (water wheels, Fr. *roues hydrauliques;* Ger. *Wasserräder;*) or engines with pistons, *water-pressure engines,* (Fr. *machines à colonnes d'eau;* Ger. *Wassersäulen-maschinen.*) Water wheels are essentially "the wheel and axle," with water as power. Pressure engines consist of a column of water, pressing on a movable piston.

Water wheels are either *vertical,* the axle of the wheel being horizontal, or they are *horizontal,* the axle of the wheel being vertical.

Vertical water wheels, concerning which we shall first treat, are either overshot, (Fr. *roues en dessus;* Ger. *Oberschlägige,*) or breast wheels, (Fr. *roues de côté;* Ger. *Mittelschlägige,*) or undershot, (Fr. *roues en dessus;* Ger. *Unterschlägige.*) The water comes on to the wheel near the top or summit, in overshot wheels; near the middle or level of the axle, in breast; and near the bottom in undershot wheels. In the first, the water's weight is chiefly the source of mechanical effect, whilst in undershot wheels it is the inertia of the water, and in breast wheels, the *weight and inertia* both that are usually effective. Undershot wheels sometimes hang freely between boats in a wide stream, and sometimes in a confined course, which is either straight or curved. Breast wheels are generally hung in a curved channel or course. It is, perhaps, necessary to distinguish from the above-named vertical wheels, Poncelet's wheel, in which the water acts by pressure in its ascent and descent on curved buckets.

§ 83. *Bucket Wheels.*—All vertical water wheels consist of an axle of wood or iron, with two journals or gudgeons—of two or more annular *crowns* or *shroudings*—of a set of arms connecting the shrouding with the axle, and of a series of *cells* or *buckets* between the shrouding—and, lastly, of a *flooring,* which reaching from crown

to crown on their under side, forms a close cylinder. The buckets divide the annular space bounded by the shroudings on the flooring into a series of compartments, which, when the buckets are placed more tangentially than radially, form *water troughs* or *cells*. This latter is the general construction of the buckets of overshot and breast wheels, which are thus distinct from the simple *floats* of undershot wheels. For overshot wheels, the water is led on to the wheel by a trough or channel having a regulating sluice, and falls thence into the second or third cell from the summit of the wheel. If, then, the wheel be once in motion, each cell gets partially filled with water as it passes the discharge of the water trough or lead, and retains the water till near to the bottom of the wheel, when it falls out, so that there is always a certain number of cells filled with water on *one side* of the wheel, and this keeps the wheel continuously revolving. Overshot wheels have been constructed for falls varying from 8 to 50 feet, and sometimes even up to 64 feet in height, and for quantities of water varying in every degree up to 50 cubic feet of water per second. It is often more advantageous to put up two or three smaller wheels, than one very large one; for the weight of the parts becomes inconvenient.

The *fall* of a water wheel should be measured as *between the surface of the water at the pentrough*, or regulating sluice, and the surface of water in the *tail race*, the depth of which latter will depend of course on the quantity of water, and on the breadth, and the inclination of the race. In order to lose as little of the effect as possible, the bottom of the wheel should be as near as possible to the surface of the race, so that the height from the surface of water in the pentrough to the bottom of the wheel may also serve as a true measure of the *height of fall*. If there be any risk of *backwater* in the race, the wheel must be hung at an extra elevation accordingly.

§ 84. *Construction of Water Wheels.*—Water wheels are made of wood or of iron, or of both these materials combined. The manner of uniting the axle and arms together is various. In the case of wooden wheels, they are either strapped or bolted on to the side of a square axle, as shown in Fig. 171, or they are let into the axle by morticing, or passed through it. The latter construction is bad, and only applicable to light wheels. The arms of the framed wheel, Fig. 171, may be strengthened by braces or auxiliary arms. Such wheels of 20 to 50 feet diameter are erected for pumping water, for driving ore mills, &c., in the Freiberg mining district. A is the axle, B and C are the journals or gudgeons, DE, FG, &c., are the main arms, HM, HL are the auxiliary arms; DFG, and $D_1F_1G_1$ are the shroudings of the wheel; K is the pentrough end. The crowns are two rings of wood composed of 8 to 16 pieces of 3 to 5 inch-thick segments. The whole is put together with screw bolts. There are cross tie-bolts for uniting the two crowns. The interior of the crowns are *grooved* out to receive the buckets. The open wheel N is a part of the mechanism for transmitting the motion.

Fig. 171.

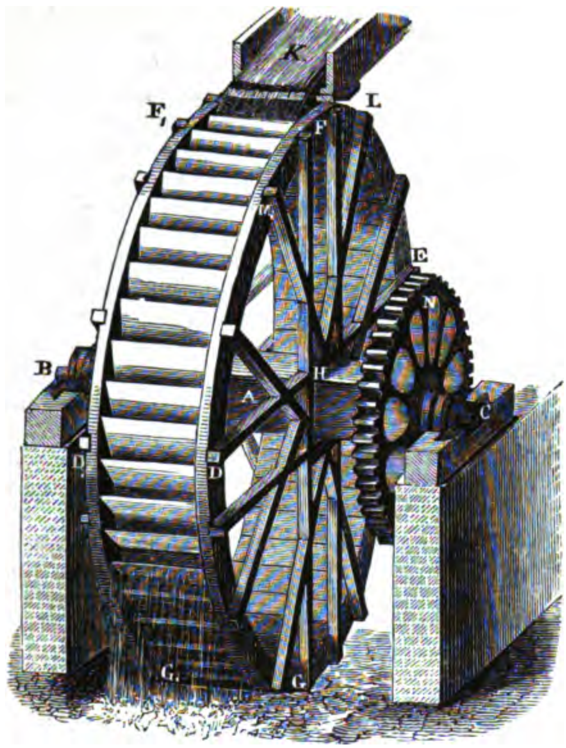
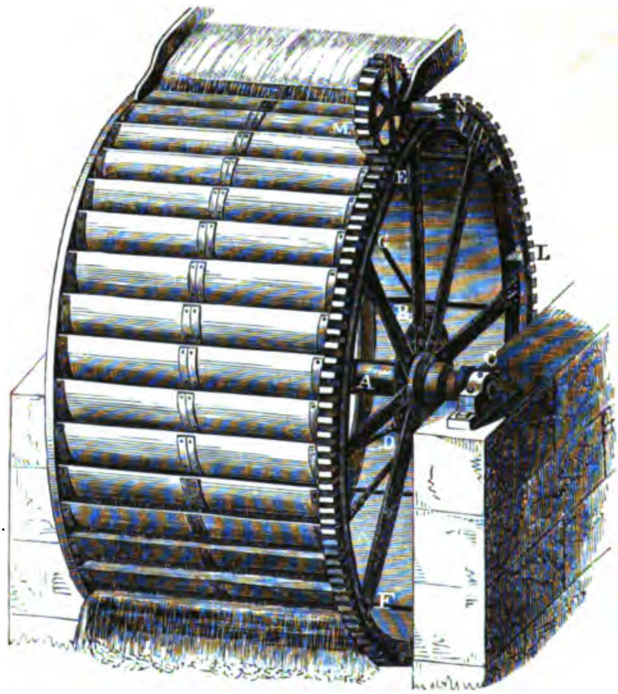


Fig. 172 is an iron water wheel. Cast iron discs, or *naves BD*, are set on the axle *AC*, and to these the arms are attached by bolts. An intermediate ring or crown is introduced when the wheel becomes more than 7 or 8 feet wide, and this has either a separate set of arms or diagonal arms, as shown by *BG*, &c., brought from this to the nave of the outer crowns. Through-bolts are introduced to bind the whole firmly together. The prime mover in the train of mechanism is often, as shown in Fig. 172, a toothed wheel, forming the periphery of an outside crown *ELF*, and this works into a pinion on a lying shaft *MN*. In practice, this pinion should be rather below than above the level of the axle, and on the side on which the water is. The buckets are of *sheet iron*, and bolted to ribs of *angle iron*, *cast on* the inner surface of the crowns, or *fastened to* them.

§ 85. *Dimensions of Parts.*—The axle, the gudgeons, and the arms of the wheel, must have dimensions proportioned to the weight and power of the wheel. To find these, the principles and rules of the third section of the first volume are to be applied. The dimensions of the axle may be determined either in reference to the moment of inertia of the wheel and the resistance to torsion of the axle,

Fig. 172.



or in reference to the weight of the wheel, and the resistance of the axle to transverse strain. In Vol. I. § 211, it has been shown, that in the case of a solid round cast iron axle of radius $= r$, acted upon by the statical moment of an effort P equal to Pa, that $Pa = 12600\, r^3$, where r and a are expressed in inches. Hence $r = \sqrt[3]{\frac{Pa}{12600}}$ inches $=$ the radius of axle; and if a be expressed in feet, then the diameter of the axle

$$d = \sqrt[3]{\frac{8 \,.\, 12\, Pa}{12600}} = \sqrt[3]{\frac{4\, Pa}{525}} = 0{,}197 \sqrt[3]{Pa} \text{ inches.}$$

But the mechanical effect corresponding to the moment Pa, u, being the number of revolutions of the wheel per minute, is

$L = Pv = P \frac{\pi u a}{30}$ feet lbs., or, in horses' power, $L = \frac{P \,.\, \pi u a}{30 \,.\, 550}$,

hence $Pa = \frac{16500\, L}{\pi u}$, and

$$d = 0{,}197 \sqrt[3]{\frac{16500}{\pi}} \,.\, \sqrt[3]{\frac{L}{u}} = 3{,}34 \sqrt[3]{\frac{L}{u}} \text{ inches.}$$

But for greater security, we generally make $d = 6{,}12 \sqrt[3]{\frac{L}{u}}$ inches.

If the axle be square, the side of the square

$$s = \sqrt[3]{\frac{3\pi}{8\sqrt{2}}} \,.\, d = 0{,}94\, d, \text{ i. e., } s = 5{,}75 \sqrt[3]{\frac{L}{u}} \text{ inches.}$$

If the axle be made hollow, the formulas given in Vol. I. § 209 and § 210 are to be used with the above co-efficients. Wooden axles should be from 3 to 4 times as large in diameter as iron axles.

If the toothed wheel, transmitting the power of the water wheel, be an integral part of it, as in Fig. 172, the axle undergoes a less torsion-strain by the moment of the power, and, therefore, its dimensions should be determined in reference to the weight of the wheel. For this we may make use of the formulas given in Vol. I. § 202, $Q\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right) = \frac{K}{6} bh^2$, in which we substitute for Q, G the weight of the water wheel, c the breadth of the wheel, l the length of the axle, and l_1 and l_2 the distance of the centre of the wheel from the two gudgeons. Hence for a square axle:

$$h = b = s = \sqrt[3]{\frac{6\,G}{K}\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right)}.$$

And if for $\frac{K}{6}$ we put 1000 lbs. as a minimum, and expressing l, l_1, and l_2, and c in feet, we get for square cast iron axles:

$$s = 0{,}229 \sqrt[3]{G\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right)} \text{ inches,}$$

and, on the other hand, for round cast iron axles:

$$d = s\sqrt[3]{\frac{16}{3\pi}} = 1{,}193 \,.\, s = 0{,}272 \sqrt[3]{G\left(\frac{l_1 l_2}{l} - \frac{c}{8}\right)}.$$

Wooden axles must be made at least as large again.

The diameter of the gudgeon d_1 is deduced from the well-known formula given in Vol. I. § 196, $Pl = \frac{\pi}{4} r^3 K$, substituting in it for $r = \frac{d_1}{2}$, and l the length of the gudgeon, which is generally about equal to d_1, its diameter. Hence we should have for the diameter $d_1 = \sqrt{\frac{32}{\pi K}} \,.\, P$, for which we may put in practice $d_1 = 0.48 \sqrt{P}$, P being the pressure on the gudgeon. Buchanan's rule is $d_1 = 0{,}241 \sqrt[3]{P}$ inches.

The arms of the wheel must evidently be of strength sufficient to resist the moment of rotation. If this moment be again taken $= Pa$, and the number of the arms in each set of arms of the wheel $= n$, so that for a double set of arms the total number of arms $= 2\,n$, then the moment which a single arm has to resist $= \frac{Pa}{2n}$. If, now, $b =$ the breadth, and h the thickness of an arm, and if the length of the arm be equal to the radius of the wheel $= a$, then, from Vol.

I. § 196, we have $\frac{Pa}{2n} = bh^2 \frac{K}{6}$, or, as b is made $= mh$, or, in iron generally, $\frac{1}{5}h$, and in wood, $\frac{5}{7}h$, *i. e.*, $\frac{Pa}{2n} = mh^3 \frac{K}{6}$, and hence the thickness of the arms sought, measured in the direction of the plane of revolution, is: $h = \sqrt[3]{\frac{3\,P\,a}{m\,n\,K}}$. If we introduce the effect, and number of revolutions of the wheel, then, for cast iron arms, $h = 10{,}4 \sqrt[3]{\frac{L}{nu}}$ inches. And, as the diameter of the axle was found $d = 6{,}12 \sqrt[3]{\frac{L}{u}}$, we have also $h = \frac{1{,}7\,d}{\sqrt[3]{n}}$, or $\frac{h}{d} = \frac{1{,}7}{\sqrt[3]{n}}$, and, therefore, for 4, 6, 8, 10, 12, 16 arms, the values of $\frac{h}{d} = 1{,}08,\ 0{,}94,\ 0{,}85,\ 0{,}79,\ 0{,}75,\ 0{,}67$, and from h, we deduce the breadth b, measured in the direction of the axis.

For wooden arms $h = 13{,}6 \sqrt[3]{\frac{L}{nu}}$, and hence we can deduce $b = \frac{5}{7}h$.

According to Rettenbacher, the number of arms in a set, or to one crown (of which there are always two at least), is $n = 2\left(\frac{a}{3} + 1\right)$. If a wheel be 8 feet wide, or wider, the number of sets of arms should not be less than three.

Example. A cast iron water wheel, weighing 35.000 lbs, gives an effect of 60 horse-power, making 4 revolutions per minute; required, the dimensions of its principal parts. The diameter of a solid axle is $d = 6{,}12 \sqrt[3]{\frac{60}{4}} = 13{,}2$ inches, and that of its gudgeons $d_1 = 0{,}048 \sqrt{\frac{35000}{2}} = 6\frac{1}{2}$ inches, which might be made 7 inches. Buchanan's formula gives $d_1 = 0{,}241 \sqrt[3]{17500} = 6\frac{1}{2}$ inches. For the arms, supposing two sets of 12 each, the thickness $h = \frac{1.7 \times 13{,}2}{\sqrt[3]{12}} = 10$ inches nearly, and the breadth $b = \frac{1}{5}\,10 = 2$ inches (h being in the direction of the plane of revolution).

§ 86. *Axles and Gudgeons.*—We must make special allusion to the manner of putting the gudgeons in the axles, and to the *plummer* blocks on which they rest. For wooden axles, oak, or larch, or beech, answers exceedingly well. They are dressed into polygons, when the arms are to be framed on the axle, and they are squared when the arms are to be morticed through, or into the axle. The

Fig. 173.

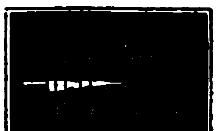


Fig. 174.

gudgeons are either spiked in, as shown at *Z*, Fig. 173, or they are *hooped*, as shown in Fig. 174. Also, plate or flat ends are used, as in Fig. 175 (and these are the most common), or rings, as at Fig. 176, or compound gudgeons, as at Fig. 177. To strengthen the

Fig. 175.

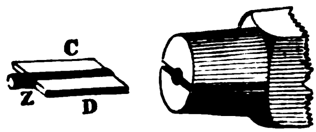


Fig. 176.

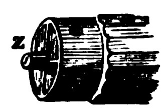
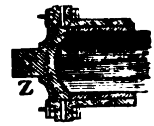


Fig. 177.



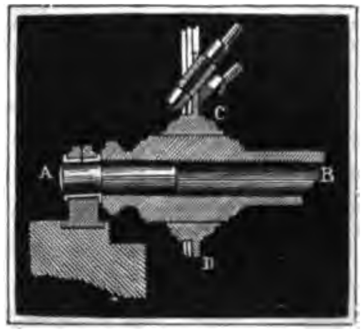
neck of the axle, to prevent its splitting, it is dressed off conically, and three iron rings, ¼ to ½ inch thick, and 1½ to 3 inches broad, are driven on while hot. The plates in the flat gudgeon ends are from 1 to 3 inches thick, and about an inch narrower than the diameter of the axle. The ring attachment is convenient, when a spur wheel is to be placed at the neck of the axle; the compound gudgeon is applied when much *wear* is anticipated, because the end plates are easily removed and renewed. Cast iron axles are either hollow or solid, either round or polygonal in section, and sometimes *ribbed* or feathered to increase their stiffness.

For solid axles the gudgeon is generally in one piece with the axle. Fig. 178 is a simple round axle, Fig. 179 is a feathered axle,

Fig. 178.

Fig. 179.

Fig. 180.



and Fig. 180 is the end of a hollow iron axle, with a gudgeon put in, and an arm plate or nave set upon it.

The gudgeons rest on supports termed *plummers* or *plumbing* blocks, which, to afford a permanent seat for the wheel, are placed on substantially founded walls. The plumbing block is lined with a brass or other movable seat for the gudgeon. These seats are either of *brass* (hence termed generally *brasses*), or of gun-metal (8 parts copper, 1 part tin), or of white metal; sometimes they are of wood, though seldom.

The gudgeons rest on a wooden block in Fig. 171. Fig. 181 is a

simple, uncovered, cast iron block. Fig. 182 is an open block, with a metal seat, or lining, and Fig. 183 is a close or covered block with

Fig. 181.

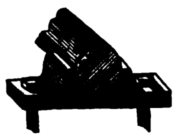


Fig. 182.

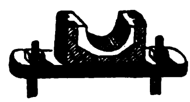
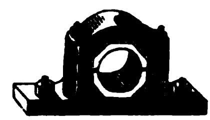


Fig. 183.



metallic lining. The plumbing blocks are bolted down by means of bolts and sole plates to the walls or beams on which the wheel is to rest. The cover of blocks is always provided with a hole, through which grease can be supplied. The inside of the cover is sometimes grooved, so that the grease diffuses more readily over the gudgeon. And, wherever it is desired to reduce the resistance from friction to a minimum, a grease cup, affording a *constant supply*, is placed in communication with a hole in the plumbing-block cover.

§ 87. *The Proportions of Water Wheels.*—The first or main element of a water wheel is the velocity of the circumference v, or the number of revolutions u. It will be seen in the sequel, that overshot wheels should have a very small velocity. Many wheels have a velocity of 10 feet per second, but 5 feet is more suitable, yet under $2\frac{1}{2}$ feet is not advisable. The velocity c of the water entering the wheel, should depend on the velocity of the wheel, and is either equal to this, or greater in a certain proportion. For creating the velocity c, a fall or height of head, AB (Fig. 184) $= h_1 = \frac{c^2}{2g}$ is necessary, leaving of the total fall $AF = h$, only the fall on the wheel $= BF = h_2 = h - h_1 = h - \frac{c^2}{2g}$.

Fig. 184.

As even in the case of the most perfect discharge, 6 per cent. of *vis viva* is lost, it is advisable to take it as 10 per cent. in this case, and, therefore, to put the effective fall required to bring the water on to the wheel with suitable velocity $h_1 = 1{,}1 \cdot \frac{c^2}{2g}$, and hence $h_2 = h - 1{,}1 \cdot \frac{c^2}{2g}$. From the fall on the wheel h^2, we deduce the semi-diameter of the wheel $CF = CS = a$, by assuming the angle

$SCD = \Theta$, by which the point of entrance of the water D deviates from the summit S as given.

Then $h_2 = CF + CB = a + a \cos. \Theta = (1 + \cos. \Theta)\, a$, and hence, inversely, $a = \frac{h - h_1}{1 = \cos. \Theta}$. From the radius of the wheel a, and the velocity v at the circumference, the number of revolutions per minute $u = \frac{30\, v}{\pi\, a}$.

When u is given, we can determine a and v. As $v = \frac{\pi u a}{30}$, and $c = \varkappa \frac{\pi u a}{30}$, in which $\varkappa$ is a given ratio $\frac{c}{v}$, we have:

$$(1 + \cos. \Theta)\, a = h - \frac{1{,}1}{2g} \times \left(\frac{\varkappa \,.\, \pi\, u\, a}{30}\right)^2,$$

and hence $a = \frac{h - 0{,}000193\, (\varkappa\, u\, a)^2}{1 + \cos. \Theta}$, and the solution of this quadratic equation gives:

1. $a = \frac{\sqrt{0{,}000772\, (\varkappa\, u)^2\, h = (1 + \cos. \Theta)^2} - (1 + \cos. \Theta)}{0{,}000386\, (\varkappa\, u)^2}$, and

hence:

2. $v = \frac{\pi u a}{30} = 0{,}1047 \,.\, u\, a.$

Example 1. For a fall of 30 feet, a wheel is to be constructed to have 8 feet velocity at circumference, and taking on the water, at 12° from the summit with twice the above velocity. What is the radius of wheel required, and what the number of revolutions? $c = 2 \times 8 = 16$ feet, and hence $h_1 = 1{,}1 \times 0{,}0155 \times 16^2 = 4{,}36$ feet, and $a = \frac{30 - 4{,}36}{1 + \cos.\, 12^\circ} = \frac{25{,}64}{1{,}978} = 12{,}9$ feet; lastly, $u = \frac{30 \times 8}{\pi \times 12{,}9} = 5{,}92$.

Example 2. If, inversely, the number of revolutions be 5, then for the above fall, and other proportions $\varkappa = 2$, and the radius of the wheel:

$$a = \frac{\sqrt{2{,}316 + 3{,}9125} - 1{,}978}{0{,}0386} = \frac{0{,}5177}{0{,}0386} = 13{,}41 \text{ feet.}$$

Again, the velocity at the circumference $v = 0{,}1047 \times 5 \times 13{,}41 = 7{,}02$ feet, the velocity at entrance $= 14{,}04$ feet, and lastly, the height of fall due to this latter velocity $= h_1 = 1{,}1 + 0{,}0155 \times 14{,}04^2 = 3{,}47$ feet.

§ 88. The proportions of the wheel, in reference to *depth* of the shrouding and width of the wheel, are important. The depth of the crown (or *water space*) is made 10 to 12 inches, and sometimes even 14 to 15 inches, and this proportion is chosen, because the water in a wheel with *shallow* shrouding, acts with greater leverage than it would do on a wheel of equal radius with deeper crowns. As to the width or breadth of the wheel, it depends on the capacity to be given to the wheel. If d be the depth of crowns, and e the width of the wheel (or distance between the internal surfaces of the crowns), then the section of the annular space above the flooring of the wheel is $= d\, e$, and if v be the velocity at the middle of the crown's depth, the capacity presented to the water, per second, is $d\, e \,.\, v$. But this cannot be considered equal to the quantity of water delivered

on the wheel, because a certain portion of this capacity is taken up by the substance of the buckets, and it is also inexpedient to fill up the buckets to the brim. We must, therefore, put $d e v = \varepsilon Q$, in which equation $\varepsilon > 1$; ε is usually $= 3$ to 5, the former when the buckets are filled rather in excess, the latter when they are deficiently filled. The width of wheel is, however, now determined:

$$e = \frac{\varepsilon Q}{d v}, \text{ or as } v = \frac{\pi a u}{30}, \text{ hence } e = \frac{30 \varepsilon Q}{\pi u a d} = 9{,}55 \frac{\varepsilon Q}{u a d},$$

and taking $\varepsilon = 4$, then $e = 38{,}2 \dfrac{Q}{u a d}$. That wheels of very great diameter may not be too narrow, it is advisable to assume $\varepsilon = 5$.

The number of buckets n is another important element in the construction of water wheels. The more cells there are, the longer will the water be retained on the wheel. But this number has its limits, because the buckets occupy space, taken from the capacity of the wheel, and the more the capacity is diminished for a given quantity of water delivered on the wheel, the sooner the water will leave it. As iron, that is sheet iron buckets, are much thinner than those of wood, we may adopt a greater number of iron buckets, than we should do of wooden buckets. We may follow the rule, to place the buckets at such a distance from each other, that at that point where the wheel begins to *spill*, or lose its water, the bucket next above, ABD, Fig. 185, shall not dip into the water of the one below at B, for if we put the buckets closer than this, the upper bucket diminishes the capacity of that under it, and so what we gain in one respect is lost in another. The number of buckets is generally made $n = 5\,a$ to $6\,a$, or according to Langsdorf $n = 18 + 3\,a$; in which expressions a is the radius of the wheel in feet: or the distance between any two buckets is made $= 7\left(1 + \dfrac{d}{10}\right)$ inches. From the given, or thus found number of buckets n, we have the angle of subdivision β, *i. e.*, the central angle between two adjacent buckets, $\beta = \dfrac{360^\circ}{n}$.

Fig. 185.

Example. Suppose an overshot wheel of 15 feet radius, having 1 foot depth of crown and taking 10 cubic feet of water per second, makes 5 revolutions per minute, the width of the wheel must be $38{,}2 \dfrac{10}{5 \,.\, 15 \,.\, 1} = 5{,}1$ feet, and the distance between two buckets is to be $7\left(1 + \dfrac{12}{10}\right) = \dfrac{77}{5} = 15{,}4$ inches, and hence the number of buckets $= \dfrac{2 \,.\, \pi \,.\, 15 \,.\, 12}{15{,}4} = 73$, or 72 for the sake of easier division of the circle. The angle of subdivision is $\beta = \dfrac{360}{72} = 5^\circ$.

§ 89. *Form of Buckets.*—The form of the cells or buckets is of much consequence to the efficiency of water wheels. The buckets must have such form and position, that the water may enter freely,

remain in them to as near the bottom of the wheel as possible, but no further. By the various forms adopted, these requirements are more or less perfectly fulfilled. The two requirements are in fact often, to a certain extent, incompatible; for if the cells be made very close, the entrance, as well as the exit of the water, becomes much impeded. If the buckets be merely plane-boards, as shown at AD, Fig. 186, the entrance of the water is quite free certainly, but then it leaves the cells too soon, so that there is a great loss of mechanical effect. To prevent this too early loss of water, the bucket would have to be very long, and the angle ADE, at which the bucket inclines to the radius CE, very large, *i. e.*, nearly a right angle. As this is a practical difficulty in construction, it is preferred to make the bucket in two parts, or by a second piece DB, to give the bucket a *bottom* or *flooring* of its own. The bottom DB is sometimes termed the start, or *shoulder*, and the outer piece BA, the *arm*, or *wrist*. The former is generally placed in the direction of the radius, sometimes at right angles to the outer piece, or arm. The circle passing through the *elbow* B, made by the junction of the shoulder and *arm*, is termed the *division circle*. In the older construction of wheels, this circle is generally found placed at $\frac{1}{3}$ of the depth of the shrouding from the interior, or the sole of the wheel. As, however, the capacity of a cell is greater the wider the shoulder-blade DB (Fig. 187) is, or the greater the angle ABE (which we term the *elbow angle*), we now usually find the division circle in the middle of the depth of the shrouding. The capacity of a cell will then depend only on the width or position of the arm. The simplest construction of buckets, is to make the end A of the arm AB start from the prolongation of

Fig. 186.

Fig. 187.

Fig. 188.

the shoulder next above it D_1B_1, or, by letting the arm be included between the sides of the division angle $\beta = \frac{360^\circ}{n}$. But this construction does not close or cover the cells sufficiently, except for very shallow shrouding, and, therefore, the usual plan, for wheels up to 35 to 40 feet diameter, is to let the *arm* extend over $\frac{3}{4}$ of the dimension angle, or the arc EA is made $= \frac{3}{4}\ EE_1$, Fig. 188. From the radius

$CA = a$, and the central angle $ACB = \beta_1$, included by the *arm*, we can easily find the elbow angle $ABE = \delta$. In the triangle ACB as $CB = CE - BE = a - \frac{b}{2}$, then:

$$tang.\ \delta = \frac{a\ sin.\ \beta_1}{\frac{b}{2} - a\,(1 - cos.\ \beta_1)}.$$

Example. A 30 feet wheel, shrouding 10 inches deep, is to have (according to Langsdorf's rule) $18 + 3 \times 15 = 63$ buckets, or, say 64, and each is to extend over $\frac{5}{4}$ of the division angle. What will be the elbow angle? We have:

$$\beta = \frac{360}{64} = 5\tfrac{5}{8}{}^\circ, \text{ hence } \beta_1 = \tfrac{5}{4} \times \tfrac{45}{8} = 7\tfrac{1}{32}{}^\circ = 7^\circ, 1', 52'', 5, \text{ and}$$

$$tang.\ \delta = \frac{15\ sin.\ 7^\circ, 1', 52'', 5}{\frac{5}{12} - 15\,(1 - cos.\ 7^\circ, 1', 52'', 5)} = \frac{36\ .\ 0{,}12241}{1 - 36 \times 0{,}00752} = \frac{4{,}40678}{0{,}72928},$$

hence $\delta = 80^\circ, 30'$.

§ 90. The position of the *arm* of the bucket may likewise be determined, by adopting as a rule, that the least section of a cell shall be somewhat greater than the section of the water coming on to the wheel. If the cells come exactly under the water-*jet*, then this construction would permit the water to enter freely, and the air to escape unhindered. This may be done as follows. From a point of division B_1, Fig. 189, in the division circle (or *line of pitch of the buckets*) with a radius greater by one or two inches than the thickness of the water-jet, or layer, describe a circle, and from the next adjacent point of division B, draw a tangent BA to this circle. This is the position of the arm of the bucket required, for then the least width B_1N of the cell ABD_1 is equal to the radius of that circle. But in order to determine the thickness of the layer of water coming (from the pentrough) on to the wheel, we must know the fall h_1 from the surface of the water in the pentrough to the point B_1N; also the quantity of water, and the width e of the wheel.

Fig. 189.

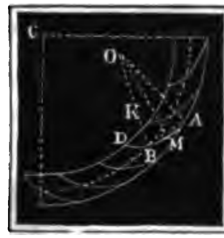
As $Q = d_1 e \sqrt{2gh_1}$, therefore, the thickness in question $d_1 = \frac{Q}{e\sqrt{2gh_1}}$, and to allow of free escape of air, we make $d_1 = B_1N = \frac{Q}{e\sqrt{2gh_1}} + 1$ or 2 inches. If equal spaces for the exit of air and entrance of water be allowed, then $d_1 = \frac{2\,Q}{e\sqrt{2gh_1}}$ would be the equation to be satisfied.

Buckets in *three parts*, as $AEBD$, Fig. 190, give, *cæteris paribus*, more capacity than those in two parts, without any greater contraction. There is, therefore, mechanical advantage in this form, though it be more expensive to execute. Curved buckets are best of all, as shown in Fig. 191, and this is the great advantage of sheet iron, to which this form can readily be given. If the section of bucket is to

be composed of two segments of circles, then it is only necessary to find the position of the *arm* of a bucket, by any of the planes above

Fig. 190.

Fig. 191.



given—at its bisection M (Fig. 191), to erect a perpendicular, and from any point O at will to describe an arc with the radius OB, and from any point K in it, to describe another arc to complete the bucket ABD of a suitable form.

Example. If, in the wheel of our example to § 88, the jet or layer of water falling on the wheel has 2½ feet fall, then as:

$$Q = 10, \text{ and } e = 5{,}1,\ d_1 = \frac{0{,}1265 \,.\, 10}{5{,}1 \,.\, \sqrt{2{,}5}} = \frac{1{,}265}{8{,}064} = 0{,}157 \text{ feet.}$$

If now we allow an equal thickness for the exit of the air, then the least distance of two buckets becomes 0,314 feet, or 3¾ inches.

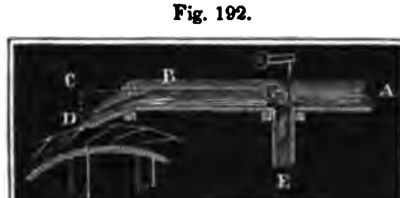
Remark. To find the elbow angle δ, in that construction of bucket which is based on the thickness of the layer of water, let us put:

$$\delta = 180^\circ - CBA = 180^\circ - CBB_1 - B_1BA = 180^\circ - \left(90 - \frac{\beta}{2}\right) - \phi = 90^\circ + \frac{\beta}{2} - \phi,$$

but $\sin. \phi = \frac{B_1 N}{B B_1} = \frac{d_1}{2\, a_1 \sin. \frac{\beta}{2}}$, when d_1 is the least distance between two buckets, and a_1 the radius of the *division circle*. For the last example, $\beta = 5^\circ$, $d_1 = 0{,}314$, $a_1 = 14{,}5$ feet, hence $\sin. \phi = \frac{0{,}314}{29 \sin 2\frac{1}{2}^\circ} = \frac{0{,}314}{1{,}265} = 0{,}2482$, hence $\phi = 14^\circ, 22'$, and $\delta = 90^\circ + 2^\circ, 30' - 14^\circ\, 22' = 78^\circ, 8'$.

§ 91. *Sluices, Pentroughs,* or *Penstocks.*—The method of bringing the water on the wheel is of no small importance. Either the water falls freely out of the lead or trough, or it is pent up by a sluice, or pentrough, or penstock, before entering the wheel. In the former case, the velocity of entrance depends on the inclination of the trough or the height of fall. In the second case, it may be regulated by adjusting the height of head created, and, therefore, this latter method should be preferred. Fig. 192 shows a trough without a regulating sluice; but there is a *waste board* at F by which the *quantity* of water can be regulated. If the water flows along the

Fig. 192.



trough with a velocity c_1, and if the fall from the end of it to the centre of the cell $= h_1$, the velocity

$$c = \sqrt{2 g h_1 + c_1^2} = \sqrt{2 g h_1 + \left(\frac{Q}{F}\right)^2},$$

if Q be the quantity of water, and F the sectional area of the water coming on the wheel.

The penstock (Fr. *vannes;* Ger. *Spannschutze*) is either vertical, horizontal, or inclined. Fig. 193 shows the arrangement of a hori-

Fig. 193.

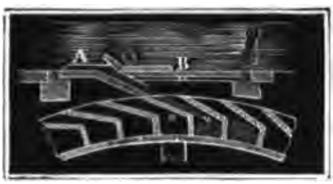
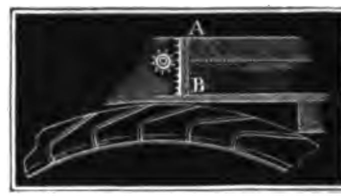


Fig. 194.



zontal sluice, and Fig. 194, that of a vertical sluice. The construction of inclined sluices as shown in Figs. 195 and 196. The one,

Fig. 195.

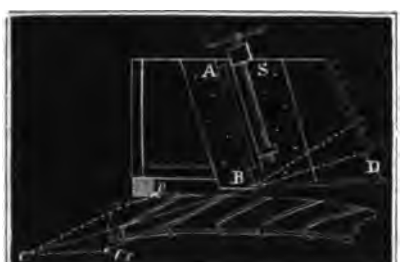


Fig. 196.

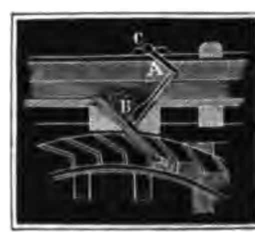


Fig. 195, is the arrangement general in the Freiberg district, the sluice being raised and depressed by means of a screw *S*. In Fig. 196, a simple lever is used for these purposes. It is a general rule for these penstocks, to make them as smooth as possible inside, and to round off the edges of the orifice, so as to adapt it to the form of the *contracted vein*, that the resistance may be the least possible. If the water, after passing the sluice, fall quite freely, and if we can place the plane of the orifice at right angles to the jet of water, it becomes then advisable to make the orifice as in a *thin plate*, but in that case, care must be taken that partial contraction does not occur, for this gives rise to an obliquity of the jet (Vol. I. § 319).

In the discharge from penstocks, the velocity of discharge is deduced from the height h_1 by the formula $c_1 = \phi \sqrt{2 g h_1}$, and if h_2 be the height of fall after passing the orifice, to the centre of the cell, then the velocity of entrance $c = \sqrt{c_1^2 + 2 g h^2} = \sqrt{2g (\phi^2 h_1 + h_2)}$. If we take the velocity co-efficient $\phi = 0{,}95$, then $c = \sqrt{2g (0{,}95 h_1 + h_2)}$.

We see from this, that for equal falls the velocity of entrance must be very nearly equal, whether it flow on freely, or be discharged from a sluice, on to the wheel.

§ 92. That the water may enter unimpeded into the wheel cells, it must not come in contact with the bucket at the outer circumference, but nearer to the inner circumference or bottom of the cells. Hence, not only must the outer edge of the buckets be sharpened off, but the layer of water AC, Fig. 197, must be so directed that its velocity may be decomposed into two others, one of which is in the direction of the velocity of the wheel $Av = v$, and the other in the direction AB of the *arm* or *wrist* of the bucket. As we may assume the direction of the outer element of the bucket—the velocity at the outer circumference of the wheel v, at right angles to the radius AC of the wheel—and the velocity c of the water coming on to the wheel, to be given, we shall have the required direction of the water layers if we draw through v a parallel to AB, and with c as radius, describe an arc from A as centre, and draw from A to the intersection of the arc with the parallel, the straight line Ac, or by calculation as follows:

Fig 197.

The angle which the velocity v of the circumference makes with the outer element of the bucket $AB = v\,AB = \phi$, may be deduced from the elbow angle $ABE = \delta$, and the division angle $ACB = \beta$, by the equation $\delta = ACB + BAC = \beta_1 + 90^\circ - \phi$, and hence $\phi = 90^\circ - (\delta - \beta_1)$.

From ϕ, v and c we have the angle $c\,AB = \psi$, by which the direction of the layer of water must deviate from that of the arm of the bucket, in order that the water may enter the cells unimpeded: for $\frac{sin.\ \psi}{sin.\ \phi} = \frac{v}{c}$, and, therefore,

$$sin.\ \psi = \frac{v\ sin.\ \phi}{c} = \frac{v\ cos.\ (\delta - \beta_1)}{c}.$$ (See Vol. I. § 32.)

Again; the angle $c\,AH$ of the direction of the water layer to the horizon, is $\nu_1 = \phi - \psi + \theta$, θ being as above the angle ACS, by which the point of entrance of the A water on the wheel, deviates from the summit S.

The relative velocity $Ac_1 = c_1$ with which the water enters the cells is $c_1 = \frac{c\ sin.\ (\phi - \psi)}{sin.\ \phi}$.

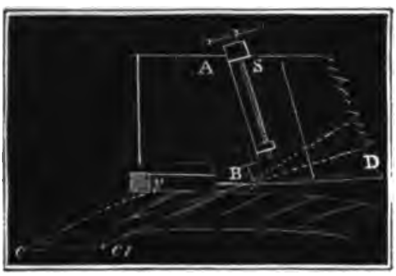
Example. Suppose a water wheel, the velocity at the circumference of which $v = 10$ feet, the velocity of the water $c = 15$ feet, the elbow angle $= 70\frac{1}{2}^\circ$, the division angle $\beta_1 = 4\frac{1}{2}^\circ$, and the point of entrance of the water deviates 12° from the summit: then $\phi = 90^\circ - (70\frac{1}{2}^\circ - 4\frac{1}{2}^\circ) = 24^\circ$, and, therefore, $sin.\ \psi = \frac{10}{15}\ sin.\ 24^\circ = 0{,}27116$, hence $\psi = 15^\circ, 44'$. Thus, that the water may enter unimpeded, the deviation of the layer

must be $15\frac{3}{4}°$ from that of the arm or outer element of the bucket. The angle of inclination to the horizon, or $\nu_1 = 24° - 15\frac{3}{4}° + 12° = 20\frac{1}{4}°$, and the relative velocity is $c_1 = \frac{15\, sin.\, 8°.\, 16'}{sin.\, 24°} = 5{,}303$ feet.

Remark. It is generally considered, in older works on this subject, that the water layer should *enter the wheel in the direction of the arm of the bucket;* but this rule is only true when $v = 0$, or $\delta - \beta_1 = 90°$, *and these cases never occur.* The deviation ψ is of course very small for a wheel revolving slowly, but never so small as to allow of our assuming it as 0. When the water enters in the direction of the outer element of the bucket, the bucket strikes against the water, and throws it before it with a velocity $v\, sin.\, \phi$, by which *vis viva* is lost, and water spilt.

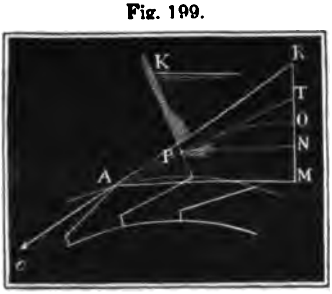
§ 93. That the water may reach the wheel with the direction required, either the sluice-opening is laid close up to the point at which the water is to enter the wheel, and the sluice is set at right angles to the direction of the layers of water, or an additional trough is laid in the required direction of the layer, or the sluice is so placed, that the direction of the parabolic curve, formed by the water in its free descent, may be that required. The Fig. 198 shows the pen-trough used in the Freiberg district, in which the bottom piece *BD*, and the lower part of the sluice-board, are set obliquely to the direction of the water layer, so that each makes an angle of about $14\frac{1}{2}°$ with the direction of the axis of it.

Fig. 198.



In order to find the direction of the sluice-board, when part of the water falls freely into the wheel, we have to recur to the theory of projectiles, given in Vol. I. § 38, &c. From the velocity $Ac = c$, Fig. 199, and the angle of inclination $RAM = \nu_1$ of the required direction of the layer to the horizon, the vertical co-ordinate MO of the apex of the parabola is: $x_1 = \frac{c^2\, sin.\, \nu_1^{\,2}}{2g}$, and, on the other hand, the horizontal co-ordinate:

Fig. 199.



$$AM = y_1 = \frac{c^2\, sin.\, 2\nu_1}{2g}.$$

If, now, we wish to place the sluice-aperture at any point P of this parabolic curve, and if we know the height $MN = a$, of this point above the point of entrance A, then for the co-ordinates of this point $ON = a$, and $NP = y$, we have the formulas: $x = x_1 - a$, and

$$y = y_1 \sqrt{\frac{x}{x_1}} = y_1 \sqrt{\frac{a}{x_1}},$$

and for the angle of inclination $TPN = \nu$, which the parabola makes with the horizon on this point,

$$tang.\ \nu = \frac{TN}{PN} = \frac{2\ ON}{PN} = \frac{2x}{y}.$$

The plane PK of the sluice-board must be set at right angles to the tangent PT; and thus we find the required position of the sluice board, if we set off the abscissa ON in the opposite direction OT, draw PT, and erect a perpendicular to it PK.

If the sluice-aperture be set at the apex of the parabola, then the sluice-board will have to be vertical.

The velocity of discharge at P is $c_0 = \sqrt{c^2 - 2\,g\,a}$, and the corresponding theoretical pressure height $h_0 = \frac{c^2}{2g} - a$, or the effective height $= 1{,}1\left(\frac{c^2}{2g} - a\right)$, when the orifice is nearly rounded. The breadth of the sluice-orifice is made a little less than the breadth of the wheel.

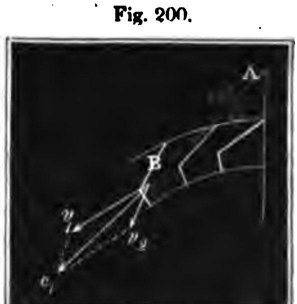
Example. For velocity $c = 15$ feet, and angle $\nu_1 = 20\frac{1}{4}°$ (see example to last paragraph), the co-ordinates of the parabola's apex are $x_1 = 0.0155\ .\ 15^4\ (sin.\ 20\frac{1}{4}°)^2 = 0{,}43$ feet, and $y_1 = 0{,}0155\ .\ 15^2\ .\ sin.\ 40\frac{1}{2}° = 2{,}33$ feet. If, now, the centre of the sluice-aperture is to be 4 inches $= 0{,}333$ feet above the point of entrance, then the co-ordinates from the centre of the opening:

$$x = 0{,}43 - 0{,}33 = 0{,}1,\ y = 2{,}33\ \sqrt{\frac{0.1}{0{,}43}} = 1{,}11 \text{ feet, and}$$

$tang\ \nu = \frac{0{,}2}{1{,}11} = 9°, 58'$, and hence the inclination of the sluice-board to the horizon is $= 90° - \nu = 90° - 9°, 58' = 80°, 2'$.

§ 94. *Effect of Impact.*—In the overshot wheel, the water acts in some degree by impact, but chiefly by its weight. We determine the effect of the shock, by deducting from the whole effect corresponding to the *vis viva* which the water entering the wheel possesses, the mechanical effect retained by the water when it leaves the wheel, and that lost by the oscillatory and eddying motion of the water in the cells. The velocity of the water leaving the wheel may be assumed as equal to the velocity v_1 of the wheel in the division circle, and hence the mechanical effect retained in this water is $\frac{v_1^2}{2g} Q\gamma$. The mechanical effect lost by the oscillation and eddying motion of the water may be put equal to $\frac{v_1^2}{2g} Q\gamma$, where v_2 is the velocity suddenly lost by the water entering the wheel. If, therefore, c_1 be the velocity Bc_1, Fig. 200, of the water entering the wheel, the mechanical effect still inherent in its *vis viva* is:

Fig. 200.



$$L_1 = \left(\frac{c_1^2 - v_1^2 - v_2^2}{2g}\right) Q\gamma.$$

But the velocity c_1 may be decomposed into two others $Bv_1 = v_1$, and $Bv_2 = v_2$, of which v_1 is exactly the velocity retained by the water as it moves on with the wheel, and, therefore, v_2 is the velocity *lost*. If we put the angle $c_1 Bv_1$, which the direction of the entrance velocity c_1 of the water makes with a tangent $B v_1$ (the direction of the velocity of the circumference) $= \mu$, then we have $v_2^2 = c_1^2 + v_1^2 - 2\, c_1\, v_1 \cos. \mu$, and, therefore, the mechanical effect in question:

$$L_1 = \left(\frac{c_1^2 - v_1^2 - c_1^2 - v_1^2 + 2\, c_1\, v_1 \cos. \mu}{2g}\right) Q\gamma =$$

$$\frac{(c_1 \cos. \mu - v_1)\, v_1}{g}\, Q\gamma, \text{ or as } \frac{1}{g} = 0{,}031,$$

and $\gamma = 62{,}5$, $L = 2{,}008\, (c_1 \cos. \mu - v_1)\, v_1\, Q$ feet lbs.

It is evident that the mechanical effect of impact is so much the greater, the greater c_1 is, and the less μ; and by comparing with Vol. I. § 386, it follows that this effect is a maximum when $v_1 = \frac{1}{2}\, c_1 \cos. \mu$. The maximum effect corresponding to this latter ratio is $\frac{1}{2}\, \frac{c_1^2 \cos. \mu^2}{2g}\, Q\gamma$; or when $\mu = 0$, or $\cos. \mu = 1$, then $L = \frac{1}{2} \cdot \frac{c_1^2}{2g} Q\gamma$. As $\frac{c_1^2}{2g}$ is the fall due to the velocity c_1, it follows, *that, in the most favorable case, the effect of impact is only half the available effect.* Hence the least possible part of the fall should be spent to produce impact, as much as possible being employed as weight. Suppose, for instance, we make $c_1 \cos. \mu = v_1$, therefore, $c_1 = \frac{v_1}{\cos. \mu}$, we sacrifice a height of fall $\frac{v_1^2}{2g \cos. \mu^2}$, without having any mechanical effect in return, but if we make $c_1 = \frac{2\, v_1}{\cos. \mu}$, we expend four times that fall, viz: $4 \cdot \frac{v_1^2}{2g \cos. \mu^2}$, and yet we have only:

$$\tfrac{1}{2} \cdot \frac{4\, v_1^2}{2g} Q\gamma = 2 \cdot \frac{v_1^2}{2g} Q\gamma,$$

and lose thereby the amount of fall represented by: $\left(\frac{4}{\cos. \mu^2} - 2\right) \frac{v_1^2}{2g}$, and even if we assume $\mu = 0$, or $\cos. \mu = 1$, the loss of fall is $2 \cdot \frac{v_1^2}{2g}$, or double as much as when we avoid all shock, or bring the water on to the wheel with the velocity with which the wheel revolves. Again, we perceive *that the efficiency of the wheel will be greater the less v_1 is, or the slower the wheel revolves.* It is true that the capacity of the wheel, its width e, and, therefore, its height, must be greater as the velocity of revolution v is less; and as the journals of a wheel must be of greater diameter the heavier the wheel is, and as the moment of friction increases as the radius of the journal, the mechanical effect consumed by the journal friction

in the case of the wheel revolving slowly, may be greater than in one moving more rapidly; and hence we perceive that it by no means follows as a matter of course, that the slower a wheel revolves, the greater its efficiency will be.

§ 95. *Effect of the Water's Weight.*—The cells of a water wheel, when filled, form an annular water space AB, Fig. 201, which is termed the water arc, as the water enters at the upper part of this arc, and leaves it at the lower end, its height h is the effective fall, and, therefore, the mechanical effect given off by the weight of water $= h \cdot Q\gamma$. The height of the water arc may be subdivided into three parts. The first part HM lies above the centre of the wheel, and depends on the angle $SCA = \theta$, by which the point of entrance deviates from the vertical passing through the summit of the wheel. If, again, we put the radius of the wheel $CA = a$, the height of the upper part of the water arc $MH = a \cos. \theta$. The second part MK lies below the centre of the wheel, and depends upon the point D, at which the wheel begins to lose water, or to *spill*. If we put the angle MCD by which this point lies below the centre of the wheel $= \lambda$, then this second height $MK = a \sin. \lambda$. The

Fig. 201.

third part includes the arc DB, in the course of which the wheel empties each bucket in turn. If we put MCB, the angle by which the point B, at which the buckets are emptied, deviates from M the centre of the wheel $= \lambda_1$, then the height $KL = a (\sin. \lambda_1 - \sin. \lambda)$. Whilst now in the two upper parts of the arc, the water has its entire effect, it communicates only a part of its mechanical effect to the wheel in this third part, because here it gradually quits the wheel, and, therefore, the total effect of the water's weight must be represented by $a (\cos. \theta + \sin. \lambda) Q\gamma + a (\sin. \lambda_1 - \sin. \lambda) Q_1 \gamma$, when Q_1 is the mean quantity of water effective in the lower division of the water arc.

If we combine with this, the effect of the impact of the water, we have the total mechanical effect of an overshot water wheel:

$$L = Pv = \left(\frac{(c_1 \cos. \mu - v_1) v_1}{g} + a (\cos. \theta + \sin. \lambda)\right) Q\gamma$$
$$+ a (\sin. \lambda_1 - \sin. \lambda) Q_1 \gamma;$$

or, if we put the height $a (\cos. \theta + \sin. \lambda)$ of the part of water arc taking up the entire effect of the water $= h_1$, and the remaining part $a \sin. \lambda_1 - \sin. \lambda) = h_2$, and the ratio $\frac{Q_1}{Q} = \varkappa$, then:

$$L = Pv = \left(\frac{(c_1 \cos. \mu - v_1) v_1}{g} + h_1 + \varkappa h_2\right) Q \gamma,$$

and the force at circumference of the wheel:

$$P = \left(\frac{(c_1 \cos. \mu - v_1) v_1}{g} + h_1 + \varkappa h_2\right) \frac{Q}{v} \gamma.$$

Example. The velocity of entrance of the water on an overshot wheel of 30 feet diameter is $c_1 = 15$ feet, the velocity v_1 of the division circle $= 9\frac{1}{4}$ feet. The angle by which the direction of the water layer deviates from the direction of motion of the wheel at the point of entrance, is $8\frac{1}{2}°$, and the deviation of this point from the summit of the wheel is $12°$. The deviation of the point where the wheel begins to lose water from the centre of the wheel $\lambda = 58\frac{1}{2}°$, and the deviation of the lowest point in the water arc from the centre of the wheel, or $\lambda_1 = 70\frac{1}{2}°$. Lastly, the quantity of water going on to the wheel $Q = 5$ cubic feet per second, and $\varkappa = \frac{Q_1}{Q}$ is assumed as $\frac{1}{2}$. Required the effect of the wheel. First, the effective *impact fall* $= 0{,}031\ (15 \cos. 8\frac{1}{2}° - 9\frac{1}{4})\ .\ 9\frac{1}{4} =$ 1,60 feet; and the effective *weight fall* is:
$15\ (\cos. 12° + \sin. 58\frac{1}{2}°) + \frac{1}{2}\!\!/\ (\sin. 70\frac{1}{2}° - \sin. 58\frac{1}{2}°) = 15\ (1{,}8307 + 0{,}0450 = 28{,}14$ feet, and hence the total effect of the wheel is $(160 + 28{,}14)\ .\ 5\ .\ 62{,}5 = 9256$ feet lbs. $= 17$ horse power, and the force at the division circle is $\frac{9256}{9\frac{1}{4}} = 1000$ pounds, nearly.

§ 96. We easily perceive from this, that for the exact determination of the effect of the weight of the water on an overshot wheel, it is essential to know the two limits of the arc in which the wheel loses its water, and the ratio $\varkappa = \frac{Q_1}{Q}$, of the mean quantity of water contained in the cells in this part of the water arc, to that originally received by them. On this subject we must now endeavor to ascertain the necessary rules.

If there be n buckets in the wheel, and if it make u revolutions per minute, there are presented $\frac{nu}{60}$ cells per minute to receive the quantity of water Q, and, therefore, into each cell there goes the quantity $V = Q \div \frac{nu}{60} = \frac{60\ Q}{nu}$. If e be, as hitherto, the width of the wheel, then the section of the prism of water in any cell $= F_0 = \frac{V}{2} = \frac{60\ Q}{n\ u\ e}$.

If now $DEFG$, Fig. 201, be the cell at which the water begins to spill, then the section: $F_0 =$ segment $DEF +$ triangle DFG, or as the triangle $DFG =$ triangle $DFN -$ triangle GN, then $F_0 =$ segment $DEF +$ triangle $DFN -$ triangle DGN. If we put the area of the segment $DEF = S$, and that of the triangle $DFN = D$, then the triangle $DGN = S + D - F_0$; but as the triangle DGN is also equal to $\frac{DN\ .\ NG}{2} = \frac{1}{2} d^2 \ tang.\ \lambda$ nearly, we have approximately (and the more accurately the greater the number of buckets), $tang.\ \lambda = \frac{S + D - F_0}{\frac{1}{2} d^2}$. Thus the angle $MCD = \lambda$, corresponding to the point at which the wheel begins to empty itself, is determined.

Each cell will have emptied itself when the outer element of the bucket becomes horizontal; if, therefore, the angle CBO, which this outer element, or the wrist of the bucket, makes with the radius $= \lambda_1$, then λ_1 gives us the angle MCB, which fixes the point where the cells have emptied themselves. In order, therefore, to find the effect of the water on the discharging arc, let us divide the height $KL = a$ $(sin.\ \lambda_1 - sin.\ \lambda)$ into an even number of equal parts, indicate the position of the bucket for each of these points of division, draw horizontal lines through the sections of the water in the cells for each of these positions, and reckon the areas $F_1, F_2, F_3 \ldots F_n$ of these sections. The mean value F of these may be determined by the Simpsonian rule, putting

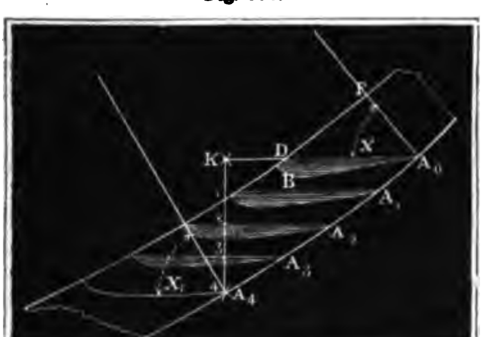
$$F = \frac{F_0 + F_n + 4(F_1 + F_3 + \ldots + F_{n-1}) + 2(F_2 + F_4 + \ldots F_{n-2})}{3n}.$$

and from this we get the ratio of the mean quantity of water in a cell in the discharging arc to the quantity in a cell before it begins to empty itself:

$$x = \frac{Q_1}{Q} = \frac{F}{F_0} = \frac{F_0 + F_n + 4(F_1 + F_3 + \ldots + F_{n-1}) + 2(F_2 + F_4 + \ldots F_{n-2})}{3nF_0}.$$

Example. There are 300 cubic feet of water per minute supplied to a water wheel 40 feet in diameter, making 4 revolutions per minute. What is the effect of such a wheel? If we suppose the depth of the shrouding to be 1 foot, then the width of the wheel $= \frac{4 \cdot 300}{\pi \cdot 40 \cdot 1 \cdot 4} = \frac{30}{4\pi} = 2,4$ feet. If there be 136 buckets on the wheel, the quantity of water in each cell: $V = \frac{300}{4 \cdot 136} = \frac{75}{136} = 0,5515$ cubic feet, and, hence, the section: $F = \frac{0,5515}{2,4}$ square feet $= \frac{144 \cdot 0,5515}{2,4} = 33,09$ square inches. By accurate measurement on the buckets themselves, as they are represented in Fig. 202, the

Fig. 202.



area of the segment A_0BD is 24,50 square inches, and that of the triangle $A_0FD = 102$ square inches, hence for the commencement of discharge:

$tang.\ \lambda = \frac{24.50 + 102 - 30.09}{\frac{1}{2} \cdot 144} = \frac{93,41}{72} = 1,2973$, and, therefore, $\lambda = 52°, 22\frac{1}{2}'$. The angle at which the wrist of the bucket meets the radius is $\lambda_1 = 62°, 30'$, and, therefore, the height KA_4 of the part of the discharging arc retaining water is $= a\ (sin.\ \lambda_1 - sin.\ \lambda)$ $= 20\ (0,8870 - 0,7920) = 1,79$ feet. If within this height we delineate three relative positions of a bucket, we find by measurement and calculations the section of the water

space in the bucket for these positions: $F_1 = 24{,}50$; $F_2 = 14{,}48$, and $F_3 = 6{,}60$ square inches. As, now, the section at the commencement is $F_0 = 33{,}09$, and at the end it is $F_4 = 0$, we shall find the ratio:

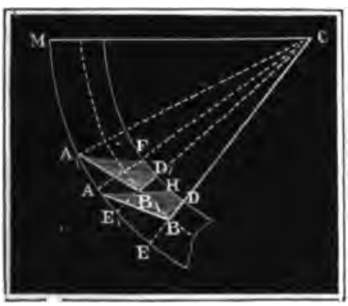
$$\varepsilon = \frac{F}{F_0} = \frac{33{,}09 + 4\,(24{,}50 + 6{,}60) + 2\,.\,14{,}48}{12\,.\,33{,}09} = \frac{15{,}5375}{33{,}09} = 0{,}469.$$

If, again, the water enter the wheel at 10^0 below the summit, and with a velocity $c_1 = \frac{v_1}{cos.\,\mu}$, so that the water acts without shock, then the whole mechanical effect given off by the wheel, neglecting the friction of the axle, is:

$$L = a\,[cos.\,\theta + sin.\,\lambda + 0{,}469\,(sin.\,\lambda_1 - sin.\,\lambda)] \times 5 \times 62{,}25$$
$$= 20\,(0{,}9848 + 0{,}7920 + 0{,}469\,.\,0{,}085)\,6600 = 1{,}8167\,.\,6225 = 11308 \text{ feet lbs.} = 23{,}5$$
horse power.

§ 97. *Number of Buckets.*—As we have above indicated, the capacity of the wheel to hold water should be made as great as possible, or the buckets should retain the water as long as possible, so that, *cæteris paribus*, the maximum effect of the fall of water is obtained when the buckets are placed so close, that the water surface AH, Fig. 203, in the bucket beginning to empty itself, is in contact with the bucket $A_1B_1D_1$ next above it. If we take this condition as basis, we can deduce a formula for determining the number of buckets. From the angle of discharge $MCF = FAH = \lambda$, and the depth of the shrouding $AF = d$, we have, approximately, $FH = d\,.\,tang.\,\lambda$. If, now, we assume the *division circle* to be at half the depth of the shrouding, we may then put:

Fig. 203.



$$D_1H = D_1F = \tfrac{1}{2}\,FH = \tfrac{1}{2}\,d\,tang.\,\lambda.$$

If, again, we introduce the angle of division $ECE_1 = \beta$, and the bucket angle $ACE = \beta_1$, we get the angle $ACE_1 = \beta_1 - \beta$, and, approximately, the arc $D_1F = a_1\,(\beta_1 - \beta)$. By equating the two values of D_1F we have $a_1\,(\beta_1 - \beta) = \frac{1}{2}\,d\,tang.\,\lambda$, and, therefore,

$$\beta = \beta_1 - \tfrac{1}{2}\,\frac{d}{a_1}\,tang.\,\lambda.$$

If the thickness of the buckets $s = 0$, then of course the number of buckets would be

$$n = \frac{360^0}{\beta^0} = \frac{2\pi}{\beta} = \frac{2\pi}{\beta_1 - \frac{1}{2}\frac{d}{a_1}\,tang.\,\lambda};$$

but as the space occupied by the buckets is something considerable, we must take it into calculation, and thus make the division angle greater by an amount corresponding to an arc $\frac{s}{a_1}$, or we must take:

$$n = \frac{2\,\pi}{\beta_1 + \frac{s}{a_1} - \frac{d}{2\,a_1}\,tang.\,\lambda}.$$

If we introduce $tang.\ \lambda = \frac{S + D - F_0}{\frac{1}{2} d^2}$, and $F_0 = \frac{60\ Q}{n\ u\ e}$, then we get:

$$2\pi = n\left(\beta_1 + \frac{s}{a_1} - \frac{S + D}{a_1\ d} + \frac{60\ Q}{n\ u\ a_1\ d\ e}\right),$$

and, therefore, the required number of buckets:

$$n = \frac{2\ \pi\ u\ a_1\ d\ e - 60\ Q}{[\beta_1\ a_1\ d + s\ d - (S + D)]\ u\ e}.$$

If we put $d\ e = \frac{4\ .\ 60\ Q}{2\ \pi\ u\ a_1}$, then we have more simply:

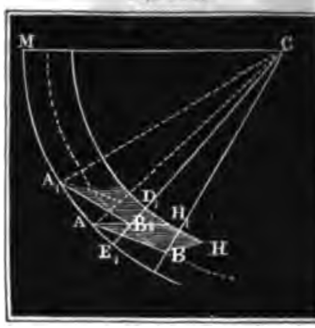
$$n = \tfrac{3}{2}\ .\ \frac{\pi\ a_1\ d}{\beta_1\ a_1\ d + s\ d - (S + D)},$$

and if we take $D = \frac{1}{2}\ \beta_1\ a_1\ d$, then:

$$n = \tfrac{3}{2}\ .\ \frac{\pi\ a_1\ d}{\frac{1}{2}\ \beta_1\ a_1\ d + s\ d - S}.$$

It is also easy to perceive that the angle of discharge λ is still more increased, when (as is represented in Fig. 204) the bucket immediately following that which is about to discharge, comes in contact with the surface of the water AH_1, in that bucket, with a *flat surface* instead of a *corner;* or, when the *wrist* of the bucket is not set in a radial direction, but in such a position, that, shortly before discharge commences, it is horizontal. In this case, the segment or triangle $S = A_1\ B_1\ D_1$, is increased by a triangle $B_1\ D_1\ H$, and hence $tang.\ \lambda = \frac{S + D - F_0}{\frac{1}{2}\ d^2}$, and, therefore, also the angle of discharge λ becomes greater.

Fig. 204.



Iron buckets are always rounded at the corner B.

Example. What number of buckets should be put in an overshot wheel of 40 feet diameter, and 1 foot in width, giving $\beta_1{}^0 = 4^\circ$ or $\beta_1 = 0{,}06981$, $S = 24{,}5$ square inches $= 0{,}17014$ square feet, and the thickness of the buckets $s = 1$ inch $= 0{,}0833$ feet. According to our formula:

$$n = \tfrac{3}{2}\ \frac{\pi\ .\ 19{,}5}{\frac{1}{2}\ .\ 0{,}06981\ .\ 19{,}5 + 0{,}0833 - 0{,}17014} = \frac{29{,}25\ \pi}{0{,}5939} = 155,$$

which, for the sake of facility of division, we may take 152.

Remark. The construction of bucket shown in Fig. 204, has another advantage, viz.: that for it a less width of wheel is necessary, as it is impossible to make the wheel space = four times the capacity of the water space (see Vol. II. § 92). If we here introduce $S = \frac{1}{2}\ a_1\ \beta\ d_1 + \frac{1}{2}\ d^2\ tang.\ \lambda$, and as $S = \frac{1}{2}\ d^2\ tang.\ \lambda + F_0 - D$, and $D = \frac{1}{2}\ a_1\ \beta_1\ d$, we obtain: $tang.\ \lambda = \frac{\frac{3}{2}\ a_1\ \beta_1\ d - F_0}{\frac{1}{2}\ d^2}$, and hence:

$$2\ \pi = n\left(\beta_1 + \frac{s}{a_1} - \beta_1 + \tfrac{1}{2}\ .\ \frac{60\ Q}{n\ u\ a_1\ d\ e}\right), \text{or } 2\ \pi = \frac{n\ s}{a_1} + \tfrac{1}{2}\ .\ \frac{60\ Q}{u\ a_1\ d\ e}.$$

If we neglect the thickness of the buckets, then $e = \frac{1}{2}\ .\ \frac{60\ Q}{2\ \pi\ u\ a_1\ d}$, or a much less width of wheel than was assumed at § 92. We see from this that we should approximate as nearly as possible to the limits of bucket construction, of which we have now been treating.

§ 98. *Effect of Centrifugal Force.*—For equal velocity of the circumference, small wheels make a greater number of revolutions than large; but the uniform motion of the machine, or the nature of the work to be done, as sawing, hammering, grinding, &c. &c., require a certain velocity of the wheel. Hence small wheels frequently make a great number of revolutions per minute. But at such high velocities, the *centrifugal force* of the water comes into play to such a degree that the inclination of the surface of the water in the buckets to the horizon is considerable, and, therefore, the discharge commences much earlier than would be the case if the wheel were moving slowly. We have found (Vol. I. § 274) that the surfaces of the water in the buckets are a series of cylindrical hollows, the common axis of which O, Fig. 205, runs parallel to the axis of the wheel, and lies at a height: $CO = k = \frac{g}{\omega^2} = g \cdot \left(\frac{30}{\pi u}\right)^2 = \frac{2850}{u^2}$ above the axis C of the wheel. This distance increases, therefore, *inversely* as the *square of the number of revolutions*, and becomes small for a great number of revolutions. Hence we at once perceive that the water-surface is horizontal only at the summit and at the bottom of the wheel, and that at a given point M above the centre of the wheel the deviation from horizontal is a maximum.

Fig. 205.

The deviation $HAW = AOC = \chi$ for any point A, at a distance $ACM = \lambda$ from the centre of the wheel, is:

$$tang.\ \chi = \frac{AH}{OH} = \frac{a\ cos.\ \lambda}{k + a\ sin.\ \lambda}.$$

For a point A_1 above M, λ is negative, and hence:

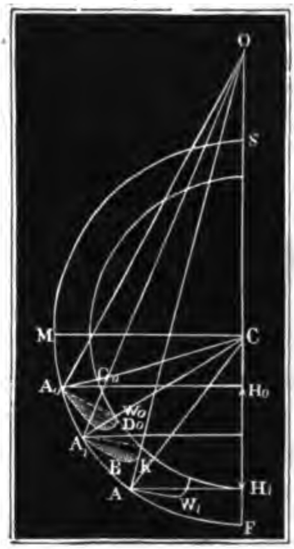
$tang.\ \chi = \frac{a\ cos.\ \lambda}{k - a\ sin.\ \lambda}$. If we lay off a tangent OA from O, we have in the point of contact A_1, that point at which the deviation from horizontality is greatest, or where χ is a maximum, and $= \lambda$, $sin.\ \chi$ being, however, $= \frac{a}{k}$.

Example 1. For a wheel, making 5 revolutions per minute, $k = \frac{2850}{25} = 114$ feet, the radius a being $= 16$ feet, and the discharging angle $\lambda = 50°$. Then:
$tang.\ \chi = \frac{16\ cos.\ 50°}{114 + 16\ sin.\ 50} = \frac{10,285}{136,260}$, therefore. $\chi = 4°, 39'$, so that the surface of the water deviates in this case $4\frac{3}{4}$ degrees from the horizontal.

Example 2. For a wheel, making 20 revolutions, $k = \frac{2850}{400} = 7,125$. If, therefore, $a = 5$, $\lambda = 0°$, then $tang.\ \chi = \frac{5}{7,125}$, hence $\chi = 35°, 3'$. At an angle of 44°, 34′ above the centre of the wheel, the deviation is as much as 44°, 34′.

§ 99. If, now, we take into consideration the effect of centrifugal force, as is obviously necessary in the case of wheels revolving rapidly, the formula we above found for the arc of discharge must be replaced by others. Let A_0 (Fig. 206) be the point at which discharge commences, $MCA_0 = H_0A_0G = \lambda$ the angle of discharge, $H_0A_0W_0 = A_0OC = \chi$ the depression of the water's surface below the horizon, or the angle $GA_0W_0 = \lambda + \chi$, and the $\triangle A_0GW_0 = \frac{1}{2} d \cdot d\ tang. (\lambda + \chi) = \frac{1}{2} d^2\ tang. (\lambda + \chi)$. If we now put the contents of the segment $ABD_0 = S$, that of the $\triangle AGD = D_0$, and the section of the body of water $= F$, then $F_0 + \frac{1}{2} d^2\ tang. (\lambda + \chi) = S + D$, are, therefore:

Fig. 206.



1. $tang. (\lambda + \chi) = \frac{S + D - F_0}{\frac{1}{2} d^2}$.

But $\frac{sin.\ AOC}{sin.\ OAC} = \frac{CA}{CO}$, *i. e.*,

$$\frac{sin.\ \chi}{sin.\ [90^\circ - (\lambda + \chi)]} = \frac{a}{k},$$

and hence follows:

2. $sin.\ \chi = \frac{a\ cos. (\lambda + \chi)}{k}$.

When by the first formula, the value of $\lambda + \chi$, and by the second, the value of χ the depression, have been found, we obtain by subtraction of the two angles $\lambda = (\lambda + \chi) - \chi$.

At the end A_1 of the angle of discharge, the outer end of the bucket coincides with the water's surface A_1W_1, and, therefore, $CA_1W_1 = \lambda_1 + \chi$ at this point = the known angle δ depending on the form of bucket. Hence

$sin.\ \chi_1 = \frac{a\ cos.\ \delta}{k}$ and $\lambda_1 = \delta - \chi_1$, that is, the angle by which the end of the arc of discharge deviates from the centre of the wheel.

Fig. 207.

If the height $H_0H_4 = h_4 = a\ (sin.\ \lambda_1 - sin.\ \lambda)$, Fig. 207, of the arc of discharge, be divided into 4 or 6 equal parts, and the filling of the bucket for corresponding positions be determined, we can again find the ratio

$x = \frac{Q_1}{Q} = \frac{F}{F_0}$ of the mean contents of the buckets during the discharge, to the contents before discharge commences, and so calculate the mechanical effect of the water in the arc of discharge. For this the above formula must be used inversely. In this case λ is given, hence:

$tang.\, \chi = \frac{a \cos.\lambda}{k + a \sin \lambda}$, and $F = S + D - \frac{1}{2} d^2 \, tang.\, (\lambda + \chi)$.

If the water does not fill the entire segment, or if $F < S$, or $\frac{1}{2} d^2$ $tang.\, (\lambda + \chi) > D$, then we must put:

$$F = \text{segment } ABD - \triangle ADK,$$

and in the case of straight buckets:

$$F = S - \tfrac{1}{2} d_1^2 \, . \, \frac{\sin.\,(\lambda + \chi - \delta_1)\sin.\,\delta_1}{\sin.\,(\lambda + \chi)},$$

in which d is the diagonal AD and δ_1 is the angle DAC included between this and the radius CA.

Example. A small wooden water wheel (Fig. 207), 12 feet high, 1 foot depth of shrouding, and 4 feet wide, receives 1080 cubic feet of water, when making 17 revolutions per minute; required the mechanical effect produced by it. Here, $a = 6$, $d = 1$, $e = 4$, $a_1 = 5{,}5$, $Q = \frac{1080}{60} = 18$, $u = 17$; allowing 24 buckets,

$\beta^0 = \frac{360^0}{24} = 15^0$, and $F_0 = \frac{1080}{24 \,.\, 17 \,.\, 4} = \frac{45}{68} = 0{,}662$ square feet. If, again, $D = 0{,}652$, and $S = 0{,}373$, then $tang.\, (\lambda + \chi) = \frac{0{,}373 + 0{,}652 - 0{,}662}{\frac{1}{2}} = 0{,}363 \times 2 = 0{,}726$, therefore, $\lambda + \chi = 35^0, 59'$. But $CO = k = \frac{2850}{17^2} = 9{,}86$ feet, and hence

$\sin.\chi = \frac{6 \cos.\, 35^0, 59'}{9{,}86} = 0{,}4924$, hence $\chi = 29^0, 30'$, and $\lambda = 6^0, 29'$. Thus the discharge commences in this case at only $6\frac{1}{2}^0$ below the centre of the wheel. To find the point at which the discharge is complete, we have in the present case (in which water hangs in the bucket, although the water's surface touches the outer extremity of the bucket), to put in the formula $\sin.\, \chi_1 = \frac{a \cos \delta}{k}$, instead of a, the radius of the division circle $a_1 = 5{,}5$, and instead of δ, the angle formed by the outer element of the bucket and the radius, and which is here $79^0, 14'$. Hence: $\sin.\, \chi_1 = \frac{5{,}5 \cos.\, 79^0, 14'}{9{,}86}$, therefore, $\lambda = 5^0, 59'$, and the second angle of discharge $\lambda_1 = 79^0, 14' - 5^0, 59' = 73^0, 15'$. Hence, the height of the arc of discharge, $h_2 = a_1 \sin.\, \lambda_1 - a \sin.\, \lambda = 5{,}5 \sin.\, 73^0, 15' - 6{,}0 \sin.\, 6^0, 29' = 5{,}2666 - 0{,}6775 = 4{,}589$ feet. Dividing this height into 4 equal parts, we determine by delineation, by measurement, and calculation, the corresponding three intermediate values of F. The results arrived at are $F_1 = 0{,}501$, $F_2 = 0{,}409$, and $F_3 = 0{,}195$, and, therefore, the required ratio of the sections:

$$x = \frac{F}{F^0} = \frac{0{,}662 + 4\,(0{,}501 + 0.195) + 2 \,.\, 0{,}409}{12 \,.\, 0{,}662} = 0{,}537,$$

and the mechanical effect produced by the water during the descent of the arc of discharge: $L_1 = x \,.\, h_1\, Q\gamma = 0{,}537 \,.\, 4{,}589 \,.\, 18 \,.\, 62{,}5 = 27{,}55$ feet lbs. If the water fell with a velocity of 20 feet, 20 degrees under the summit of the wheel, in such a direction that it deviated 25^0 from the tangent at the point of entrance, then the remaining effect of the water's pressure:

$L_2 = (5{,}5 \cos.\, 20^0 + 6 \sin.\, 6^0, 29')\; 18 \,.\, 62{,}5 = 5{,}854 \,.\, 1120 = 6556$ feet lbs.

and the effect of impact, the velocity in the division circle v_1 being $\frac{11 \,.\, \pi \,.\, 17}{60} = 9{,}791$ feet is $L_3 = 0{,}031\,(20 \cos.25^0 - 9{,}791)\; 9{,}791 \,.\, 18 \,.\, 62{,}5 = 2{,}611 \times 1120 = 2974$ feet lbs, and hence the whole mechanical effect produced is:

$$L = L_1 + L_2 + L_3 = 12305 \text{ feet lbs.}$$

§ 100. *Friction of the Gudgeons.*—No inconsiderable portion of the mechanical effect of overshot wheels is lost in the mechanical effect absorbed by *friction on the gudgeons.* Let the weight of the water wheel, together with the water in the buckets, $= G$, the radius of the gudgeon $= r$, then the friction is $= f\,G$, and the velocity at the periphery of the axis $= v = \frac{\pi\,u\,r}{30}$, and hence the mechanical effect consumed by the friction of the gudgeons $= f\,G\,v = \frac{\pi\,u\,r}{30} f\,G$ $= 0{,}1047\,.\,u\,f\,G\,r$. For well turned gudgeons on good bearings $f = 0{,}075$, when oil or tallow is well supplied, or $f = 0{,}054$, when a constant supply of best oil is kept up. In ordinary circumstances of the application of a black lead unguent, $f = 0{,}11$. The weight G of the wheel must be determined by admeasurement for each case. For wheels of 18 to 20 feet in height, the weight has been found to be from 800 to 1000 times the number of effective horse power in pounds. The wooden wheels of Freiberg, 35 feet in height, weigh, when saturated with water, nearly 44000 lbs. Being 20 horse power, this makes upwards of 2000 lbs. per horse power for the weight of the wheel.

The effective power L of a wheel increases as the weight of the wheel increases, as the proportion of the bucket filled $\varepsilon = \frac{Q}{d\,e\,v}$ increases, and as the number of revolutions u increases, so that, inversely, $G = \frac{\iota\,L}{\varepsilon\,u}$, in which ι is a co-efficient to be ascertained from experience. According to Rettenbacher, a small iron wheel, the buckets of which are filled $\frac{1}{4}$, the horse power being 6,8, $\iota = 3432$ lbs.

For the Freiberg wooden wheels of 20 horse power, $\iota = 2750$ lbs., so that, for a first approximation, we may use the formula: $G = 3000\,\frac{L}{\varepsilon\,u}$ pounds.

The strength of the gudgeons depends on the weight of the wheel G, and thus the weight has a twofold influence on the friction (Vol. II. § 87). We have given the formula:

$2\,r = 0{,}048\,\sqrt{G}$ inches $= 0{,}00045\,\sqrt{G}$ feet, for the strength of gudgeons, and, therefore, we may here put $Gr = 0{,}00142\,\sqrt{G^3}$, and hence the mechanical effect consumed by the friction at the gudgeons

$$= 0{,}1047\,uf\,.\,0{,}00142\,\sqrt{G^3} = 0{,}00015\,uf\,\sqrt{G^3}$$

Example. What amount of mechanical effect is consumed by the friction of a wheel of 25000 lbs. weight, with gudgeons of 6 inches diameter, the wheel making 6 revolutions per minute. Assuming $f = 0{,}08$, then $f\,G = 0{,}08 \times 25000 = 2000$ lbs., and the statical moment of this $= f\,G\,r = \frac{1}{4}\,.\,2000 = 500$ feet lbs., and the mechanical effect $= 0{,}1047\,.\,6\,.\,f\,G\,r = 314$ feet lbs.

Remark. The gudgeon friction of a water wheel may be increased or diminished according to the manner in which the mechanism for transmitting its power is applied to it. If, as in Fig. 208, the power P and the resistance Q act on the same side of the wheel, then the friction on the gudgeons is diminished by an amount equal to the resist-

ance Q, so that the friction is less; but if the power and resistance work on opposite sides (as in Fig. 209), then the pressure on the gudgeons is increased by an amount equal to

Fig. 208.

Fig. 209.

the resistance Q, and, therefore, the friction is as much greater as in the other case it was less. If in the former case the leverage CB of the resistance be made equal to the leverage of the power CA, so that the transmission is effected by a toothed wheel on the periphery of the water wheel, as shown in Fig. 171, then the effect of the power on the gudgeon is counterbalanced by the resistance (not if the pinion wheel be in the position shown in the diagram, but when this is placed *below the centre* of the wheel).

§ 101. *Total Effect.*—The total effect of an overshot water wheel may be put:

$$L = \left(\frac{(c_1 \cos. \mu - v_1)\, v_1}{g} + h_1 + x\, h_2\right) Q\, \gamma - f\, \frac{r}{a}\, Gv,$$

or when the water enters in a direction nearly tangential, and with a velocity equal to that of the wheel:

$$L = (h_1 + x\, h_2)\; Q\, \gamma - f\, \frac{r}{a}\, Gv.$$

Hence for a given weight of wheel, the *total effect*, as well as the mere water power effect, is a maximum when $v = 0$, or when the wheel revolves with the least possible velocity. This condition does not hold good in practice: for the dimensions and weight of water wheels depend on the power they give, and on their velocity, and are so much the greater, the greater the effect or power, and the less the velocity of the wheel.

§ 102. *Useful Effect of Wheel.*—Smeaton, Nordwall, Morin, and others, have experimented on the efficiency of overshot water wheels; but there is still room for further experimental inquiry, especially as to the efficiency of well-constructed large wheels; because the effect of these is not accurately known, and, as the author has had occasion to verify, their efficiency is generally *under estimated*.

Smeaton made experiments with a model wheel of 75 inches circumference having 36 buckets, and found for 20 revolutions per minute a maximum of useful effect of 0,74. D'Aubuisson mentions, in his work on hydraulics, that for a wheel of 11½ metres = 32 feet diameter, with a velocity of 8½ feet per second, the efficiency was found to be 0,76. Weisbach has tested the stamp-mill wheels at

Freiberg, generally 7 metres or 22′ — 9″ high, and 3 feet wide, having 48 buckets, and making 12 revolutions per minute, and found an efficiency of 0,78. The *pumping* and *winding* wheels of 35 feet diameter, making 5 revolutions per minute, give an efficiency of 0,80, and often higher. It is also quite ascertained that well-constructed wheels of greater diameter than the above, give 0,83 efficiency, the losses being 3 per cent. for shock at entrance, 9 per cent. by too early discharge, and 5 per cent. for gudgeon friction.

Small wheels always give a *less* efficiency than larger, not only because they make a greater number of revolutions, but because they have a smaller water arc. The most complete experimental inquiry on water wheels is that of M. Morin, "*Expériences sur les Roues hydrauliques à aubes planes, et sur les Roues hydrauliques à augets*, Metz, 1836."* Of these experiments we can here only allude to those made on three small-sized wheels. The first of these was a wooden wheel 3,425 metres = 10,6 feet diameter, with 30 buckets, and giving for a velocity of the circumference of 5 feet per second, an efficiency of 0,65, and the co-efficient ν = 0,775. The second wheel was only 2,28 metres in diameter (7,47 feet)—of wood, with 24 curved plate buckets. With a velocity of 5 feet per second, the efficiency of this wheel was = 0,69, and the co-efficient of velocity or of fall = ν = 0,762. The third was a wooden wheel for a tilt-hammer, 4 metres diameter (13 feet), with 20 buckets, and 1 metre of *impact* fall to the summit of the wheel. The velocity being 5 feet per second, the efficiency was 0,55 to 0,60, and the velocity being $11\frac{1}{2}$ feet per second, its efficiency was not more than 0,40, which was further reduced to 0,25, when the velocity rose to 4 metres or 13 feet per second, for then the centrifugal force was such that the water could not properly enter the buckets.

Morin deduces from his experiments that for wheels of less than 6′ — 6″ diameter, having a maximum velocity of 6 feet per second at the periphery, and for wheels of more than 6′ — 6″ diameter, having a maximum velocity of 8 feet per second, the co-efficient ν of the *pressure fall* averages 0,78, and, therefore, the *useful effect* of these overshot wheels, exclusive of friction on the gudgeons, is:

$$Pv = \left(\frac{c\,(\cos.\,\mu - v)\,v}{g} + 0{,}78\,h\right) Q\,\gamma,$$

h being the height of the point of entrance above the foot of the wheel, or 0,78 h represents the mean height of the arc holding water. This co-efficient ν = 0,78 is, however, only to be used when the co-efficient ϵ representing the extent of filling of the buckets is under $\frac{1}{2}$; it must be made 1,65 when ϵ amounts to nearly $\frac{2}{3}$. In the case of wheels of great diameter ν is certainly higher. For the Freiberg wheels, for example, it is 0,9. Morin further deduces, that when wheels have a greater velocity of revolution than 6′ — 6″ per second, or if the buckets be more than $\frac{2}{3}$ filled, a definite co-efficient ν for

[* The author was apparently unacquainted with the experiments made by a committee of the Franklin Institute.—Am. Ed.]

the water-arc cannot be given, because, then, small variations or deviations in v and ϵ have considerable influence on the amount of the useful effect. It must, however, be remarked, that it is not the velocity, but the number of revolutions u (Vol. II. § 98), which determines this limit: for high wheels with $6' - 6''$ velocity at circumference, give a great and tolerably well-ascertained useful effect.

[*American Experiments.*—The most important of the deductions from the experiments on water wheels, made in 1829–30, by the Committee of the Franklin Institute, using a wheel 20 feet in diameter, and with a head and fall varying from $20\frac{3}{4}$ to 23 feet, may be stated as follows:—

1. "*In running a large overshot wheel to the best advantage*, 84 *per cent. of the power may be calculated upon for the effect.*"

2. "*The velocity of the overshot wheel bears a constant ratio for maximum effects to that of the water entering the buckets, this ratio being at a mean about* ,55 *or* $\frac{9}{16}$*ths.*"*

3. "*Without diminishing the ratio of effect to power more than* 2 *per cent., we may so arrange a high overshot wheel as to increase the velocity of its periphery from* $4\frac{1}{2}$ *to* $6\frac{1}{8}$*th, and probably even to* $7\frac{1}{2}$ *feet per second.*"†

As the quantity of work done by a given wheel, when the ratio of effect to power is the same, must evidently depend on the velocity of the wheel, it must be advantageous to run the wheel with the highest velocity within which that ratio can be kept constant, or nearly so, that is, from 6 to 7 feet per second.

The ratio of effect to power with "centre buckets" was found to be .78 of that with "elbow buckets," owing to the water sooner leaving the former than the latter.

When air-vents are used they involve a loss of effect with centre buckets, but scarcely vary the action of elbow buckets.‡

4. With a wheel 15 feet in diameter "84 *per cent. of the power expended may, as before, be relied on for the effect*," but when the *heads* bore to the *falls*, or heights of wheel, a proportion as great as 1 to 5 or 1 to 4, the ratio of effect to power was reduced as low as ,80 and even ,75.

An overshot wheel of 10 feet in diameter gave with heads of water above the gate varying from ,25 to 3,75 feet, a ratio decreasing from ,82 to ,67 of the power; and with a wheel 6 feet in diameter, the ratios, under like variations of the head of water above the gate, varied from ,83 to ,64. The same general law of a decrease of ratio of *effect* to *power*, according as the proportion of *head* to the

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* [Mr. Elwood Morris (see Journal Franklin Institute Vol. IV., 3d series, p. 222) ascertained, by direct experiment on three excellent overshot flouring-mill wheels, with all the modern improvements, that, calculating by the whole head and fall, while they ran at their usual pace, and with everything in order, they required "788 *cubic feet of water falling one foot per minute, to grind and dress one bushel of wheat in an hour.*" This is an expenditure of power = 49,250 feet lbs. per minute = $1\frac{1}{2}$ horse power.—Am. Ed]

† Journ. Frank. Inst., 3d series, Vol. I., pp. 149 and 154.

‡ Ibid., p. 221.

§ Ibid., p. 223.

total head and fall increases, may be traced in the action of different wheels.

Thus the wheel having a diameter of
15 feet, and mean proportion of head to head and fall = ,063 gave ratio of effect to power = ,841
20 feet, and mean proportion of head to head and fall = ,067 gave ratio of effect to power = ,838
6 feet, and mean proportion of head to head and fall = ,072 gave ratio of effect to power = ,801
10 feet, and mean proportion of head to head and fall = ,079 gave ratio of effect to power = ,795.

A still further increase of proportion until the *head* was 45 per cent. of the *head and fall* gave, in the case of the 6 feet wheel, a ratio of effect to power only ,604. Indeed, it is easy to understand that all that head of water above the bottom of the bucket which exceeds what is necessary to give the water the same velocity as that of the wheel, can act only by creating impact, and, therefore, must be considered so much of a head destined to produce, not pressure, but percussion, and the co-efficient of effect of any water delivered under such increased head, must be undershot co-efficient, which experiment proved to be ,285.]

§ 103. *High-Breast Wheel.*—The overshot wheel frequently receives the water lower than the point we have above indicated, at a point somewhat nearer the centre of the wheel. These are called by the French *roues par derrière*, by the Germans *rückenschlägige Rädar*. The lead or water-course, or the pentrough, passes above the wheel in the case we have discussed. For high-breast wheels the pentrough is below the summit of the wheel, or the diameter of the wheel is greater than the total water-fall. In the overshot wheel, the wheel revolves in the direction in which the water is led on to it. In the high-breast wheel, it revolves in the opposite direction. High-breast wheels are erected more especially when the level of the water in the tail-race and pentrough are much subject to variation; because the wheel revolves in the direction in which the water flows from the wheel, and, therefore, *backwater* is less disadvantageous, and because penstocks or sluices can be applied that admit of an *adjustable* point of entrance of the water on the wheel, or of maintaining a given height between the water in the pentrough and in the tail-race; and even for different conditions of the water-course, the same velocity of discharge and of entrance of the water can be maintained. Penstocks, or sluices for these wheels are represented in Figs. 210 and 211, in which the shuttle *AB* is made to slide or fold as an apron, to open more or fewer apertures as required. In Fig. 210 *AB* is made concentric with the circumference of the wheel, in order that the aperture *A* may direct the water into the wheel for all positions of the apron. The motion of the sluice or apron is effected by the pinion and rack *AD* and *C*. In Fig. 211, the water flows over the top *A* of the sluice-board, which is adjusted in a manner similar to that above described. In

order, however, that the water may come on to the wheel in a proper direction, a set of stationary *guide*-buckets EF, is laid between

Fig. 210.

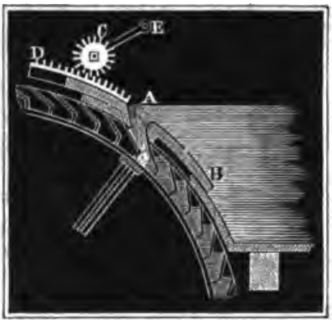
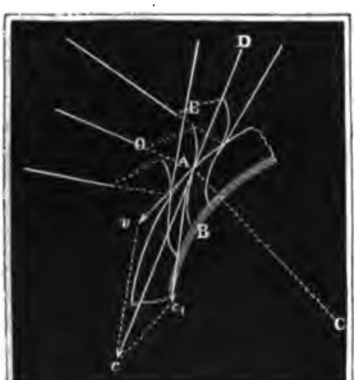


Fig. 211.

the wheel and the sluice-board, and this latter slides over them. The *guide-buckets* must have a certain position, that the water may not strike on the outer end of the wheel-buckets. If Ac_1, Fig. 212, be the direction of the outer element of the wheel-bucket, and if Av, represent in magnitude and direction the velocity of this element A, then, exactly as in Vol. II. § 92, the required direction Ac of the water entering the wheel is obtained by drawing vc parallel to Ac_1, and making Ac equal to c, the velocity of the water entering, as deduced from the height of the water's surface above A. If h be the depth of A below the surface of the pen-trough, then $c = 0{,}82 \sqrt{2gh}$ at least, as in the discharge through short additional tubes (Vol. I. § 323), and when the guide-buckets are rounded off on the upper side, then $c = 0{,}90 \sqrt{2gh}$. If the guide-curves be made straight, then they are to be laid in the direction CAD, but if curved buckets AE are adopted, which has the advantage of *gradually* changing the direction of the water's flow, then they are made tangents to AD at A, by raising AO perpendicular to AD, and describing an arc AE from O as a centre.

Fig. 212.



As for different depths, the pressure (h) is different, and the velocity due (c) is also different, the construction must be gone through separately for each guide-bucket. The velocity of entrance is usually 9 to 10 feet, and the velocity of the wheel is $\frac{1}{2}$ c to $\frac{2}{3}$ c at the most. The construction is to be gone through for the *mean* level of the

water in the trough, that the deviation in cases of high and low water may not be excessive.

From this kind of sluice the air escapes less readily than in others, and, therefore, the sluice is made *narrower* than the wheel, or the wheel-buckets are specially *ventilated*, that is, the floor of the wheel is perforated with air-holes. It is not advisable to make the buckets too close, but rather to keep the water in the wheel by an apron, than by making the angle of discharge too great, for in this latter case the guide-buckets extend over too large an arc of the wheel, or become long and contracted, and so occasion loss of fall.

As to the efficiency of high-breast wheels, it is at least equal to that of the ordinary overshot. From the advantageous manner in which the water is laid on to them, it is not unfrequently greater than in overshot wheels having the same general proportions. For a wheel 30 feet high, having 96 buckets, the entrance of the water being at a point 50° from the summit, the velocity at circumference being 5 feet per second, and that of the water's entrance 8 feet, Morin found the efficiency $\eta = 0,69$, and the height of the arc holding water $= 0,78 \, . \, h$.*

§ 104. *Breast Wheels.*—These wheels are either ordinary bucket wheels, or they are wheels with paddles or floats confined by a stone curb or wooden mantle (Vol. II. § 83). As by a too early discharge of the water from the buckets, the greatest loss of fall or of mechanical effect takes place in the lower half of the wheel, it is evident, that *cæteres paribus*, breast wheels must have a smaller efficiency than overshot or high breast wheels. Upon these grounds the fall must be carefully preserved for breast wheels, or the water kept on the wheel to the lowest possible point. Hence the angle of discharge for their buckets is made great, or the water is even introduced from the inside of the wheel, as shown in Fig. 213, or, as is the better plan, the wheel is enveloped by a mantle or curb, and the buckets or *paddles* are set more or less radially. The curb must not be at more than from ½ an inch to an inch from

Fig. 213.

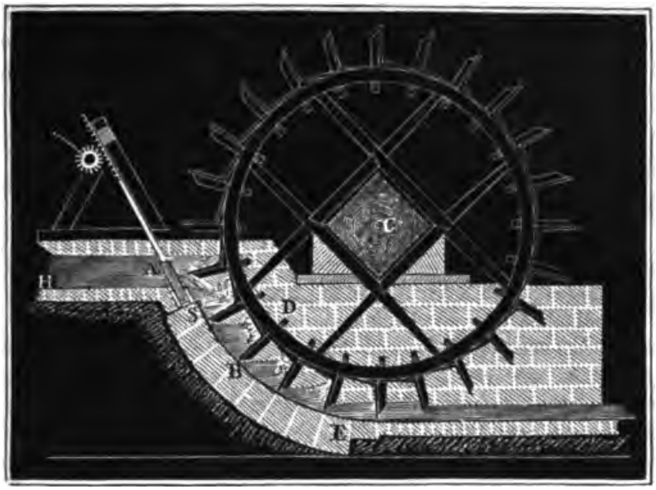
* [With a high breast wheel 20 feet in diameter, water let on 17 feet above the bottom of the wheel, under a head of 9 inches, the Committee of the Franklin Institute found that elbow buckets gave a ratio of effect to power of ,731 at a maximum, and centre buckets ,653. Admitting the water on at a height of 13 feet 8 inches, the elbow buckets gave ,658 and the centre buckets ,628. At 10,96 feet above the bottom of the wheel, the water produced on elbow buckets with a head of 4,29 feet, a ratio of ,544, and centre buckets ,329, with the gate 7 feet above the bottom of the wheel, and a head of 2 feet of water, this "low-breast" wheel gave a ratio of ,62 for elbow buckets and ,531 for centre buckets. At a height of 3 feet 8 inches above the bottom of the wheel, and one foot head above the bottom of the gate, elbow buckets gave ,555, centre buckets ,533.—Am. Ed.]

the wheel, that as little water as possible may escape through the intermediate space. As the buckets or floats of wheels inclosed by a curb are not intended for holding water, they may be placed radially, but, in order that they may not *throw up* water from the tail-race, it is advisable to give the outer element of the float or bucket such a position that it may leave the water vertically. As to the number of floats, it is advantageous to have them numerous, not only because by this means the loss of water by the *play* left between the wheel and the mantle or curb is smaller, but, also, because, by putting them close together, the *impact*-fall is rendered less, and the *pressure*-fall increased. The distance between two floats is generally made equal to the depth of the shrouding d, or it is taken at from 10 to 15 inches, or one of the rules above given (Vol. II. § 88) may be applied for fixing the number of buckets. It is, however, essential that breast wheels be well ventilated, so that the air can escape outwards, because in them the stream of water laid on is nearly as deep as the whole distance between two buckets. Air-holes or slits must, therefore, be left in the floor to prevent the air from interfering with the water's entrance. This is so much the more necessary in these wheels, as they are usually arranged to be filled to $\frac{1}{2}$ at least, even $\frac{2}{3}$ of their capacity. Breast wheels are generally adopted for falls varying from 5 to 15 feet, and for supplies of from 5 to 80 cubic feet per second.

Remark. Theoretical and experimental researches on the subject of breast and undershot wheels, with the water laid on from the inside, have been made in Sweden, and are given in detail in a work entitled "*Hydrauliska Försök*, etc., of Lagerhjelm, of Forselles och Kallstenius, Andra Delen, Stockholm, 1822." Egen describes a wheel of this kind in his "*Untersuchungen über den Effect einiger Wasserwerke*, &c, Berlin, 1831." The efficiency of the wheel was not more than 59 per cent., although the fall was 13,42 feet. A wheel on the same model was erected in France, but only 6′ — 6″ in diameter. M. Mallet experimented on this wheel (see "*Bulletin de la Société d'Encouragement*, No. 282)," and found its efficiency 60 per cent. It would appear, therefore, as Egen justly observes, that these wheels are seldom to be adopted. They can have only a limited width, and cannot be so substantial as those receiving the water on the outside.

§ 105. *Overfall Sluices.*—The mode of *laying on* the water to breast wheels is very various. The overfall sluice, the guide-bucket sluice, and the ordinary penstock are in use. The water is seldom allowed to go on quite undirected. In the *overfall* sluices, shown in Figs. 214 and 215, the water flows over the *saddle-beam*, or *lip* A of the sluice; but that it may flow in the required direction, the lip is rounded, or a rounded guide-bucket AB, Fig. 215, is appended to it. This guide-bucket AB, Fig. 216, is curved in the form of the parabola, in which the elements of water lying deepest move; for if it were more curved, the water-layer would not lie to it, and if it were less curved, the width of the guide, and, therefore, the friction, would be unnecessarily increased, and the water would reach the wheel less in the direction of a tangent. According to the theory of discharge over weirs (Vol. I. § 317), if e_1 be the breadth of the weir, h_1 the height above the sill or lip HA, Fig. 216, and μ the coefficient of discharge, then $Q = \frac{2}{3} \mu\, e_1\, h_1 \sqrt{2gh_1}$; but if the quantity

Fig. 214.



laid on Q, and breadth of the aperture e_1 (which is only 3 or 4

Fig. 215.

Fig. 216.

inches less than the width of the wheel e) be given, then the head for the discharge:

$$h_1 = \left(\frac{\frac{3}{2}\,Q}{\mu\, e_1 \sqrt{2g}}\right)^{\frac{2}{3}} = ,3093 \left(\frac{Q}{\mu\, e_1}\right)^{\frac{2}{3}}.$$

Again, the velocity c of the water entering the wheel at B is determined by its proportion $\varkappa = \frac{c}{v}$ to the velocity of the wheel, and, hence, the fall necessary for communicating this velocity:

$$HM = h_2 = \frac{c^2}{2g} = \frac{(\varkappa\, v)^2}{2g}, \text{ or,}$$

on account of absorption of fall by discharge, $h_2 = 1,1 \,.\, \frac{(\varkappa\, v)^2}{2g}$: $\varkappa$ is

generally made $= 2$, and, therefore, $h_2 = 4{,}4\,\frac{v^2}{2g}$. From h_1 and h_2 we deduce the height AM of the lip of the guide-curve, $k = h_2 - h_1$, and if the total fall $HD = h$, there remains for the head available as weight on the wheel $MD = EF = h_3 = h - h_2$. Again, we have from the theory of projectiles, the angle of inclination TBM of the guide-curve's end to the horizon determined by the formula

$$k = \frac{c^2 \sin. \alpha^2}{2g}, \text{ therefore, } \sin. \alpha = \sqrt{\frac{k}{h_2}} = \sqrt{\frac{h_2 - h_1}{h_2}},$$

and the length or projection MB of the guide-curve is:

$$MB = l = \frac{c^2 \sin. 2\,\alpha}{2g} = h_2 \sin. 2\,\alpha.$$

Lastly, if the very desirable condition of bringing the water *tangentially* on to the wheel is to be fulfilled, the radius of the wheel $CB = CF = a$ is determined by the equation:

$$a\,(1 - \cos. \alpha = h - h_2, \text{ therefore, } a = \frac{h - h_2}{1 - \cos. \alpha}.$$

Inversely, the central angle $BCF = \alpha_1$ of the water arc is determined by $\cos. \alpha_1 = 1 - \frac{k - h_2}{a}$, and when the latter condition is not fulfilled, or α_1 is not made $= \alpha$, then the deviation of the direction of the water entering the wheel from the direction of the motion of the bucket on which it impinges: $\delta = \alpha_1 - \alpha$.

Example. Suppose for a breast wheel the quantity of water laid on by an overfall sluice, $Q = 6$ cubic feet, the total fall $h = 8$ feet, and the velocity of the periphery $= 5$ feet, also the ratio of the buckets filling $= \frac{1}{2}$, then for a depth of wheel $= 1$ foot, the requisite width on the breast $e = \frac{4}{3} \cdot \frac{Q}{dn} = \frac{5 \cdot 6}{2 \cdot 1 \cdot 5} = 3$ feet. And if the breadth of the aperture be made $2\frac{3}{4}$ feet, and $\mu = 0{,}6$, then the height at which the water stands $h_1 = 0{,}3093\left(\frac{6}{0{,}6 \cdot \frac{11}{4}}\right)^{\frac{2}{3}} = 0{,}76$ feet. If we take $s = \frac{8}{5}$, then the fall necessary to generate the velocity c with which the water enters the wheel:
$c = \frac{8}{5} \cdot 5 = 8$ feet, $h_2 = 1{,}1 \cdot 0{,}0155 \cdot 8^2 = 1{,}1$ feet nearly, and, therefore, the height of the lip of the guide $k = 1{,}1 - 0{,}76 = 0{,}34$ feet $= 4{,}08$ inches. Again, for the angle of inclination of the end of the guide-curve: $\sin. \alpha = \sqrt{\frac{0{,}34}{1{,}1}} = 0{,}5539$, and hence $\alpha = 33°$, $38'$, and the *breadth* of the lip of the guide-curve $l = 1{,}1 \sin. 67°, 16' = 1$ foot nearly. That the water may enter the wheel tangentially, the radius of the wheel would have to be

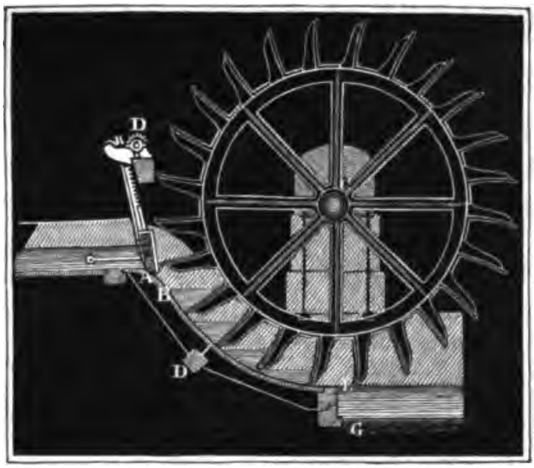
$$a = \frac{h - h_2}{1 - \cos. \alpha} = \frac{8 - 1{,}1}{1 - \cos. 33°, 38'} = 41{,}06 \text{ feet};$$

but if we limit the size of the wheel to 25 feet diameter, or make $a = 12.5$, then the central angle α_1 of the water-arc is $\cos. \alpha_1 = 1 - \frac{8 - 1{,}1}{12{,}5} = 0{,}45$, or $\alpha_1 = 63°, 16'$, and the deviation of the direction of motion of the water from that of the wheel at the point of entrance: $\alpha_1 - \alpha = 63°, 16' - 33°, 38' = 29°, 38'$.

§ 106. *Penstock Sluices.*—Fig. 217 shows the manner of laying on the water on a breast wheel by the ordinary penstock. The sluice-board, which is placed as close to the wheel as possible, is made of such thickness and form at the lower edge, as obviates

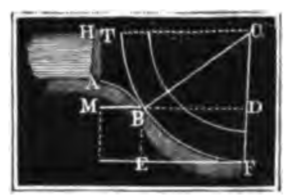
contraction. For the same reason the end AB of the bottom of the

Fig. 217.



pentrough or lead, is formed with a parabolic lip. The height $BE = DF = h_2$, Fig. 218, of the end of the curve depends on the total fall h, and on the velocity height

Fig. 218.



$$h_1 = 1{,}1 \cdot \frac{c^2}{2g} = 1{,}1 \cdot \frac{x^2 v^2}{2g},$$

and is determined by the formula $h_2 = h - h_1$: hence, as $cos.\, \alpha = \frac{CD}{CB} = \frac{a - h_2}{a}$, the corresponding central angle $BCF = \alpha = 1 - \frac{h - h_1}{a}$. If, therefore, the water is to be taken on tangentially, the inclination TBM of the layer of water to the horizon is to be put $= \alpha$, and, hence, we determine the co-ordinates $MA = k$ and $BM = l$ of the apex of the parabola A by the formulas $k = \frac{c^2\, sin.\, \alpha^2}{2g}$, and $l = \frac{c^2\, sin.\, 2\,\alpha}{2g}$.

But it is not necessary to set the aperture exactly in the apex of the parabola, it may be placed in any other point of the parabolic arc, *provided that the axis of the aperture is tangential to the parabola* (Vol. II. § 93).

A third method of laying on the water, consists in a penstock with guide-curves, or buckets, Fig. 219. This arrangement is particularly applicable when the water in the pentrough is subject to great variation of level. The apparatus shown in Fig. 219 consists of two sluice-boards A and B, each of which can be separately adjusted, and thus, not only the *head*, but also the *orifice* of discharge, is regulated. It is not possible to lay the water on to the wheel

tangentially by means of the guide-buckets DE; but we can approximate to within 20 to 30 degrees of this. The water flows out between the guides, according to the law for the *discharge through short additional tubes*, and, therefore, the co-efficient μ may be taken as ,82, or when the bottom of the sluice-board is accurately rounded on the inside $\mu = 0,90$. Hence the co-efficient of resistance is greater in this case than in overfall sluices, or in the ordinary penstock. Assuming $\mu = 0,85$, the height necessary for producing the velocity c is:

Fig. 219.

$$h_1 = \left(\frac{1}{0,85}\right)^2 \cdot \frac{c^2}{2g}$$

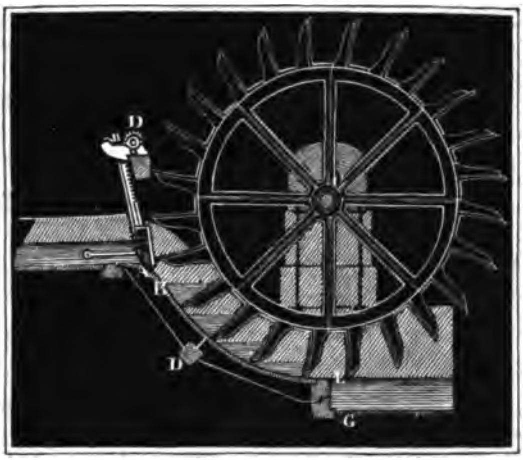
$= 1,384 \frac{c^2}{2g}$, and hence the portion of the total height h remaining in water-arc is: $h_2 = h - h_1 = h - 1,384 \frac{x^2 v^2}{2g}$. In the case of a variable supply of water, the arrangements are adapted to the average supply, by laying the outer end M of the centre guide-curve at the height h_2 above the foot of the wheel F. In order to place all the guides at the same angle of circumference as the wheel, the normal position of which is 3 inches from the guides, they are drawn tangential to a circle KL, concentric with the wheel, the position of which is determined by the direction EK of the first guide-curve.

§ 107. *Construction of the Curb or Mantle, and of the Wheel.*—The mantle or curb by which breast wheels are inclosed, in order to retain the water in them as long as possible, is formed either of masonry or of wood, and sometimes of iron. The object of the curb is the better fulfilled the less the play between the outer edge of the buckets and the cylindrical surface of the curb, as the water escapes by whatever free space is left. The play amounts to ½ an inch in the best constructed wheels; but it is not unfrequently as much as an inch, or even 2 inches. When the wheels are of wood and the curb likewise, a play of ½ an inch is an insufficient allowance, because the curb is apt to loose its symmetry, and then friction between the buckets and the curb might ensue. For iron wheels and stone curbs, the chances of deformation are small, and, therefore, a very small amount of play is sufficient. When the wheels fit too closely in their mantles, stray pieces of wood or ice floated

on to the wheel may have very injurious consequences. It is, therefore, necessary to have a screen in front of the sluices to keep back all stray matters.

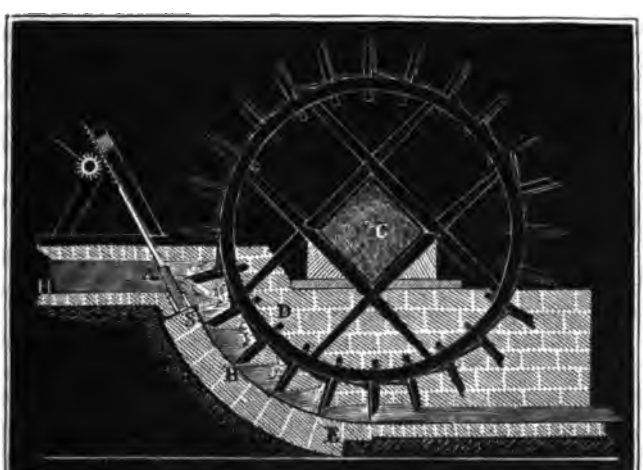
Stone curbs are constructed of properly selected and carefully dressed stone, or of brick, either being laid in good hydraulic mortar or cement. Wooden curbs *AE*, Fig. 220, are composed of beams

Fig. 220.



A, *D*, *E*, of larch or oak, carefully planked with deals curved as required. The bed of the curb is inclosed by side walls, so that lateral escape of water is prevented. If the water can flow off by the tail-race with the velocity with which it is delivered from the

Fig. 221.



wheel, then the curb may be finished flush with the bottom of the race, as at E, Fig. 221; but if the water flows with a less velocity, then the race should be cut out deeper, as at AE, Fig. 220, so as to avoid all risk of back water.

The construction of these wheels differs essentially from that of overshot wheels, in respect of the buckets of the latter forming cells, whilst in the former these are mere paddles or floats; and this gives rise to a different mode of connecting the buckets with the rim of the wheel. The Germans distinguish *Staberäder* and *Strauberäder*. The former have *shroudings* like the overshot wheels, to support the floats; in the latter, the floats or buckets are chiefly supported by short arms or cantilevers, which project radially from the circumference of the wheel. Fig. 219 is a wheel with shrouding, Figs. 220 and 221 are *Strauberäder*. Fig. 221 is a wooden wheel, and Fig. 220 an iron wheel. Very narrow float wheels have only a single narrow ring by which the floats are attached on to cantilevers. In wooden wheels the supports for the floats are passed between the two sides of a compound ring forming the shrouding. In iron wheels, on the other hand, they are either cast in one piece with the segments of the wheel, or they are bolted on to these. The buckets or paddles are usually of wood, and are nailed or screwed to the above-described supports. The floor of the wheel is placed on the *outside* of the rings or shrouding, and does not close the wheel completely, slits being left for the escape of the air, as is represented in Fig. 215, in which DE is the wheel-paddle, composed of two pieces, EF a piece of the bottom of the wheel, and G the air-slit or ventilator.

§ 108. *The Mechanical Effect of Curb Wheels.*—The mechanical effect produced by wheels hung in a curb or mantle consists, as in overshot wheels, of the mechanical effect produced by the impact, and that by the pressure or weight of water. The formula representing the efficiency of each is the same, save that the determination of the loss of water requires a different calculation, inasmuch as in the one case the water is lost by a gradual emptying of the buckets in the arc of discharge, and, in the case now in question, the water escapes through the space necessarily left between the wheel and the curb. We have, therefore, to determine how, and in what quantity, the water escapes through this space, and hence deduce the loss of effect to the wheel. If we put, as for overshot wheels, the velocity with which the water goes on to the wheel at the division line $= c_1$, the velocity of the wheel in this line or circle $= v_1$, and the angle $c_1 E v_1$, Fig. 222, between the directions of these velocities $= \mu$, then the effect of impact:

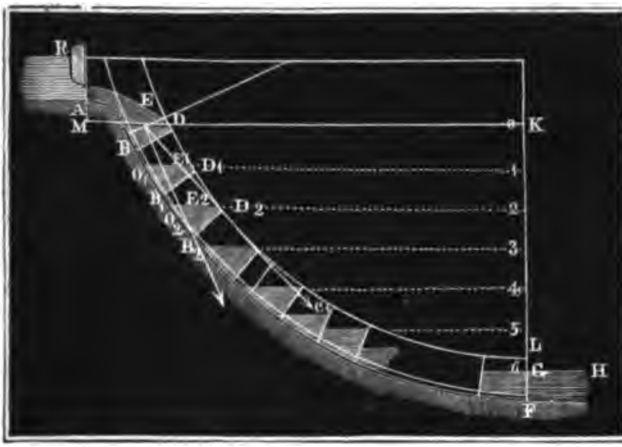
$$= \frac{(c_1 \cos. \mu - v_1) v_1}{g} \cdot Q \gamma.$$

If, further, GK, the difference of level between the point of entrance E, and the surface of the water in the tail-race, $GH = h_1$, we have (neglecting loss of water through free space) the effect of the weight of the water $= h_1 Q \gamma$, and hence the total effect is, as before:

$$L = Pv = \left(\frac{(c_1 \cos. \mu_1 - v_1)\, v_1}{g} + h_1\right) Q\gamma.$$

From the theoretical effect given by the formula, we have to deduct the loss by the escape of water. This loss affects chiefly the effect

Fig. 222.



of the water's weight, as the water escapes uninterruptedly, as any given bucket or float BD descends successively to the position B_1D_1, B_2D_2, &c., to the lowest point FL. The free space forms an orifice of discharge through which the water escapes under a variable head BE, B_1E_1, B_2E_2. If we put $e =$ the width of the wheel, and the breadth of the free space $= s$, the area of the orifice of discharge $= se$, and if we put for the *head* BE, B_1E_1, &c., z, z_1, z_2, &c., and ϕ for the co-efficient of discharge, then the quantity escaping in any instant of time t, through the free space:

$$Q_1 = \phi e s t \sqrt{2g\, z_1} + \phi e s t \sqrt{2g\, z_2} + \phi e s t \sqrt{2g\, z_3} + \ldots$$
$$= \phi e s t \sqrt{2g}\,(\sqrt{z_1} + \sqrt{z_2} + \sqrt{z_3} + \ldots)$$

If n be the number of positions of a float assumed, the mean escape from each cell:

$$= \phi e s t \sqrt{2g}\left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots + \sqrt{z_n}}{n}\right),$$

or for one second:

$$= \phi e s \sqrt{2g}\left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots \sqrt{z_n}}{n}\right).$$

But the whole of the cells, or the water-arc, corresponds to the fall h_1, and, hence, the loss of mechanical effect may be put:

$$L_1 = \phi e s \sqrt{2g}\left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots + \sqrt{z_n}}{n}\right) h_1 \gamma.$$

There is a slight loss of water on each side of the wheel, as there must be a free space of 1 to 2 inches here. If we put arcs BO,

$B_1 O_1$, &c., of the curb covered with water, equal to l_1, l_2, &c., then the water escaping by the sides, as it were through a series of notches or small weirs:

$$= 2 \cdot \tfrac{2}{3}\, \phi\, s\, l_1 \sqrt{2g\, z_1},\ 2 \cdot \tfrac{2}{3}\, \phi\, s\, l_2 \sqrt{2g\, z_2}, \text{ \&c.,}$$

and, therefore, the corresponding loss of mechanical effect:

$$L_2 = \tfrac{4}{3}\, \phi\, s \sqrt{2g} \left(\frac{l_1 \sqrt{z_1} + l_2 \sqrt{z_2} + \ldots l_2 \sqrt{z_n}}{n}\right) h_1 \gamma.$$

§ 109. *Losses.*—There is a still further loss of effect, when the surface of the water in the lowest cell does not correspond with the surface of the tail-race, as represented in Fig. 223. For in this case the water flows from the cell $BD D_1 B_1$, as soon as the float BD passes the end of the curb F, and acquires a velocity due to the height $FM = h_2$, in addition to the velocity v of the wheel. This height h_2 is variable, but its mean value is evidently $\frac{1}{2} h_2$, and, therefore, the head to which the velocity of the water flowing from the wheel is due is not $\frac{v^2}{2g}$, but $\frac{v^2}{2g} + \frac{1}{2} h_2$. We have already deducted the loss due to the height $\frac{v^2}{2g}$ in estimating for impact, and we have, therefore, only $\frac{1}{2} h_2\, Q\, \gamma$ to deduct from the effect found. From this we see, that a sudden fall from the end of the circle should be adopted only in cases where back-water is to be feared.

Fig. 223.

There are still other sources of loss of effect in breast wheels, such as the friction of the water on the curb, and the resistance of the air to the motion of the floats; but these are comparatively of slight importance.

§ 110. *Formula for Total Effect.*—We shall now give the formula for the total effect of breast wheels, leaving out of consideration the loss of effect by escape of water at the sides, as also the loss from friction of the water and resistance of the air; but allowing for the escape through the free space between the wheel and curb, and for the friction of the gudgeons. The formula will then stand thus:

$$L = Pv = \frac{(c_1 \cos. \mu - v_1)\, v_1}{g}\, Q \gamma + h_1\, Q \gamma - \phi\, e\, s\, w\, h_1 \gamma - f\, G \frac{r}{a}\, v,$$

in which w is substituted for the mean velocity of discharge:

$$\sqrt{2g} \left(\frac{\sqrt{z_1} + \sqrt{z_2} + \ldots + \sqrt{z_n}}{n}\right),$$

and r for the radius of the gudgeons. Hence:

$$L = \left[\frac{(c_1 \cos. \mu_1 - v_1)\, v_1}{g} + h_1 \left(1 - \frac{\phi\, e\, s\, w}{Q}\right)\right] \cdot Q \gamma - \frac{r}{a} f\, G\, v.$$

or, introducing ϵ the co-efficient of the bucket's filling, generally $= \frac{1}{2}$, and $\frac{e}{Q} = \frac{1}{\epsilon\, d\, v}$, we have:

$$L = \left[\frac{(c_1 \cos.\mu_1 - v_1)\,v_1}{g} + h_1\left(1 - \frac{\phi\, s\, w}{s\, d\, v_1}\right)\right] Q\gamma - f\,G\,\frac{r}{a}\,v.$$

If we put the total fall, measured from the surface of the water in the pentrough to the surface of the tail-race $= h$, then, instead of h_1, we may introduce $h - 1{,}1 \cdot \frac{c_1^2}{2g}$, and then:

$$L = \left[\frac{(c_1 \cos.\mu - v_1)\,v_1}{g} + \left(h - 1{,}1 \cdot \frac{c_1^2}{2g}\right)\left(1 - \frac{\phi s w}{s d v_1}\right)\right] Q\gamma - f\,G\,\frac{r}{a}\,v.$$

In order to find the value of the velocity of entrance c_1, for which the effect is a maximum, we have only to consider when the expression

$$\frac{c_1 v_1 \cos.\mu}{g} - 1{,}1 \cdot \frac{c_1^2}{2g}\left(1 - \frac{\phi\, s\, w}{s\, d\, v_1}\right)$$

is a maximum. Putting $\dfrac{2\, v_1 \cos.\mu}{1{,}1\left(1 - \frac{\phi\, s\, w}{s\, d\, v_1}\right)} = k$, then the expression to be made a maximum becomes $kc_1 - c_1^2$. But we know from Vol. I. § 386, that this becomes a maximum for $c_1 = \frac{k}{2}$, and, therefore, it is evident that the effect will be a maximum when the velocity of entrance of the water $c_1 = \dfrac{v_1 \cos.\mu}{1{,}1\left(1 - \frac{\phi\, s\, w}{s\, d\, v_1}\right)}$.

If, from the necessarily small value of μ, we put $\cos.\mu = 1$, and assume also, that there is no loss in the discharge from the sluice, then $c_1 = \dfrac{v_1}{1 - \frac{\phi\, s\, w}{s\, d\, v_1}}$; and hence we perceive that the velocity of entrance must be made greater than the velocity at the circumference of the wheel, and this so much the more as the free space s is greater. The loss by escape of water is not, as an average, more than 10 to 15 per cent., or $\frac{\phi\, s\, w}{s\, d\, v_1} = \frac{1}{10}$ to $\frac{3}{20}$, and, therefore, the velocity of entrance of the water $c_1 = \frac{10}{9}\, v_1$ to $\frac{20}{17}\, v_1$. In practice, however, c_1 is made $= 2\, v_1$, or the wheel revolves with *half the velocity the water has acquired at entering the wheel*, because in this way the loss is not much, and the water's entrance is more easily regulated.

If we introduce $c_1 \cos.\mu = 2\, v_1$, or $v_1 = \frac{1}{2}\, c_1 \cos.\mu$, into the equation above found, we get:

$$L = \left[\frac{v_1^2}{g} + \left(h - 1{,}1 \cdot \frac{4\, v_1^2}{2g \cos.\mu^2}\right)\left(1 - \frac{\phi\, s\, w}{s\, d\, v_1}\right)\right] Q\gamma - f\,\frac{r}{a}\,G\,v,$$

$$= \left[-\left(\frac{2{,}2}{\cos.\mu^2} - 1\right)\frac{v^2}{g} + h\left(1 - \frac{\phi\, s\, w}{s\, d\, v}\right)\right] Q\gamma - f\,\frac{r}{a}\,G\,v.$$

From the factor $\left(1 - \frac{\phi\, s\, w}{s\, d\, v}\right)$, we learn that the maximum effect

does not take place here when $v = 0$; for even when $v = \frac{\phi s w}{s d}$ the whole effect of the water's weight is lost by the water escaping through the free space.

§ 111. *Efficiency of Breast Wheels.*—Morin has made a number of experiments on breast wheels of good construction. He has compared his experimental results with those of the theoretical formula:

$$Pv = \left(\frac{(c \cos. \mu - v) v}{g} + h_1\right) Q \gamma,$$

and has found a tolerable agreement between the two to subsist, when the formula is corrected by a co-efficient x, or if we put:

$$Pv = x \left(\frac{(c \cos. \mu - v) v}{g} + h_1\right) Q \gamma.$$

One wheel on which Morin experimented was of cast iron, with wooden buckets placed obliquely to the sluice, and turning in a close-fitting iron curb. The wheel was 21 feet 4 inches in diameter, and 4′—10″ wide; the fall was 5′—6″, there were 50 floats, and the velocity of revolution was from 3′—4″ to nearly 8 feet per second, the velocity of the water from a well-constructed sluice being from 9 feet 2″ to 10 feet 6″. The co-efficient x was found to be about 0,75, and the efficiency, including the friction of the axles, nearly 0,60. A second iron wheel, experimented upon by M. Morin, was hung in a well-fitting sandstone curb. It was 13 feet diameter, and 13 feet wide. There were 32 floats, the fall being 6 feet 6″. So long as the speed of the wheel did not differ more than from 47 to 100 per cent. from that of the water entering it, that is within speeds varying from 1′—8″ to 6 feet, the co-efficient x remained nearly constant, viz.: $= 0{,}788$, and the efficiency of the wheel was $= 0{,}70$. A third wheel was almost entirely of wood, and hung in a close-fitting curb. Its height was 20 feet, and it had 40 floats. Worked with a common sluice, the co-efficient $x = 0{,}792$, and with an *overfall sluice* this rose to 0,809. The efficiency, however, was in the first case only 0,54, and in the second 0,67. If from these results we adopt a mean value, we get for breast wheels with penstock sluice:

$$L = 0{,}77 \left(\frac{(c \cos. \mu - v) v}{g} + h_1\right) Q \gamma,$$

and for those with overfall sluice:

$$L = 0{,}80 \left(\frac{(c \cos. \mu - v) v}{g} + h_1\right) Q \gamma,$$

from which, however, the mechanical effect consumed by the friction of the axles has to be deducted. The greater efficiency of the overfall-sluiced wheels arises from the water entering more slowly than in the case of the penstock, and, hence, there is no loss by impact.

It follows, besides, from Morin's experiments, that the efficiency diminishes if the water fills more than from $\frac{1}{2}$ to $\frac{2}{3}$ of the space be-

tween the floats, and that the efficiency does not vary much for variations of the angular velocity of the wheel from 1′ — 8″ to 6′ — 6″ per second.

Egen made experiments on a breast wheel 23 feet in diameter, and 4½ feet wide. There were two peculiarities in this wheel. The 69 well-ventilated buckets, were constructed exactly as in overshot wheels; and the sluice was in two divisions, of which, according to the state of supply of water, the upper or under one could be drawn. Although the mantle fitted very closely, the efficiency of this wheel was, at best, only 0,52, and as an average, with 6 cubic feet water per second, and 4 revolutions per minute, the efficiency was only 0,48.

Experiments with a breast wheel are described in the "*Bulletin de la Société Indust. de Mulhouse*, L. XVIII." The wheel was of wood, 5 metres or 16,4 feet in diameter, and 13 feet wide, made in three divisions on 2 centre shroudings. The curb started from a parabolic saddle-beam 8 inches in height, and the water was laid on by an overfall sluice 8 inches high. Thus the velocity of the water was about 8,8 feet, and the angular velocity of the wheel from 5 feet to 6′ — 6″. The buckets were filled from ⅙ to ⅜, and the efficiency increased as the buckets were more filled. When the buckets were quite filled, the efficiency was 0,80; when half-filled, it was 0,73; and with less water, it was only 0,52. The experiments on the efficiency of the wheel for different degrees of filling of the buckets, were easily and precisely made in this case, from the circumstance that the water could be laid on each division of the wheel separately.*

Example. Required the calculated proportions of a breast wheel, being given $Q = 15$ cubic feet per second, $h = 8\frac{1}{2}$ feet, and the velocity of revolution 5 feet. We shall assume the depths of the floats or of the shrouding to be 1 foot, and suppose the buckets to be filled to ⅓ their contents. The width of the wheel is, hence, $e = \frac{2Q}{d\,v} = \frac{30}{1.5} = 6$ feet. Assume also that the water enters with double the velocity of rotation, then $c = 2 \,.\, 5 = 10$ feet, and the fall required to generate the velocity

$$h_1 = 1{,}1\,\frac{c^2}{2g} = 1{,}1 \,.\, 0{,}00155 \,.\, 100 = 1{,}705 \text{ feet.}$$

Deducting this impact fall from the total fall, there remains for the height of the curb, or for the fall during which the water's weight alone acts, $h^2 = h - h_1 = 8{,}5 - 1{,}705 = 6{,}795$ feet. We shall adopt a large wheel, that the water may not fall too high into it. Making the radius $a = 12$ feet, and the radius of the division line $= 11{,}5$ feet. The water revolving with the velocity of the wheel, we shall suppose to be carried to the bottom of the curb, as represented in Fig. 224. The central angle α of the curb EG, or the angle by which the points of entrance of the water E deviates from lowest point F, is

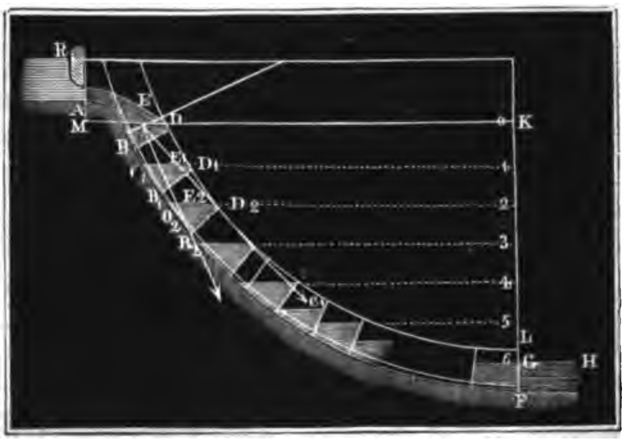
$$\cos.\,\alpha = 1 - \frac{h_2}{a_1} = 1 - \frac{6{,}795}{11{,}5} = 0{,}4092,$$

and, hence, $\alpha = 65°, 50′$. We shall assume that the direction Ec_1 of the water deviates 20 degrees from the direction Ev_1 of the wheel's motion at the division

* [For the efficiency of breast wheels, with elbow and centre buckets, see above (p. 182, note). The Franklin Institute committee found, with a 15 feet wheel and elbow buckets, taking the water 10,46 feet above the bottom of the wheel, an efficiency from 612 to 677, and laying in on 7 feet from the bottom, the efficiency varied, with different heads, from ,568 to ,631.—Am. Ed.]

circle, and refer the velocity of 5 feet, in like manner, to the division or *pitch* circle. We then have the co-ordinates of the summit of the parabolic saddle, $AM = k = \frac{c_2 \sin. (45^\circ\, 50')^2}{2g} = 0,8$ feet, and $ME = l = \frac{c_2}{2g} \sin. 91^\circ\, 4' = 1,55$ feet; according to which dimensions the construction of Fig. 224 has been carried out. The height of the water

Fig. 224.



AR above the sill is $h_1 - k = 1,705 - 0,8 = 0,905$, and if we put the height of the orifice $= x$, $Q = \mu\, e\, x \sqrt{2g\left(0,905 - \frac{x}{2}\right)}$

$$\therefore x = \frac{Q}{\mu e \sqrt{2g\left(0,905 - \frac{x}{2}\right)}} = \frac{15}{0,9 \,.\, 6 \,.\, 8,02 \sqrt{0,905 - \frac{x}{2}}}$$

$$= \frac{5}{14,43 \sqrt{0,905 - \frac{x}{2}}} = \frac{0.35}{\sqrt{0,905 - \frac{x}{2}}}$$: and, hence, $x = 0 \,.\, 4$ feet.

The theoretical effect of this wheel is $L =$

$$\left(\frac{(c \cos. \mu - v)\, v}{g} + h_2\right) Q\, \gamma = \left(\frac{(10 \cos. 20^\circ - 5)\, 5}{32,2} + 6,795\right) . \; 15 \,.\, 62,25 = 7244 \text{ feet}$$

lbs., and the whole available effect is $8,5 \times 933$ feet lbs. $= 7930$ feet lbs. We have now to deduct the loss by the escape of water through the free space between the curb and the wheel. Assuming the *play* to be 1 inch $= \frac{1}{12}$ feet, then the area of the slit by which the water can escape is $\frac{1}{12} \,.\, 6 = \frac{1}{2}$ square feet. In order now to find the mean velocity w, with which the water passes through this aperture, the height of curb KG is to be divided into 6 equal parts, and the position of the buckets for each point so found, accurately delineated, as is done in Fig. 224, and the heads or pressures measured. Commencing at the top, we have: $z = 0,80$, $z_1 = 0,80$, $z_2 = 0,80$, $z_3 = 0,80$, $z_4 = 0,67$, $z_5 = 0,48$ and $z_6 = 0$. From this we have the mean of the square roots of these quantities $=$

$$\frac{\frac{1}{2} \,.\, 0,894 + 0,894 + 0,894 + 0,818 + 0,693 + 0}{6} = 0,7736,$$

and, hence, the mean velocity of escape of the water $= 8,02 \times 0,7736 = 6,188$. The mechanical effect corresponding to this is $L_1 = \phi\, e\, s\, w\, h_1\, \gamma$, in which $\phi = 0,7$ the co-efficient of discharge, therefore,

$$L = 0,7 \,.\, \tfrac{1}{2} \,.\, 6,188 \times 6,79 \times 62,25 = 916 \text{ feet lbs.}$$

The loss by escape at the *sides* of the wheel may be calculated by the formula given § 108. It will be found $= 180$ feet lbs. So that the total loss by the escape of water $= 1096$ feet lbs; deducting this from 7244 feet lbs., there remain 6148 feet lbs.

effective. The escape of water in this wheel we see involves a loss of 15 per cent. of the mechanical effect of the fall. By the friction of the water and the resistance of the water, the loss is about 160 feet lbs., or about $2\frac{1}{2}$ per cent. There remains, therefore, 5988 feet lbs.

If now we take the weight of the wheel $G \doteq \frac{3000\ L}{\varepsilon\, u}$, ε, the ratio of filling the buckets, being $\frac{1}{2}$, we have $u = \frac{30\ .\ 5}{\pi\ n} = \frac{25}{2\ \pi} = 4$ and $L = \frac{5988}{550} = 11$ lbs. feet nearly; then, the weight of the wheel $= \frac{3000 \times 12}{\frac{1}{2}\ .\ 4} = 1650$ lbs. Hence, the radius of the gudgeons $r = 0{,}002\ \sqrt{8250} = 0{,}182$ feet, and from this we get the mechanical effect absorbed by friction $= \frac{r}{a}\ f\ G\ v = \frac{0\ .\ 19}{11\ .\ 5}\ .\ 0{,}1\ .\ 16500\ .\ 5 = 136$ feet lbs. Making this further deduction, there remains 5852 feet lbs. $= 10{,}6$ feet lbs., and, lastly, the *efficiency* of the wheel $\eta = \frac{5852}{7930} = 0{,}74$.

§ 112. *Undershot Wheels.*—Undershot wheels usually hang in a channel made to fit as closely to the wheel as possible, so that water may not escape without producing its effect. Hence, the application of a channel having a curb concentric with the wheel is considered better than a straight channel tangential to the wheel. The curb allows of some of the effect of the weight of the water being availed of. The calculation for such a wheel as is represented in Fig. 225, when the curb *AB* embraces 3 to 4 floats at least, is identical with these for the breast wheels last considered. The rules for construction of undershot wheels, correspond too with those for breast wheels. The floats are usually put in radially; but sometimes they are inclined upwards towards the sluice, that they may carry no water up with them on the opposite side. These floats are not unfrequently composed of two equal pieces *AD* and *BD*, Fig. 226, so that the angle $ADB = 100^\circ$ to 120°. This arrangement allows of ample openings being left in the *flooring* of the wheel, without fear of the water flowing through the sluice. The cells or buckets are allowed to fill from one-half to two-thirds of their capacity, or $\varepsilon = \frac{1}{2}$ to $\frac{2}{3}$. To prevent overflow of the water inwards, or, in order to have greater capacity, the depth of the wheel, *i. e.*, of the shrouding, is made from 15 to 18 inches. The laying on the water *tangentially* is more rarely done than in breast wheels. The sluice-board is *inclined* in order that the sluice-

Fig. 225.

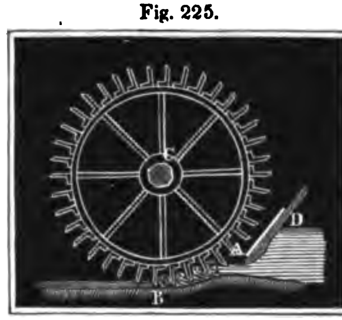
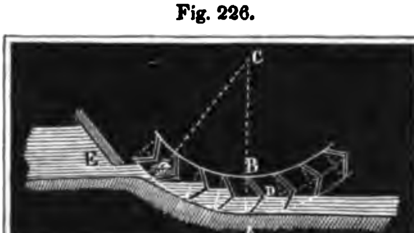


Fig. 226.



aperture may lie as close to the wheel as possible. The lower edge should be rounded off to prevent *partial contraction* of the vein of water.

§ 113. The effect of undershot wheels is less than that of breast wheels, the fall available as weight being greater in the latter. The *half* of the fall is necessarily lost when it acts by impact, whereas the loss by escape of water acting by its weight on those wheels does not amount to $\frac{1}{4}$ of the whole. Experiment has satisfactorily established this. The wheel with which Morin experimented was 19'.6" in diameter, $5\frac{1}{4}$ feet wide, and had 36 radial floats. The sluice was inclined at an angle of $34\frac{1}{2}^\circ$ to the horizon, and the sluice-aperture was $2\frac{1}{2}$ feet back from the commencement of the curved course. The total fall was 6'—8", and the head on the sluice-aperture 4'—7". There was, therefore, a fall of about 1'—8" through which the water's weight acted. The velocity of the circumference of the wheel was from 6'—6" to 13'—0": and the velocity of the water on reaching the wheel from 16 to 18 feet. As long as $\frac{v}{c}$ did not exceed 0.63, the efficiency η was 0.41 as a mean: but when $\frac{v}{c}$ varied between the limits 0.5 and 0.8, then the mean efficiency η was only 0.33.

Retaining our former notation, we have, for the effect of this wheel, exclusive of friction of gudgeons,

$$Pv = 0{,}74\left(\frac{(c-d)\,v}{g} + h_1\right) Q\,\gamma,$$

in the first case; and

$$Pv = 0{,}60\left(\frac{(c-v)\,v}{g} + h_1\right) Q\,\gamma$$

in the second.

A second wheel with which Morin experimented, was about 13 feet high, 2'—8" wide, 11,8 inches deep, and had 24 floats. The water was laid on by a vertical sluice, and reached the wheel through a straight course 2'—8" long. This channel and the curb were of sandstone, and the free space left amounted to only 0,2 of an inch. The mean fall was 3 feet. The head of water on the sluice-aperture varied from 6 to 18 inches. Experiments were made at various velocities of rotation. For small velocities the efficiency was very small. For the mean velocity of 5 feet it was a maximum, and, when the velocity of the water's arrival on the wheel was not much different from this, a maximum efficiency 0,49 was obtained. For ratios of velocities $\frac{v}{c} = \frac{1}{2}$ and $\frac{v}{c} = \frac{1}{4}$, the mean value of κ was found to be, as for the former wheel 0,74. Hence

$Pv = 0{,}74\left(\frac{(c-v)\,v}{g} + h_1\right) Q\,\gamma$, is the formula for this case also.

Morin puts together the results of his experiments on wheels con-

fined in mantles or curbs as follows. Wheels in which $h_1 = \frac{1}{4}h$ $x = 0,40$ to $0,45$. When $h_1 = \frac{2}{6}h$, $x = 0,42$ to $0,49$. When $h_1 = \frac{3}{6}h$, $x = 0,47$, and when $h_1 = \frac{3}{4}h$, $x = 0,55$.

Example. Required, the effect of an undershot wheel, 15 feet in diameter, and making 8 revolutions per minute. The fall 4 feet, and the quantity of water 20 cubic feet per second, $v = \frac{\pi u a}{30} = \frac{\pi . 8 . 15}{60} = 6,283$ feet, and supposing the velocity of the water to be double this, then the pressnre of the water in front of the sluice, or what we have termed the *impact*-fall $= 1,1 \frac{c^2}{2g} = 1,1 \times 0,0155 \times 12,56^2 = 2,689$ feet, and, there therefore remains as fall, through which the water acts by its weight, $h_1 = 4 - 2,689 = 1,311$ feet, and hence the theoretical effect $= (0,031 . 6,283^2 + 1,311)\ 20 . 62,5 = (1,263 + 1,311)\ 1245 = 3264$ feet lbs. In this case, $h_1 = \frac{1,311}{4}\ h = 0,33\ h$, and, therefore, the co-efficient x may be taken 0,42, and hence the effect $L = 0,42 . 3264 = 13708$ feet lbs. from which, however, the gudgeon-friction has to be deducted.

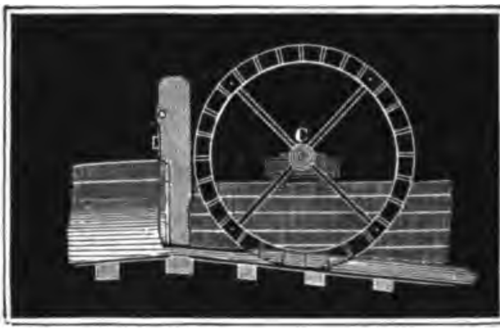
§ 114. *Wheels in Straight Courses.*—When the undershot wheel is hung in a straight course, the effect is a minimum; because the water produces its effect by impact alone, and a considerable quantity escapes unused. These wheels are only adopted for falls of less than 4 feet, and where *water power is of value* the Poncelet-wheel, or turbines, are now invarially preferred. They are made from 12 to 24 feet in diameter, with 24 to 48 floats, usually radial, but sometimes placed with a slight inclination towards the sluice. The breadth or depth of the floats should be made about three times the thickness of the layer of water coming through the sluice, because the water in contact with the wheel retains only 35 to 40 per cent. of the velocity of the water before impact, when the greatest effect is produced; and, therefore, the stream of water flowing along as the wheel revolves is $2\frac{1}{2}$ to 3 times the thickness of the water coming from the sluice. The depth of the sluice-aperture is usually 4 to 6 inches, and thus the floats are made from 12 to 18 inches deep for the above reason. The straight course in which undershot wheels are suspended may be either horizontal as in AB, Fig. 227,

Fig. 227.

or inclined, as in Fig. 228. That as little water as possible may escape unemployed, the space between the wheel and the course

must be reduced to 1 or 2 inches at most. And hence, it is better to lay the course with a slight curvature, the floats being made so numerous, that there are always 4 or 5 floats submerged.

Fig. 228.



The penstock is set with an inclination to bring the orifice of discharge as near to the wheel as possible, and to avoid contraction. To prevent back-water, the course is made to drop suddenly some inches, at the point where the water quits the wheel. Besides this provision, arrangements for raising and depressing the wheel, or the water-course, are adopted.

Fig. 227 represents a *lift* for the wheel (called in German *Ziehpanster*). The axle *M* of the lever *MD* coincides with that of the wheel, so that the connection between the driving wheel and pinion may not be altered in raising or depressing the water wheel. All these arrangements are, however, rendered unnecessary by the adoption of the turbine, instead of undershot wheels, in all cases in which the water is liable to much variation.

§ 115. *Useful Effect of Undershot Wheels.*—Experiments on the useful effect of undershot wheels, with straight courses, have been made, but only on models, by De Parcieux, Bossut, Smeaton, Lagerhjelm, &c.

The experiments of Smeaton and Bossut are the best.* The results of the experiments are satisfactorily in agreement with each other, and confirm the theory. The mechanical effect evolved by these wheels was ascertained in all the experiments, by raising a weight by means of a cord passed round the axle of the wheels. Smeaton's experiments were made with a small wheel 75 inches in circumference, having 24 floats, each 4 inches wide, and 3 inches deep. The general conclusion at which Smeaton arrived is, that for the velocity ratio $\frac{v}{c} = 0,34$ to $0,52$, the maximum useful effect amounts to 0,165 to 0,25. Bossut's experiments were made with a wheel, 3 feet in diameter, provided with 48, with 24, and with 12 buckets, 5 inches wide, and 4 to 5 inches deep. Bossut found, as

* [See foot note and reference next page.—Am. Ed.]

theory indicates, that with 48 floats, the efficiency is greater than with 24, and with 24 greater than with 12; and he deduced from his experiments that about 25° of the wheel's circumference, or $\frac{25}{360} \cdot 48 = \frac{10}{3}$, or more than 3 floats should be in the water at the same time. From Bossut's experiments on the wheel with 48 floats, a somewhat greater efficiency results than is indicated by Smeaton's experiments, and this may probably be attributed to the greater proportional number of buckets in Bossut's model.* The mean result of the two sets of experiments gives the effect of such wheels, friction not taken into account:

$$L = 0{,}61 \frac{(c - v)\, v}{g} Q \gamma = 1{,}19\, (c - v)\, v\, Q \text{ feet lbs.}$$

This formula will only apply on the scale of practice, when the play allowed between wheel and course *is not greater than* $1\frac{1}{2}$ *inch.* Instead of Q we have Fc; in which F is the arc of the float dipping into the water; and hence we have the formula given by Christian in his "*Mécanique industrielle,*" substituting 0,76 for 0,61.

$$L = 0{,}76\, F \gamma \,.\, \frac{(c - v)}{g}\, c\, v = 1{,}48\, (c - v)\, F c\, v \text{ feet lbs.}$$

From the experiments extant, it follows also, that the maximum effect is produced for the velocity ratio $\frac{v}{c} = 0{,}4$, as indicated by theory. For greater velocities this ratio is somewhat less, and for large bodies of water the ratio is somewhat greater.

§ 116. *Partition of Water Power.*—A given *fall* of water is often divided between several wheels, not only because a single wheel would have cumbrous dimensions, but more especially for the sake of working different machines or tools independently, avoiding the coupling connections with one source of power. The question may arise as to a division of height of *fall*, or, as to partition of the quantity of water. As a general rule, we may assume that for wheels on which the water acts by its weight, a partition of the *quantity* of water, and for wheels on which the water acts by impact, a partition of the height of fall is to be preferred; for we have seen that the efficiency of overshot wheels of great diameter, is greater than that of overshot wheels of smaller diameter, or even than breast wheels; and, on the other hand, it is manifest that the loss of effect by impact, and by the escape of water through the wheels, is less when these wheels are placed one behind the other, than when they are put side by side, because the velocity due to the height corresponding to the loss of effect $\frac{(c - v)^2}{2g}$ (Vol. I. § 387), and the ratio $\frac{s}{d_1}$ of the free space to the depth of water, is less than in the latter case. For breast wheels in a curb, on which the water acts

* [The ratio of effect to power, obtained by the Committee of the Franklin Institute, was found to vary from ,266 to ,305, and the average is set down at ,285. See Journal Franklin Institute, 3d series, Vol. II., p. 2, for July 1841.—AM. ED.]

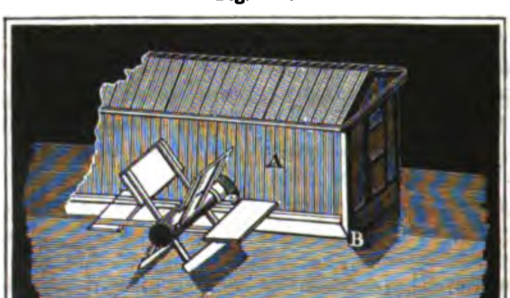
by its weight and by impact, and in which the loss of water depends mainly on $\frac{s}{d_1}$, there is no general rule for the preference of one mode of partition over the other, and the circumstances of each case must determine our choice.

§ 117. *Floating-mill Wheels.*—Wheels suspended on two boats, or barges, conveniently moored in a river, are undershot wheels without curb or *limited* course of any kind. These wheels are supported either on two boats, one of which contains the mill machinery, or one end of the axle rests on a boat, the other resting on piles driven in on shore, in which case the mill machinery is kept on shore.

The construction of boat-mill wheels differs from that of ordinary undershot wheels, inasmuch as they have no shrouding, the floats being attached directly to the arms. These wheels are made from 12 to 15 feet in diameter, and have generally only 6 or 7 floats, although 10 to 12 would constitute a better wheel. The floats are made long and very broad, that they may catch a large stream of water, for, the velocity being usually small, the *vis viva* depends in a great measure on the *mass*. Floats of 6 to 18 feet in length, and 2 feet to 30 inches broad, are usual. The floats are inclined at angles of from 10° to 20° to the radius, and dip to about one-half their breadth into the stream.

Fig. 229 represents a boat-mill (Fr. *moulin à nef;* Ger. *Schiff-*

Fig. 229.



mühle). *A* being the mill-house on the barge *B*, and *C* a wheel with 6 floats, the axle of which passes through the mill-house, and projects as far on the opposite side of the boat as the one seen in the figure does on this. The mill gear is within the house.

The effect of boat-mill wheels is less than of wheels hung in a confined course, for two reasons, viz.: the water not only escapes by the sides, and under the floats, but a considerable quantity passes through the wheel without coming into action, from the small number of buckets that dip into the water.

§ 118. The theoretical effect of a freely suspended water wheel may be represented, as for undershot wheels, by the formula

$$L = Pv = \frac{(c-v)\,v\,c}{g} F \gamma,$$

c and v being the velocities of the water and of the wheel, and F the area of the part of the float dipped, neglecting the damming up of the water upon it. This expression has to be multiplied by a co-efficient allowing for the loss of water.

§ 119. Experiments on the effect of these wheels have been made by De Parcieux, Bossut, and Poncelet, but principally on models.

Bossut's model wheel was 8 feet in diameter, had 24 floats, $6\frac{1}{2}$ inches wide, dipping $4\frac{1}{2}$ inches into the water. The velocity of the water was 6 feet per second. The result of these experiments gives $\mu = 1{,}37$ to $1{,}79$ as the co-efficient, by which the formula $L = \frac{(c-v)\,v\,c}{g} F\gamma$ is to be multiplied, and $\mu = 0{,}877$ to $0{,}706$ as the co-efficient for the formula $L = \frac{(c-v)\,v\,c}{g} F\gamma$ (*see* D'Aubuisson, "Hydraulique," § 352). The limits of the values of the co-efficients are nearer each other, in this latter case, than in the other, which was to be expected, as, from the number of buckets, the second formula is most applicable. The number of buckets should be such that 2 at least are in the water, and then the latter formula with the mean co-efficient $\mu = 0{,}8$, will apply, or,

$$L = 0{,}8 \frac{(c-v)\,c\,v}{g} F\gamma = 1{,}55\,(c-v)\,c\,v\,F \text{ feet lbs.}$$

Poncelet's observations, made on three boat-mill wheels on the Rhone, agree with this. These wheels were 8 to 10 feet long, and the floats dipped $2' - 8''$ to $2' - 9''$ into the water flowing with a velocity of from 4 to $6\frac{1}{2}$ feet per second. Poncelet cites an experiment by Boistard, and one by Christian, both of which confirm the accuracy of this formula.

Bossut's experiments, in exact accordance with theory, show that the maximum effect is obtained when $v = 0{,}4\,c$, and Poncelet's experiments on the Rhone boat wheels also give: $\frac{v}{c} = 0{,}4$.

Introducing $v = 0{,}4\,c$ into the above formula, the useful effect becomes:

$$L = 0{,}8\,\frac{0{,}6\,.\,0{,}4\,c^3}{g} F\gamma = 0{,}192\,\frac{c^3}{g} F\gamma = 0{,}384\,\frac{c^2}{2g} Q\gamma,$$

and, hence, the *efficiency* $\eta = 0{,}384$.

De Parcieux's experiments were specially directed to ascertaining the best position for the floats. The result was that an inclination of 60° to the stream is the best.

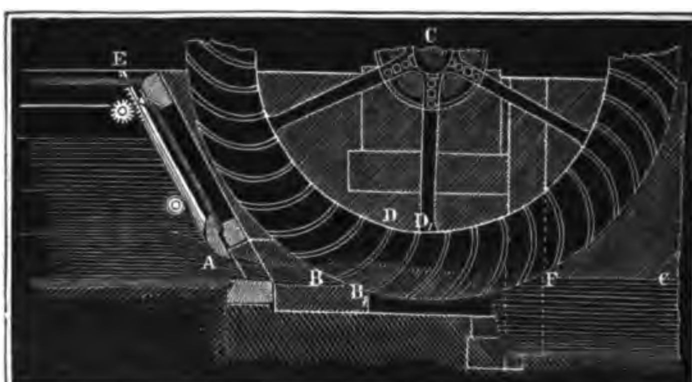
Remark. There has long been a doubt as to which of the two formulas $L = \frac{\mu\,(c-v)^2\,v}{g} F\gamma$ and $L = \frac{\mu_1\,(c-v)\,c\,v}{g} F\gamma$ is the more correct. The one is known as *Parent's* formula, the other as *Borda's*. Now, although for a wheel in an unconfined

stream acting on the floats, all the water going through the wheel does not assume the velocity of the floats, yet, considering the great extent of the floats' surface, it may certainly be presumed that the greater part of the water on impinging, takes the velocity of the floats, and, hence, the greater accordance between experiments in Borda's formula is explained. Parent's formula is founded on the assumption that the impact is proportional to the height due to the relative velocity $c - v$. (*Compare* Vol. I. § 392, where the force of impact is given $= 1,86 \frac{c^2}{2g} F \gamma$ when $v = 0$.)

§ 120. *Poncelet's Wheels.*—If the floats of undershot wheels be curved so that the stream of water runs along the concave side, pressing upon it without impact, the effect produced is greater than when the water impinges at nearly right angles against straight buckets.

Poncelet introduced these wheels. They are of very advantageous application for low falls under 6 feet, because their effect is much greater than that of undershot wheels with or without a curb. For greater falls, breast wheels with a well-formed circle excel them, and as their construction is more difficult, they are not applied for greater falls than 6 feet. Poncelet has treated of these wheels in a special work, entitled "*Mémoire sur les Roues hydrauliques à aubes courbes, mues par-dessous*, Metz, 1827." Fig. 230 represents the

Fig. 230.

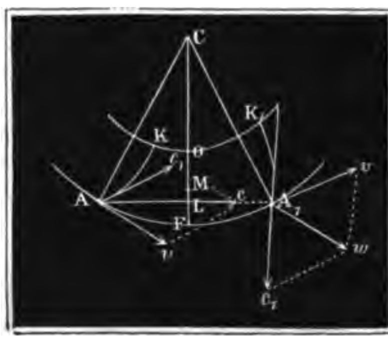


general arrangement of these wheels: AE is an inclined sluice-board; AB is the stream of water entering the wheel at the buckets BD and B_1D_1. FG is the surface of the tail-race. In order that nearly all the water may come into action, the wheel must have very little play in the water-course, and to prevent partial contraction, the under side of the sluice-board is rounded off: also, to prevent loss of *vis viva* by friction in the channel, the aperture of the sluice is brought close to the wheel. The first part of the course AB is inclined at $\frac{1}{10}$ to $\frac{1}{15}$. The remainder of the course, which embraces the length occupied by three buckets at least, is accurately curved concentrically with the wheel, and at the end of it, a sudden dip of 6 inches is made, and the tail-race should also be widened to guard against any liability to back-water on the wheel. Poncelet wheels

have been constructed from 10 to 20 feet in diameter, and with 32 to 48 floats of sheet iron or of wood. Wooden floats are composed of staves, like a barrel, the outer edge being sharpened, or provided with a sheet iron edge piece. Sheet iron floats are, however, much more suitable, as good construction is an essential feature in this wheel. The sluice is not drawn more than 1 foot in any case, and for falls of 5 to 6 feet, an aperture 6 inches high, or less, is arranged for.

§ 121. *Theory of Poncelet's Wheels.*—To obtain the maximum effect from these wheels, the water must go on to the buckets without impact. If $Ac = c$ (Fig. 231) be the velocity of the water going on to the wheel, and $Av = v$, the velocity of the periphery of the wheel, we then have in the side $Ac_1 = c_1$ of the parallelogram $A\,v\,c\,c_1$, the velocity of the water in reference to the wheel, both in magnitude and direction. If, therefore, we put the curved float AK tangential to Ac_1, the water will begin to ascend along it without the least shock, with the velocity c_1. If we put the angle $c\,A\,v$ by which the direction of the water deviates from that of the circumference of the wheel, or the tangent $Av = \delta$, we have for the relative velocity of the water beginning its ascent on the floats $= c_1 \sqrt{c^2 + v^2 - 2\,c\,v\,cos.\,\delta}$, and for the angle $v\,A_1\,c = \epsilon$, by which it deviates from the circumference of the wheel, or from the tangent Av, we have $sin.\,\epsilon = \frac{c\,sin.\,\delta}{c}$.

Fig. 231.



The water ascends on the float with a retarded velocity, and partakes of the velocity of rotation v of the wheel at the same time. Having ascended to a certain height, its relative velocity is lost, and it descends with an accelerated velocity, so that at last it arrives at the outer extremity A_1 with the same velocity c_1 with which it commenced its ascent. If we combine the relative velocity $A_1\,c_1 = c$, after the water leaving the wheel at A, with the velocity of the circumference $A_1\,v = v$ as a parallelogram of the velocities, we have in the diagonal $A\,w = w$ the absolute velocity of the water leaving the wheel. This velocity is

$$w = \sqrt{c_1^2 + v^2 - 2c_1\,v\,cos.\,\epsilon},$$

and, therefore, the mechanical effect, retained by the water, and lost for useful effect, is

$$L_1 = \frac{w^2}{2g}\,Q\,\gamma = \left(\frac{\sqrt{c_1^2 + v^2 - 2c_1\,v\,cos.\,\epsilon}}{2g}\right)\,Q\,\gamma.$$

If, now, we deduct this loss from the amount of effect $\frac{c^2}{2g}\,Q\,\gamma$ inhe-

rent in the water before its entrance on the wheel, we have the following expression for the theoretical effect of the wheel:

$$L = \left(\frac{c^2}{2g} - \frac{w^2}{2g}\right) Q\gamma = \left(\frac{c^2 - w^2}{2g}\right) Q\gamma = \left(\frac{c^2 - c_1^2 - v^2 + 2 c_1 v \cos. \varepsilon}{2g}\right) Q\gamma,$$

or, as $c^2 = c_1^2 + v^2 + 2c_1 v \cos. \varepsilon$, $\therefore L = \frac{2\, c_1 v \cos. \varepsilon}{g} . Q\gamma$, or, $c_1 \cos. \varepsilon = \sqrt{c_1^2 - c^2 \sin. \delta^2} = \sqrt{c^2 \cos. \delta^2 + v^2 - 2\, c\, v \cos. \delta} = c \cos. \delta - v$, and, if we put this in the above expression, we have:

$$L = \frac{2\, v\, (c \cos. \delta - v)}{g} Q\gamma.$$

We easily perceive that the effect is a maximum when $v = \frac{1}{2} c \cos. \delta$, and then $L = \frac{c^2 \cos. \delta^2}{2g} Q\gamma$. Also, the loss of mechanical effect is *null*, or the whole mechanical effect available, or $L = \frac{c^2}{2g} Q\gamma$ is got from the water when $\cos. \delta = 1$, or when $\delta = 0$.

Although it is not possible to make the angle of entrance $\delta = 0$, it follows from this that δ should not be a large angle—not more than 30°, if a good effect is desired, and it is also manifest that the velocity of rotation of the wheel should be only a little less than half the velocity of the water going on to the wheel, that the efficiency may be the greatest.

§ 122. The vertical height LO, to which the water ascends on the floats, would be $\frac{c_1^2}{2g}$ if the wheel were at rest, but as it has a velocity of rotation v, a centrifugal force arises, acting nearly in the same direction as gravity, and giving rise to an acceleration p, which may be represented by $\frac{v_1^2}{a_1}$, if a_1 be the mean radius CM, and v_1 the mean velocity of the wheel's shrouding, or the velocity of the point M. We have then:

$$(g + p)\, h_1 = \frac{c_1^2}{2}, \text{ or } \left(g + \frac{v_1^2}{a_1}\right) h_1 = \frac{c_1^2}{2},$$

and hence, the height of ascent in question $h_1 = \frac{c_1^2}{2\left(g + \frac{v_1^2}{a_1}\right)}$.

In order that the water may not pass over the top at O, it is necessary that the shrouding should have a certain depth $FO = d$, which is determined by the equation $d = LO + FL = h_1 + CF - CL = h_1 + a - a \cos. ACF = \frac{c_1^2}{2\left(g + \frac{v_1^2}{a_1}\right)} + a\,(1 - \cos. \lambda)$, where λ is the angle ACF by which the point of entrance of the water on the wheel deviates from the lowest point of the wheel F. The thickness of the stream of water d_1 is to be added to this, because the particles in the upper stratum must rise so much higher than

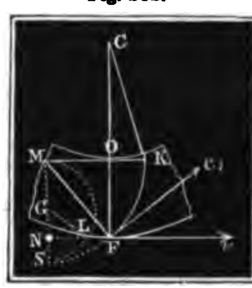
those of the lower stratum on the assumption of a mean velocity. The depth of the shrouding is, therefore,

$$d = d_1 + \frac{c_1^2}{2\left(g + \frac{v_1^2}{a_1}\right)} + a\,(1 - cos.\,\lambda).$$

The width of the wheel is equal to the width of the stream of water; or, $e = \frac{Q}{d_1\,c}$. If the capacity $d\,e\,v_1$ of the wheel be made $1\frac{1}{2}$ times that of the water laid on, then we have the equation $d\,v_1 = \frac{3}{2}\,d_1\,c$ to $2\,d_1\,c$, and hence the thickness or depth of the stream laid on $= d_1 = \frac{1}{2}\,\frac{d\,v_1}{c}$ to $\frac{2}{3}\,\frac{d\,v_1}{c}$. Another important circumstance in reference to these wheels is the determination of the points of entrance and exit of the water, that is the water arc AA_1, which it is best to set off in two equal portions on each side of the lowest point of the wheel F. The length of this arc depends on the time necessary for the ascent and descent of the water on the floats. To find this, we must know the form and dimensions of the floats. If the time $= t$, then we may put $AA_1 = 2\,\lambda\,a = v\,t$, and hence the points on either side of F, at which the water enters and quits the wheel, are at a distance $= \lambda = \frac{v\,t}{2a}$.

§ 128. In order that the water, when it has reached the highest point K, Fig. 232, may not run over, but fall back along the float, the inner end of the float K must not overhang the float when in the mean position FK; but, on the other hand, that the float may not be too long, the end K of the float must not cut the inner circumference of the shrouding at too acute an angle. Hence, it is best to give the inner end of the float a vertical position, when the float is in its mean position. Adopting a cylindrical form of float, we get the centre of the circular arc, its section, by drawing MF perpendicular to Fc_1, and OM horizontal. From the depth of shrouding $FO = d$, we have the radius:

Fig. 232.



$$FM = KM = r = \frac{d}{cos.\,\varepsilon},$$

ε being the angle $MFO = c_1\,F\,v$.

The time required for the ascent and descent of the water on the arc FK may be found in the same manner as the time of oscillation of a pendulum, by substituting for the accelerating force of gravity the sum $g + \frac{v_1^2}{a_1}$ of this acceleration, and that of centrifugal force. This time may be found exactly by the method given Vol. I. § 246,

by putting here, as there, the instant of time required to move through a small space:

$$\tau = \left(1 + \frac{h\,(1 + cos.\,\phi)}{8\,r}\right)\sqrt{\frac{r}{g}} \cdot \frac{\phi}{2n}.$$

In order to find the time for ascent and descent in the arc FK, we have to substitute for ϕ the central angle MGL, which may be determined from the angle $c_1\,F\,v = FMS = \varepsilon$, and the radius $MF = MS = r$, by the formula:

$$cos.\,\phi = -\frac{NG}{LG} = -\frac{MN - MG}{MG} = -\frac{r\,cos.\,\varepsilon - \frac{1}{2}\,r}{\frac{1}{2}\,r}$$

$$= -(2\,cos.\,\varepsilon - 1), \text{ or } sin.\,\tfrac{1}{2}\,\phi = \sqrt{cos.\,\varepsilon}.$$

We have now the time t_1 required for describing the whole arc FK, by adding together all the values of the expression:

$$\tau = \left(1 + \frac{h}{8r}\,(1 + cos.\,\phi)\right)\sqrt{\frac{r}{g}} \cdot \frac{\phi}{2\,n},$$

when for $cos.\,\phi$ we substitute in succession:

$$cos.\,\frac{\phi}{n},\; cos.\,\frac{2\phi}{n},\; cos.\,\frac{3\phi}{n} \ldots cos.\,\frac{n\phi}{n}. \quad \text{But}$$

$$cos.\,\frac{\phi}{n} + cos.\,\frac{2\phi}{n} + cos.\,\frac{3\phi}{n} + \ldots + cos.\,\frac{n\phi}{n} = \frac{sin.\,\frac{\phi}{2}\,cos.\,\frac{\phi}{2}}{\frac{\phi}{2n}}$$

$= \dfrac{sin.\,\phi}{\frac{1}{n}\,\phi}$, and hence $t_1 = \left[\dfrac{\phi}{2} + \dfrac{h}{8r}\left(\dfrac{\phi}{2} + \dfrac{\phi}{2\,n}\right.\right.$ times the sum of all

cosines from 0 to $\left.\left.\phi\right)\right]\sqrt{\dfrac{r}{g}} = \left[\dfrac{\phi}{2} + \dfrac{h}{8r}\left(\dfrac{\phi}{2} + \frac{1}{2}\,sin.\,\phi\right)\right]\sqrt{\dfrac{r}{g}}$.

If we also consider that the whole height of fall, or the diameter $MS = h$, that $MF = r$, and that $g + \dfrac{v_1^2}{a_1}$ is to be substituted for g the force of gravity, the whole time for the rise and fall of the water on the arc FK is

$$t = 2\,t_1 = [\phi + \tfrac{1}{8}\,(\phi + sin.\,\phi)]\sqrt{\frac{r}{g + \frac{v_1^2}{a_1}}},$$

and the length of the water arc AA_1 (Fig. 231), is:

$$b = 2\lambda a = v\,t = \left[\phi\,\frac{(\phi + sin.\,\phi)}{8}\right] v\,\sqrt{\frac{r}{g + \frac{v_1^2}{a_1}}}.$$

§ 122. We have now to derive rules for the arrangement and construction of Poncelet's wheels from these data. We can only assume the height of fall h, the quantity of water Q, and the number of revolutions u of the wheel, as given, and from this we have to deduce the velocity of rotation v, the radius of the wheel a, the depth

of shrouding d, the width of wheel e, and the angles δ, ϵ, λ, and the velocity c_1 of the water at the beginning of its ascent. If we attentively consider the formulas above found, we perceive that they do not admit of a direct solution of the problem, but that the method of gradual approximation must be adopted.

If we lay on the water in a horizontal direction, the deviation δ of the direction of the water-stream from the periphery of the wheel is equal to the distance λ of the point of entrance from the foot of the wheel. In the first place, we may put, as an approximation, the velocity of the water entering the wheel: $c = \mu \sqrt{2gh}$, and from this again, the velocity of rotation of the wheel $v = \frac{1}{2} c$, as also the initial velocity of the ascending water $c_1 = \frac{1}{2} c$, we have hence also an approximate value of the radius $a = \frac{30 v}{\pi u}$, and the same for the depth of shrouding $d = \frac{c_1^2}{2g} = \frac{1}{4} \cdot \frac{c^2}{2g}$, and, hence, also, we obtain an approximate value for the length of the water arc, if we put in the last formula of the preceding paragraph:

$$\phi = \pi, \frac{\phi + \sin. \phi}{8} = 0, \text{ and } r = d, \text{ then:}$$

$$2 \lambda a = \pi v \sqrt{\frac{d}{g + \frac{v^2}{a}}}, \text{ and, therefore,}$$

$$\lambda = \frac{\pi v}{2a} \sqrt{\frac{d}{g + \frac{v^2}{a}}} = \frac{\pi^2 u}{60} \sqrt{\frac{d}{g + \frac{v^2}{a}}}.$$

With the assistance of this approximate value of $\lambda = \delta$, the calculations must be repeated, using the more exact formulas, and taking for the depth of the water-stream d_1 an appropriate value of from 3 to 12 inches, according to circumstances. The head or pressure is then only $h - d_1$, and hence the velocity of the water entering the wheel is: 1. $c = \mu \sqrt{2g\,(h - d_1)}$, that of the wheel. 2. $v = \frac{1}{2} c \cos. \delta$. Again, the radius of the wheel 3. $a = \frac{30 v}{\pi u}$; for the angle ϵ made by the circumference of the wheel with the end of the float,

4. $cotg.\ \epsilon = cotg.\ \delta - \frac{v}{c \sin. \delta} = \frac{1}{2} cotg.\ \delta$, or $tang.\ \epsilon = 2\ tang.\ \delta$;

and the initial velocity of the water rising on the float.

5. $c_1 = \frac{c \sin. \delta}{\sin. \epsilon} = \frac{v}{\cos. \epsilon}$; and if, instead of $\frac{v_1^2}{a_1}$, we put $\frac{v^2}{a}$, the depth of shrouding,

6. $d = d_1 + \frac{c_1^2}{2\left(g + \frac{v^2}{a}\right)} + a\,(1 - \cos. \lambda)$:

and hence again we have the width of the wheel:

7. $e = \frac{Q}{d_1\, c}$, and the radius of the curvature of the floats:

8. $r = \frac{d}{cos.\ \varepsilon}$, and the angle ϕ:

9. $sin.\ \frac{1}{2}\,\phi = \sqrt{cos.\ \varepsilon}$, and lastly the length of the water arc,

10. $$b = 2\,\lambda\, a = \left(\phi + \frac{\phi + sin.\ \phi}{8}\right) v \sqrt{\frac{r}{g + \frac{v^2}{a}}};$$

and from this the accurate value of:

11. $$\lambda = \left(\phi + \frac{\phi + sin.\ \phi}{8}\right) \frac{\pi u}{60} \sqrt{\frac{r}{g + \frac{v^2}{a}}}.$$

Even after these values have been found, the calculations may be repeated on the more accurate foundations.

Example. It is required to ascertain the general proportions of a Poncelet undershot wheel. *Given*, the height of fall 4,5 feet, the quantity of water 40 cubic feet per second. If we make the radius $a = 2h = 9$ feet, and allow the thickness of the stream $d_1 = \frac{1}{6} h = 0{,}75$ feet, and further, $\mu = 0{,}90$, then the velocity of discharge $c = 0{,}9\sqrt{2g\,(h - d_1)} = 0{,}9 \times 8{,}02\sqrt{3{,}75} = 7{,}218 \times 1{,}936 \times 14$ feet; and, therefore, the velocity of the wheel, as also the initial velocity of the water, is approximately $v = c_1 = \frac{1}{2} c = 7$ feet. Hence the depth of shrouding is, nearly,

$d = \frac{1}{4} \cdot \frac{c^2}{2g} + d_1 = \frac{1}{4} \cdot 3{,}04 + 0{,}75 = 1{,}51$ feet, and the arc $\lambda = \delta = \frac{\pi \cdot 7}{2 \cdot 9}\sqrt{\frac{1{,}51}{32{,}2 + \frac{7^2}{9}}}$ $= 0{,}24$, and the angle λ° corresponding $= 14^\circ$, for which, however, we shall take 15°. If we now introduce this value of δ, we get, more accurately, $v = \frac{1}{2} c\ cos.\ \delta = 7\ cos.\ 15^\circ = 6{,}762$ feet, and hence, the number of revolutions $u = \frac{30\, v}{\pi\, a} = 7{,}17$. It follows, therefore, that $tang.\ \varepsilon = 2\ tang.\ \delta = 2 \cdot 0{,}26795 = 0{,}53590$, $\therefore \varepsilon = 28^\circ\ 11\frac{1}{4}'$, and, therefore, $c_1 = \frac{67{,}62}{cos.\ 28^\circ,\ 11\frac{1}{4}'} = 7{,}67$ feet. Again, we have the depth of shrouding $d = 0{,}75 + 9\,(1 - cos.\ 15^\circ) + \frac{7{,}6^2}{2\,(32{,}2 + \frac{1}{2} \cdot 6{,}76^2)} = 1{,}845$ feet; and the width of the wheel $e = \frac{40}{0{,}75 \cdot 14} = 3{,}80$ feet. The radius of curvature of the floats $r = \frac{1{,}845}{cos.\ 28^\circ,\ 11\frac{1}{4}'}$ $= 2{,}093$ feet, and $sin.\ \frac{1}{2}\,\phi = \sqrt{cos.\ 28^\circ,\ 11\frac{1}{4}'}$ $\therefore \frac{1}{2}\,\phi^\circ = 69^\circ,\ 51\frac{1}{2}'$, and $\therefore \phi^\circ = 139^\circ,\ 43'$, $\phi = 2{,}4385$, $sin.\ \phi = 0{,}6466$; and, lastly,

$$\lambda = \left(2{,}4385 + \frac{2{,}4385 + 0{,}6466}{8}\right) \times \frac{6{,}76}{18}\sqrt{\frac{2{,}093}{36{,}52}} = 2{,}824 \times 0{,}3697\sqrt{\frac{2{,}093}{36{,}52}}$$

$= 0{,}2499$, and $\lambda^\circ = 14^\circ,\ 19'$, for which $14\frac{1}{2}^\circ$ would be substituted in the actual construction of the wheel, so that the length of the water arc, or the length of the concentric curb $b = 2\,\lambda\, a = 18 \cdot 0{,}253 = 4\frac{1}{2}$ feet, or $2\frac{1}{4}$ feet on each side of the lowest point of the wheel.

§ 128. *Experiments with Poncelet's Wheels.*—Poncelet himself instituted experiments on the useful effect of his water wheels. These are minutely detailed, and their results ascertained in his work above cited.

The first experiments were made with a model wheel of 20 inches

diameter. It was of wood, had 20 floats, about $\frac{1}{16}$ of an inch thick, $2\frac{3}{4}$ inches deep, and 3 inches wide. The greatest effects were produced when the velocity of the wheel = 0,5 that of the water, as indicated by theory, and then the efficiency was 0,42 to 0,56, the former when the water stream was kept thin, the latter when this was increased, or the cells of the wheel better filled. Reckoning the efficiency by the height due to the velocity of the water, and not by the actual fall, the effect rises to 0,65 to 0,72.

Poncelet afterwards experimented on a water wheel erected on his principle, measuring the effect by means of a friction brake, and the results are very much the same as those obtained from the model. The wheel was 11 feet in diameter, and had 30 plate-iron floats of $\frac{1}{8}$ inch thickness. The shroudings, arms, and axle of the wheel were of wood. The shrouding was 14 inches deep, and 3 inches thick, the distance between them, or width of the wheel, 28 inches. For a mean head of 4′—4″, and 8 inches depth of water stream, the ratio of the velocities being 0,52, the efficiency came to 0,52, which gives 0,60, when the height due to the velocity, instead of the total fall, is made the basis of calculation. Poncelet makes the following deductions from his series of experiments.

The best velocity ratio $\frac{v}{c}$ is 0,55;* but this may vary between 0,50 and 0,60 without material diminution of the useful effect. For falls of 6′—6″ to 7′—6″, the efficiency $\eta = 0{,}5$, for falls of 5 feet to 6′—6″, the efficiency $\eta = 0{,}55$, and for falls of less than 5 feet $\eta = 0{,}60$.

The useful effect may, therefore, be represented, in the first case, by:

$Pv = 0{,}96\,(c - v)\,v\,Q$ ft. lbs., in the second:

$Pv = 1{,}06\,(c - v)\,v\,Q$ ft. lbs., and in the third:

$Pv = 1{,}15\,(c - v)\,v\,Q$ ft. lbs.

Poncelet gives the following general rules for the construction and arrangement of his wheels, deduced from his experiments. The distance between 2 floats, at their outer extremity, should not exceed 8 to 10 inches, and the radius of the wheel should not be less than 3′—4″ (1 metre), nor more than 8′—2″ ($2\frac{1}{2}$ metres). The axis of the water stream should meet the periphery of the wheel at an angle of 24° to 30°, and be inclined about 3° to the horizon. The offset at the end of the curb should be sufficient to insure the water's free escape from the wheel, and the space left between the wheel and the curb would not exceed $\frac{3}{8}$ inch.

According to the experiments, the efficiency increases with the depth of the water stream laid on, and, therefore, *cæteris paribus*, as the filling of the cells. Further experiments prove that the degree of filling of the cells is an important element in the question.

§ 124. *Recent Experiments.*—Morin has quite recently instituted

* [This is the same ratio as that found by the Committee of the Franklin Institute for the velocity of an overshot wheel with elbow buckets.—AM. ED.]

experiments with three wooden and one iron wheel, constructed on Poncelet's principle, using the friction-brake. They were made with the special object of testing the advantages of a curvilinear course for laying on the water, proposed by M. Poncelet; as also for the purpose of getting more exact information as to the influence of the relative dimensions of the wheels, for in several wheels that have been erected according to Poncelet's rule, it is found that, when the deviation from the mean velocity is considerable, the water overruns the floats. (See *Comptes Rendus*, 1845, t. XXII.) As to the curved water course, its object was to lay the whole of the water on to the wheel without impact, and not the top or bottom stratum only. When the water stream is straight *ABED*, Fig. 233, the upper layer of water *DE* meets the periphery of the wheel, as also the float, at a different angle from that at which the lower stratum does; so that if one enters without impact, the other cannot do so. If, however, we hollow out the bottom of the course as *AOB*, the water stream comes upon a smaller arc *BK*, and the difference in the direction of the periphery of the wheel and the layers of water is less, and, therefore, the impact is less than when the water stream embraces the arc *BE*.

Fig. 233.

The three wooden wheels were respectively 5′ — 3″, 8′ — 3″, and 10′ — 3″ in diameter. The diameter of the iron wheel was 9′ — 3″. The buckets were of sheet iron. The first three wheels were 16 inches wide, and the other was 32 inches. The depth of shrouding was 30 inches. It was found that wooden wheels, having very little inertia, moved unsteadily, and hence arose a loss of water. The smallest wheel revolved very unsteadily, and for a fall of 18 to 22 inches, the cells being at least half filled, the efficiency was 0,485. Had the weight of the wheel been greater, its efficiency would probably have been 0,55. The second wheel, having a fall of 30 inches, gave an efficiency = 0,60 to 0,62. The third wheel was used to make experiments on different lengths of floats. It appeared that for a fall of 22 inches, a length of 17 inches, and for a fall of 28 inches, a length of 24 inches, is too little. Poncelet's curved lead was adapted to this wheel, and it was found that the efficiency was increased, and also that the degree to which the cells are filled, might be made $\frac{2}{3}$ without inconvenience.

The experiments with the iron wheel were instituted with falls of 4 feet to $4\frac{1}{2}$ feet, and of 3 feet, the wheel being free from back-water, and with a fall of 15 inches, the wheel being *in* back-water. For sluice-openings of 6 inches, 8 inches, 10 inches, and 11 inches, the maximum efficiency was 0,52, 0,57, 0,60, and 0,62 respectively, and for variations in the number of revolutions between the limits of 12 to 21, 18 to 21, 11 to 20, and 12 to 19, the efficiency did not differ more than $\frac{1}{18}$, $\frac{1}{14}$, $\frac{1}{12}$, and $\frac{1}{8}$ from the maximum values. From the results of these experiments, it follows that, for wheels with the *hollow* water-lead, the effect is expressed by the formula:

$$Pv = 0,871 \left(\frac{c^2 - w^2}{2g}\right) Q\gamma.$$

Also, that the best velocity ratio $\frac{v}{c} = 0,5$ to $0,55$. That the same effect is produced, whether the water in the race be 5 inches below, or 8 to 10 inches above the bottom of the wheel—that the efficiency falls as low as 0,46, if the wheel be in back-water to the depth of half the depth of the shrouding. The main advantage of the new form of lead is, that the wheel may vary its velocity of rotation within wider limits, without material diminution of the efficiency. Morin considers that, for falls of 3 feet to 4 feet, a breadth of shrouding equal to the half of the radius is a good proportion to adopt, and that the capacity of the wheel should be double that corresponding to the water to be laid on, *i. e.*, the co-efficient of filling $\varepsilon = \frac{Q}{dev}$ should be made $= \frac{1}{2}$.*

Remark. It would thus appear that the capacity of the wheel treated in our last example is too small, and that it would have been better to have made $d_1 = 0,5$ feet, and $e = 5,71$ feet.

§ 125. *Small Wheels.*—Some other vertical water wheels have been applied, besides the systems we have now discussed. Very small wheels of 2 or 3 feet diameter, are moved by the pressure or impact of water.

D'Aubuisson describes, in his "Hydraulique," small impact wheels *ACB*, Fig. 234, with falls of 6 to 7 metres, often to be met with in the Pyrenees. These wheels are from 7 to 10 feet in diameter, and have 24 hollowed floats. Their effect is about ,78 of that of an overshot wheel of the same fall. The effect of such a wheel may be calculated by the theory of breast wheels above given, for these wheels are nothing more than breast wheels with a great impact fall and small height, during which the water can act by its weight. To prevent the spilling of the water, the wheels are hung in a curb with close-fitting sides. Such wheels may be very neatly made of iron, and are to be found in North Wales. This kind of wheel is very commonly employed at the forges in the Alps.

Fig. 234.

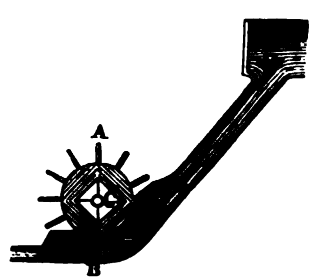


Fig. 235.

Fig. 235 represents a wheel erected by Mr. Mary, and described in the "Tech-

* [The Committee of the Franklin Institute tried curved, oblique, and elbow buckets successively on the same wheel. They found the ratio of effect to power for the curved buckets nearly equal to that for elbow buckets, while in reference to the velocity of the wheel they are much inferior. Elbow buckets gave 5.6, curved 4.2, and oblique 3.7 feet per second velocity of wheel.—Am. Ed.]

nologiste, Sept., 1845." The water here works chiefly by pressure. Belanger experimented with the wheel, and reported an efficiency of 0,75 to 0,85 for a velocity of 4 feet per second. The wheel consists of a shrouding of plate iron, 13 inches wide and 5 inches deep, and 7′ — 6″ in diameter, and having six elliptical floats strengthened by ribs.

The curb is made to fit very accurately, and sheet iron fenders, fitting close to the wheel, prevent the water in the lead from escaping into the race. The power with which such a wheel revolves, is, of course, the product of the weight of water, measured by the difference of level in the lead and in the race, by the area of the float.

Literature. The literature treating of vertical water wheels is very extensive; but there are few works upon the subject worthy of much attention, as the most of them give very superficial and even erroneous views of the theory of these wheels. Eytelwein, in his "Hydraulik," treats very generally of water wheels. Gerstner, in his "Mechanik," treats very fully of undershot wheels. Langedorf's "Hydraulik" contains little on this subject. D'Aubuisson, in his work "Hydraulique à l'usage des Ingénieurs," treats very fully of overshot wheels. Navier treats water wheels in detail in his "Leçons," and in his edition of "Belidor's Architecture Hydraulique." In Poncelet's "Cours de Mécanique appliquée," the theory of water wheels is briefly, but very clearly, set forth. In the "Treatise on the Manufactures and Machinery of Great Britain," P. Barlow has given details on the *construction* of water wheels, but has not entered into the theory of their effects, &c. Very complete drawings and descriptions of good wheels are given in Armengaud's "Traité pratique de Moteurs hydrauliques et à vapeur." Nicholson's "Practical Mechanic," contains some useful information on this subject. The most complete work hitherto published on vertical water wheels is Redtenbacher's "Theorie und Bau der Wasserräder, Manheim, 1846." Poncelet's and Morin's Memoirs have been already cited.

[The experiments of the Franklin Institute are contained in the Journal of that institution for 1831-2 (vols. 7, 8, & 9), and for 1841. In the last mentioned volume, the discussion of the results is commenced, but has not yet been completed. The committee, as originally constituted, does not appear to have given its attention to the application of mathematical reasoning to the observations made and experiments performed. Subsequent European experiments have consequently, in this respect, occupied the attention of physical inquirers to the exclusion of the American.—Am. Ed.]

CHAPTER V.

OF HORIZONTAL WATER WHEELS.

§ 126. In horizontal water wheels, the water produces its effect either by *impact*, by *pressure*, or by *reaction*, but never directly by its weight. Hence, horizontal water wheels are classified as impact wheels, hydraulic pressure wheels, and reaction wheels. These wheels are now very commonly designated by the generic term *turbines* (Ger. *Kreiselräder*).

The *impact* wheels have plane or hollow pallets, on which the water acts more or less perpendicularly. The *pressure* wheels have curved buckets, along which the water flows, and the *reaction* wheels have as their type a close pipe, from which the water discharges

more or less tangentially. Pressure wheels and reaction wheels are generally very similar to each other in construction, the essential difference between them being, that in the former the cells or conduits between two adjacent buckets are not filled up by the water flowing through them, while in reaction wheels the section is quite filled.

According to the different directions in which the water moves in the conduits of pressure and reaction wheels, two systems arise. The relative motion of the water in the conduits is either horizontal, or in a plane inclined to the horizon, and usually vertical.

In the first system, there are to be distinguished those wheels in which the water flows from the *interior* to the *exterior*, and those in which the water takes the opposite course; and in the second system, there are the distinct cases of the water flowing from above downwards, and that in which it flows from below upwards.

Horizontal water wheels in which the water flows from above downwards, are often named *Danaïdes*.

§ 127. *Impact Wheels.—Impact turbines*, as shown in Fig. 236, are the simplest, but also the least efficient form of impact wheels. They consist of 16 to 20 rectangular floats AB, A_1B_1, &c., so set upon the wheel as to incline 50° to 70° to the horizon. The water is laid on to them by a pyramidal trough EF, inclined from 40° to 20°, so that the water impinges nearly at right angles to the floats. Such wheels are employed for falls of from 10 to 20 feet, when a

Fig. 236.

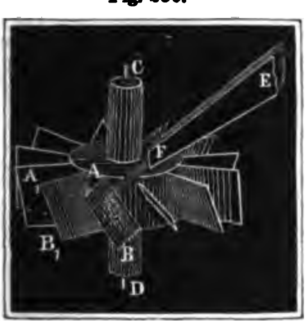
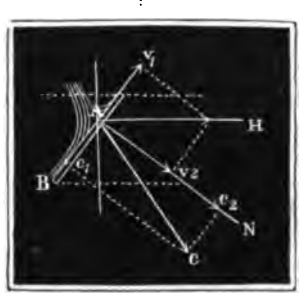


Fig. 237.



great number of revolutions is desired, and when simplicity of construction is a greater desideratum than efficiency. Wheels of this form are met with in all mountainous countries of Europe, and in the north of Africa, applied as mills for grinding corn. They are made from 3 to 5 feet in diameter, the buckets being about 15 inches deep, and 8 to 10 inches long.

The mechanical effect of these wheels is determined according to the theory of the impact of water, as follows. The velocity $Ac = c$, Fig. 237, of the water impinging, and the velocity $Av = v$ of the buckets may be each decomposed into two velocities expressed by the formulas

$c_1 = c \sin. \delta$, $c_2 = c \cos. \delta$, $v_1 = v \sin. \alpha$, and $v_2 = v \cos. \alpha$, δ being the angle $c\,AN$, by which the direction Ac of the stream of water deviates from the normal AN, and α the angle HAN at which the normal is inclined to the horizon, or by which the direction of the wheel's motion deviates from the normal, or the plane of the bucket from the vertical. The component velocity $c_1 = c \sin. \delta$, remains unchanged, as its direction coincides with that of the plane of the bucket; the component $c_2 = c \cos. \delta$, is, on the other hand, changed by impact into $v_2 = v \cos. \alpha$, as the bucket moves away in the direction of the perpendicular with this velocity. The water, therefore, loses by impact a velocity

$$c_2 - v_2 = c \cos. \delta - v \cos. \alpha,$$

and the corresponding loss of effect $= \frac{(c \cos. \delta - v \cos. \alpha)^2}{2g} Q\gamma$. If, now, we deduct from the whole available mechanical effect $\frac{c^2}{2g} Q\gamma$, the above, and further, the effect, $\frac{(c \cos. \delta - v \cos. \alpha)^2}{2g} Q\gamma$, and $\left(\frac{c^2 \sin. \delta^2 + v^2 \cos. \alpha^2}{2g}\right) Q\gamma$, which the water flowing away with the velocity $w = \sqrt{c_2 \sin. \delta^2 + v^2 \cos. \alpha^2}$, retains, the mechanical effect communicated by the wheel is

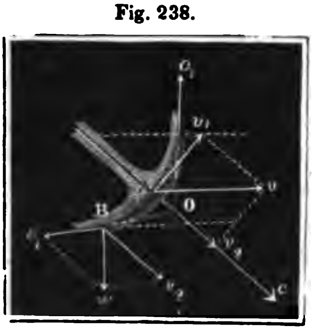
$$L = Pv = [c^2 - (c \cos. \delta - v \cos. \alpha)^2 - (c^2 \sin. \delta^2 + v^2 \cos. \alpha^2)] \frac{Q\gamma}{2g}$$

$$= \frac{(c \cos. \delta - v \cos. \alpha)\, v \cos. \alpha}{g} \cdot Q\gamma.$$

To get the maximum effect, we must make $\cos. \delta = 1$, or $\delta = 0$, or direct the stream at right angles to the bucket, and besides this, as in other similar cases already treated, we must make $v \cos. \alpha = \frac{1}{2} c$, or $v = \frac{c}{2 \cos. \alpha}$. The maximum effect corresponding, is $Pv = \frac{1}{2} \frac{c^2}{2g} Q\gamma = \frac{1}{2} h\, Q\gamma$, or the half of the entire mechanical effect available.

§ 128. The effect of impact wheels is increased by surrounding the buckets with a projecting border or frame, or by forming them like spoons, as shown in Fig. 238. Vol. I. § 385 explains the cause of this increased effect, but we may here determine the amount of this increase. As the bucket moves in the direction of the stream with the velocity $v_2 = v \cos. \alpha$, the relative velocity of the water in reference to the bucket may be put:

Fig. 238.



$$c_1 = c - v_2 = c - v \cos. \alpha,$$

and if $\beta =$ the angle $c_1\, O\, c$, by which the water is turned aside from its ori-

ginal direction, the absolute velocity of the water flowing off:

$$w = \sqrt{c_1^2 + v_2^2 + 2\, c_1\, v_2 \, cos.\, \beta}$$

$$= \sqrt{(c - v\, cos.\, \alpha)^2 + v^2\, cos.\, \alpha^2 + 2\,(c - v\, cos.\, \alpha)\, v\, cos.\, \alpha\, cos.\, \beta},$$

and hence, the corresponding loss of effect:

$$= [c^2 - 2\,(c - v\, cos.\, \alpha)\, v\, cos.\, \alpha\, (1 - cos.\, \beta)]\, \frac{Q\gamma}{2g},$$

and the effect of the wheel:

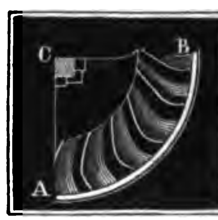
$$L = Pv = \left(\frac{c^2 - w^2}{2g}\right) Q\gamma = (1 - cos.\, \beta)\, \frac{(c - v\, cos.\, \alpha)\, v\, cos.\, \alpha}{g} \, . \, Q\gamma.$$

When the buckets are plane, $\beta = 90^\circ$, $\therefore$ $cos.\, \beta = 0$, and, therefore

$$L = \frac{(c - v\, cos.\, \alpha)\, v\, cos.\, \alpha}{g}\, Q\gamma,$$

as we have already found, though by an entirely different method of inquiry. In the case of hollow buckets, β is greater than 90°, and, therefore, $cos.\, \beta$ is negative, and hence $1 - cos.\, \beta$ is greater than 1, consequently the effect is greater than in plane buckets.

Fig. 239.



To this class of wheels belong those termed in France *rouets volants*, upon the effect of which MM. Piobert and Tardy have recorded experiments in a work entitled "Expériences sur les Roues hydrauliques à axe vertical, &c., Paris, 1840." The following are results of experiments on a small wheel of 5 feet diameter, 8 inches high, having 20 curved buckets, Fig. 239, with a fall of 14 feet (measuring from surface of water in lead to bottom of wheel), and with 10 cubic feet of water per second:

$$\text{For } \frac{v}{c} = 0{,}72,\ \eta = 0{,}16;$$

$$\text{" } \frac{v}{c} = 0{,}66,\ \eta = 0{,}31;$$

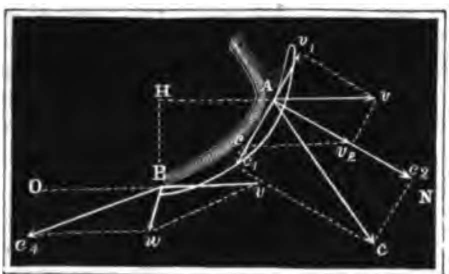
$$\text{" } \frac{v}{c} = 0{,}56,\ \eta = 0{,}40;$$

and hence, in cases in which the velocity ratio $\frac{v}{c}$ does not much differ from 0,6: $Pv = 0{,}75\,(c - v\, cos.\, \alpha)\, \frac{v\, cos.\, \alpha}{g}\, Q\gamma.$

Example. What effect may be expected from an impact turbine with hollow buckets (Fig. 239), there being 6 cubic feet of water, and a fall of 16 feet at disposition? If we neglect the depth of the wheel itself, the theoretical velocity of entrance of the water $c = \sqrt{2gh} = 8{,}02\sqrt{16} = 32{,}08$ feet, and if the inclination of the trough be assumed as 20°, the most advantageous velocity for the wheel $v = \frac{c}{2\, cos.\, \alpha} = \frac{16{,}04}{cos.\, 20^\circ} = 17$ feet and hence, from the above formula, the effect attainable is
$L = Pv = 0{,}75 \, . \, \frac{c\, v\, cos.\, \alpha - v^2\, cos.\, \alpha^2}{g}\, Q\gamma = \frac{3}{4} \, . \, 031 \, . \, (512{,}09 - 15{,}96^2) \, . \, 6 \, . \, 62{,}5 = {,}023 \times 257.4 \, . \, 375 = 2220$ feet lbs.

§ 129. *Impact and Reaction Wheels.*—If we give the buckets greater length, and form them to such a hollow curve, that the water leaves the wheel in a nearly horizontal direction, the water then not only impinges on the bucket, but exerts a pressure on it, and, therefore, the effect of the wheel is greater than in the impact wheel. The theory of such wheels is merely an extension of that given in § 127. If we conceive a normal erected at the point of entrance A, Fig. 240, and if we again put the angle $c\ AN = \delta$, and the angle $v\ AN = \alpha$, we have the lost velocity arising from impact:

Fig. 240.



$c_2 - v_2 = c\ cos.\ \delta - v\ cos.\ \alpha$, and the loss of effect corresponding $= \dfrac{(c\ cos.\ \delta - cos.\ \alpha)^2}{2g} Q\gamma$.

The velocity with which the water begins to flow down the buckets is $c_1 + c_3 = c\ sin.\ \delta + v\ sin.\ \alpha$, and if we put the height BH, through which the water descends on the bucket $= h_1$, we have the relative velocity of the water at the bottom B of the bucket:

$$c_4 = \sqrt{(c_1 + c_3)^2 + 2gh_1} = \sqrt{(c\ sin.\ \delta + v\ sin.\ \alpha)^2 + 2gh_1}.$$

But the water possesses the velocity v in common with the wheel, and, therefore, the absolute velocity of the water flowing from the wheel: $w = \sqrt{c_4^2 + v^2 - 2\ c_4\ v\ cos.\ \theta}$, where $\theta =$ the angle c_4BO, at which the lowest element of the bucket is inclined to the horizon. The loss of effect corresponding to this is:

$$\frac{w^2}{2g} Q\gamma = \left(\frac{c_4^2 + v^2 - 2\ c_4\ v\ cos.\ \theta}{2g}\right) Q\gamma.$$

If we deduct these two losses from the whole available effect, we get the useful effect communicated to the wheel:

$$L = Pv = [c^2 - (c\ cos.\ \delta - v\ cos.\ \alpha)^2 - (c_4^2 + v^2 - 2\ c_4\ v\ cos.\ \theta)] \frac{Q\gamma}{2g},$$

in which we have to substitute for c_4 the value above given.

If the water impinges at right angles $\delta = 0$, and $c_4 = \sqrt{v^2\ sin.\ \alpha^2 + 2gh_1}$, and, therefore,

$$L = [c^2 - (c - v\ cos.\ \alpha)^2 - (c_4^2 + v^2 - 2\ c_4\ v\ cos.\ \theta)] \frac{Q\gamma}{2g}$$

$$= [2\ c\ v\ cos.\ \alpha - (1 + cos.\ \alpha^2)\ v^2 - v^2\ sin.\ \alpha^2 - 2gh_1 + 2\ v\ cos.\ \theta\ .$$
$$\sqrt{v^2\ sin.\ \alpha^2 + 2gh_1}] \frac{Q\gamma}{2g}$$

$$= [(c\ cos.\ \alpha - v)\ v - gh_1 + v\ cos.\ \theta \sqrt{v^2\ sin.\ \alpha^2 + 2gh_1}] \frac{Q\gamma}{2g}.$$

In order that the water may produce its maximum effect, it should

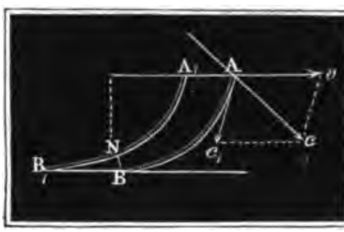
fall dead from the wheel, or w should $= 0$. This requires that $\theta = 0$, and $c_4 — v$, i. e., $v^2 \sin. \alpha^2 + 2gh_1 = v^2, \therefore v = \frac{\sqrt{2gh_1}}{\cos. \alpha}$.

§ 130. *Pressure Wheels.*—If the water is to be laid on without impact, then $v \cos. \alpha$ must $= c \cos. \delta$, and in order that the water may quit the wheel, deprived of its *vis viva*, we must have: $\theta = 0$, and $c_4 = v$, i. e. $(c \sin. \delta + v \sin. \alpha)^2 + 2gh_1 = v^2$, or $c^2 \sin. \delta^2 + 2 c v \sin. \alpha \sin. \delta + 2gh_1 = v^2 \cos. \alpha^2 = c^2 \cos. \delta^2$. If, again, we subtract from both sides: $2 c v \cos. \alpha \cos. \delta = 2 c^2 \cos. \delta^2$, then: $c^2 \sin. \delta^2 - 2 c v (\cos. \alpha \cos. \delta - \sin. \alpha \sin. \delta) + 2gh_1 = - c^2 \cos. \delta^2$, or $c^2 + 2gh_1 = 2 c v \cos. (\alpha + \delta)$, and, therefore,

$$v = \frac{c^2 + 2gh_1}{2 c \cos. (\alpha + \delta)} = \frac{g (h + h_1)}{c \cos. \phi},$$

in which h is the velocity of the water at entrance, and, therefore, $h + h_1$ the whole fall, ϕ is the angle $c A v$ between the direction of the water and that of the wheel. The theoretical effect is, in the latter case, $= (h + h_1) Q \gamma$, and the efficiency $\eta = 1$, because there is no loss from any cause. When, for such a wheel, the best velocity of rotation v has been found, we get the requisite position of the buckets by drawing through the point of entrance A, Fig. 241, a line parallel to $v c$, completing the parallelogram $A v c c_1$. The side $A c_1$, thus given, gives the relative velocity c_1, with which water begins to descend along the bucket, in magnitude and direction, and also the direction of the upper element of the bucket.

Fig. 241.



That the water may flow unimpeded through the openings BB_1, &c., the foot of the buckets must have a slight inclination to the horizon. If we put the mean radius of the wheel $= a$, and the mean length of the buckets, measured on the radius, $= l$, we may put the section of the orifice of discharge $= \overline{BN} . l = \overline{BB_1} \sin. \theta . l$, and, therefore, the section of the united orifices of the wheel $= 2 \pi a l \sin. \theta$. If, again, $c_2 =$ the relative velocity with which the water arrives at the bottom of the wheel, or, if

$$c_2 = \sqrt{c^2 + v^2 - 2 c v \cos. \phi + 2gh_1},$$

we have $2 \pi a l \sin. \theta = \frac{Q}{c_2}$, and, therefore, for the requisite angle

$$\sin. \theta = \frac{Q}{2 \pi a l c_2}.$$

Remark. According to the theory of the impact of water, or of *hydraulic pressure*, expounded in our first volume, it is not necessary that $v \cos. \alpha = c \cos. \delta$, or, which amounts to the same, that the component c_1 of the velocity, should fall in the direction of the bucket. According to Vol. I. § 43, the relative velocity c_1 of the water in reference to the bucket AB, Fig. 242, is the diagonal of the parallelogram constructed from the abso-

lute velocity of the water c and the velocity of the wheel v, taken in the opposite direction; therefore, $c_1 = \sqrt{c^2 + v^2 - 2\,c\,v\cos.\phi}$. If, now, the direction, but not the magnitude, of this velocity be changed by the shock on the bucket, we have the relative velocity at discharge, after descent through the height $BH = h_1$, $c_2 = \sqrt{c_1^2 + 2gh_1} = \sqrt{c^2 + v^2 - 2\,c\,v\cos.\phi + 2gh_1}$: lastly, that the whole effect may be taken up from the water, we have to make:

$$c_2 = v, \text{ or } c^2 + v^2 - 2\,c\,v\cos.\phi + 2gh_1 = v^2, \text{ therefore,}$$

$$v = \frac{c^2 + 2gh^1}{2\,c\cos.\phi} = \frac{g\,(h + h_1)}{c\cos.\phi}.$$

§ 131. *Borda's Turbine.*—The wheels discussed in the last paragraph, are called *Borda's* turbines, from their having been the suggestion of that distinguished officer and philosopher. Their construction is shown by Fig. 243, which is a sketch of one driving 6 amalgamation barrels, at the silver mines of Huelgoat in Brittany. The curved buckets are composed of three beech boards put carefully together, and the inner and outer casings are composed of staves, the outer one being bound by two iron hoops. The diameter of the wheel is 5 feet. The buckets 14 inches long or deep, and 16½ inches wide. There are 20 of them. The fall was 16′ — 3″, and the wheel makes 40 revolutions per minute.

Fig. 242.

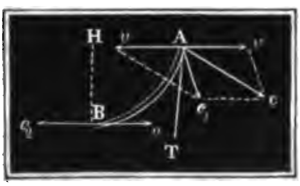
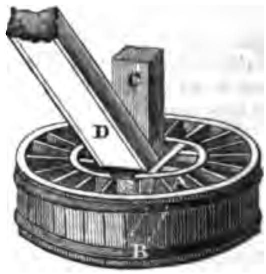


Fig. 243.



There are no good experiments on the efficiency of Borda's turbines. Borda gives 0,75 of the theoretical effect as the useful effect, or $L = 0{,}75\,.\,[h + h_1 - (c\cos.\delta - v\cos.\alpha)^2 - w^2]\,Q\gamma$. Poncelet very justly remarks that it is advisable to make the diameter and the height of the wheels as great as possible, so as to curtail the length of bucket, that is, bringing the outer and inner casings near to each other. By giving height to the wheel, the fall due to the velocity is diminished, and, therefore, the velocity of the water and of the wheel is less. By keeping the *diameter* great, the number of revolutions falls out less, and as for a larger wheel, the capacity remaining the same, the width of the wheel may be less, and then the difference of velocity of the particles of water adjacent to each other will be less.

Example. What quantity of water must be supplied to a Borda's turbine, constructed as shown in Fig. 243, which, with a fall of 15 feet, is to drive a pair of millstones requiring 2 horse power? Suppose the wheel to be 1¾ feet high, then the theoretical velocity of entrance of the water:

$$c = 8{,}02\sqrt{15 - 1{,}75} = 8{,}02\sqrt{13{,}25} = 29{,}19 \text{ feet.}$$

If the water be laid on at an angle of 30° to the horizon, then the least velocity of rotation is: $v = \frac{g\,(h + h_1)}{c\ cos.\ \phi} = \frac{32{,}2 \times 15}{29{,}19 \times cos.\ 30^\circ} = 19{,}1$. If the water enters without shock, the velocity with which it begins its descent along the bucket is:
$c_1 = \sqrt{c^2 + v^2 - 2\,c\,v\ cos.\ \phi} = \sqrt{c^2 + v^2 - c^2 - 2gh_1} = \sqrt{v^2 - 2gh_1} = 15{,}88$ feet. For the angle ψ, at which the upper element of the buckets must incline to the horizon, we have: $\frac{sin.\ \psi}{sin.\ \phi} = \frac{c}{c_1} \therefore sin.\ \psi = \frac{29{,}19}{15{,}88}\ sin.\ 30^\circ = 0{,}9189 \therefore \psi = 66^\circ, 46'$. If we give the bottom of the bucket an inclination of 25° to the horizon, we get for the absolute velocity of the water flowing away:

$$w = 2\,v\ sin.\ \frac{\theta}{2} = 2\,.\,19{,}1\ sin.\ 12\tfrac{1}{2}^\circ = 8{,}2\ \text{ft.},$$

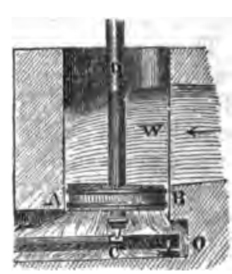
and, hence, the effect:

$$L = \tfrac{3}{4}\left(h + h_1 - \frac{w^2}{2g}\right) Q\gamma = \tfrac{3}{4}\left(15 - \frac{8{,}2^2}{2g}\right).\ 62{,}5\ Q = 654\ Q.$$

That we may have 2 horse power, or 1100 feet pounds per second, we must have $\frac{1100}{652} = 1{,}7$ cubic feet of water per second. If the mean radius (measured to the centre of the buckets) of the wheel be 2 feet, and if the water space be 6 inches wide, we get the united areas of section of the orifices of discharge at the bottom of the wheel $= 2\,\pi\,a\,l\ sin.\ \theta = \pi\,.\,4\,.\,\tfrac{1}{2}\ sin.\ 25^\circ = 2{,}65$ square feet, which is quite sufficient to pass 1,7 cubic feet of water per second, with a velocity of 19 feet.

§ 132. *Roues en Cuves.*—To this category of turbines belong those horizontal wheels enclosed in a pit or *well*, frequently met with in the south of France, and called *roues en cuves* (Ger. *Kufenräder*). They are described by Belidor in the "Architecture Hydraulique," by D'Aubuisson in his "Hydraulique," and Piobert and Tardy, in the work already cited, have given the results of experiments instituted on one of these wheels. These wheels are very similar in form to those last described (Fig. 239). They are generally 1 metre in diameter, and have 9 curved buckets. They are made of only two pieces, and are bound together by iron hoops. The axis CD (Fig. 244) stands on a pivot, the footstep of which is on a lever CO, by which the wheel may be raised and lowered as the millstone may require. The wheel is near the bottom of a well, 2 metres deep, and 1,02 metres in diameter. The water comes into the well by a lead laid tangentially to it, about 13 feet long, the breadth at the outer extremity being 2′ — 6″, and at the entrance to the well about 10 inches. The water flows in with a great velocity, acquires a rotary motion in the wheel chamber, and acts by impact and pressure on the wheel buckets, flowing through it into the tail-race. There is evidently a great loss of water in such wheels, and their efficiency is consequently small. Piobert and Tardy found an efficiency of 0,27 for a *well* wheel at Toulouse, the fall being 10 feet, with $13\frac{1}{2}$ cubic feet of water per second, and the number of revolutions $u = 100$. For $u = 120$, the efficiency η was $= 0{,}22$, and for $u = 133$, $\eta = 0{,}15$. The wheels of the Basacle mill, at Toulouse, give an efficiency of 0,18.

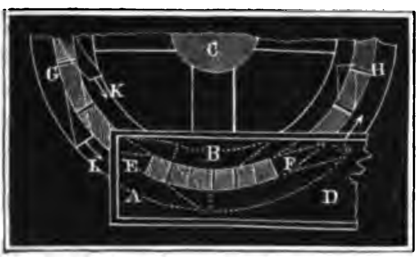
Fig. 244.



D'Aubuisson mentions that wheels of this kind have been erected recently, the wheel being put immediately under the bottom, and made of somewhat greater diameter than the well. The pyramidal trough for laying on the water is much shortened, and by these means the efficiency has been raised to 0,25. These wheels are, therefore, at best, inferior to the impact wheels already treated of.

§ 133. *Burdin's Turbines.*—M. Burdin, a French engineer of mines, proposed what he terms a "turbine à évacuation alternative." They are the best wheels of the category now under examination. They differ from Borda's wheels only in this essential, namely, that the water enters them at various points simultaneously, and that the orifices of discharge are distributed over 3 concentric rings. This latter arrangement is adopted, that the water, discharged with a small absolute velocity, may not hinder the revolution of the wheel. The first wheel of this kind was erected by Burdin for a mill at Pont-Gibaud, and is described in the "Annales des Mines, III. série, t. III." Fig. 245 represents a plan of this wheel. *ABD* is the pen-trough immediately above the wheel, having a series of orifices *EF* in the bottom, through which the water is laid on to the wheel with a slight inclination. The wheel revolving on the axis *c* consists of a series of conduits, the entrances to which make together the annular space *GBH*, which moves accurately under the arc *EF* formed by the trough-openings, so that the water passes unimpeded from the one into the other. The conduits (Fr. *couloirs*) are vertical at the upper end, and nearly horizontal, and tangential at the bottom. The lower ends are brought into *three* distinct rings, so that the third of the number of entrances only discharge in the ring vertically under them; one-third, as *K*, discharge *within*, the others, as *L*, discharge *outside* this ring.

Fig. 245.



From the experiments made on the turbine erected at Pont-Gibaud by Burdin, it appears that for 3 cubic feet of water per second, and a fall of 10,35 feet, the efficiency was 0,67. The impact turbine formerly in the same position consumed 3 times this quantity of water to produce the same effect. The diameter of the wheel was 4,6 feet, and the depth 15 inches. The number of buckets 36.

§ 134. *Effect of Centrifugal Force.*—In the turbines hitherto under consideration, the water moves nearly, if not exactly, on a cylindrical surface, and, therefore, each element of water retains the same relative position to the axis, or at least does not vary it much. But we have now to consider wheels, in which the water, besides a rotary and vertical motion, possesses a motion inwards or outwards in reference to the axis, and more or less radial. The peculiarity of

such turbines is, that their motion depends on the centrifugal force of the water, so that they might be termed centrifugal turbines. Before entering on a discussion of these wheels, it will be well to investigate the effect of the water's centrifugal force, when its motion is in a spiral line round a centre, or when the motion is radial and rotary at the same time. The centrifugal force of a body of the weight G, revolving at a distance y, with an angular velocity ω, round a given point, is $F = \frac{\omega^2 G y}{g}$ (Vol. I. § 231). If this weight moves also a small distance σ radially outwards, or inwards, then this force will have produced, or absorbed, an amount of mechanical effect represented by: $F\sigma = \frac{\omega^2 G y \sigma}{g}$. If, then, we assume that the motion commences in the centre of rotation, and continues radially outwards, so that ultimately the distance of the weight from the axis $= r$, we may ascertain the mechanical effect produced by the centrifugal force by substituting in the last formula $\sigma = \frac{r}{n} y$, introducing successively, however, $\frac{r}{n}, \frac{2r}{n}, \frac{3r}{n} \ldots \frac{nr}{n}$, and uniting the mechanical effects resulting by summation. Hence the mechanical effect in question is:

$$L = \frac{\omega^2 G r}{n g}\left(\frac{r}{n} + \frac{2r}{n} + \frac{3r}{n} + \ldots + \frac{nr}{n}\right)$$

$$= \frac{\omega^2 G r^2}{n^2 g}(1 + 2 + 3 + \ldots + n) = \frac{\omega^2 G r^2}{n^2 g} \cdot \frac{n(n+1)}{2}$$

or, as we must assume n infinite:

$$L = \frac{\omega^2 G r^2}{n^2 g} \cdot \frac{n^2}{2} = \frac{\omega^2 r^2}{2g} \cdot G = \frac{v^2}{2g} G,$$

when v is the velocity of rotation ωr of the body at the extreme point of its motion. As this mechanical effect is produced by the centrifugal force when the motion is from within *outwards*, it must be consumed when the motion is from without *inwards*. If the body does not come to the centre at the end of its motion, but remains at a distance r_1 from it, then there remains an amount of effect $\frac{\omega^2 r_1^2}{2g} G$, and the body consumes, therefore, only the effect

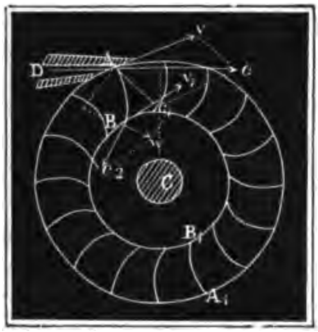
$$L = \frac{\omega^2 r^2}{2g} G - \frac{\omega^2 r_1^2}{2g} G = (r^2 - r_1^2)\frac{\omega^2}{2g} G = \left(\frac{v^2 - v_1^2}{2g}\right) G,$$

if v_1 represent the velocity of rotation at the distance r_1 or end of the motion, as v represents it at the distance r or commencement of the motion. If the motion is from within outwards, then the effect produced by centrifugal force is $L = \left(\frac{v^2 - v_1^2}{2g}\right) G$.

§ 185. *Poncelet's Turbine.*—One of the most simple horizontal wheels, in which centrifugal force influences the working, is Ponce-

let's turbine, shown in Fig. 246, in plan. This turbine has curved buckets between shroudings, and is, in fact, one of Poncelet's undershot wheels, laid on its side. The water is laid on by a trough AD nearly tangentially, and runs along the curved bucket to discharge itself in the interior. That the effect of the water on the wheel may be a maximum, it is necessary that the water should enter without shock and discharge into the interior deprived of its *vis viva*. The direction of the end of the bucket A, insuring no shock, is determined exactly as for Poncelet's undershot wheel, by constructing a triangle with the velocity v of the wheel and that c of the water entering and drawing Ac_1 parallel to the side vc. The relative velocity Ac_1 with which the water enters the wheel is: $c_1 = \sqrt{c^2 + v^2 - 2\,c\,v\,cos.\,\delta}$, δ being the angle $c\,A\,v$ by which the direction of the stream of water deviates from the tangent to the circumference of the wheel. This velocity is, however, diminished by centrifugal force during the motion of the water on the bucket, and, therefore, the relative velocity $Bc_2 = c_2$ with which the water comes to the inside of the wheel, is less than the above velocity c_1. According to the result of the investigation in the last paragraph, the water loses an amount of effect represented by $\left(\frac{v^2 - v_1^2}{2g}\right) Q\gamma$, or $\frac{v^2 - v_1^2}{2g}$ in pressure or velocity height, v being the velocity of rotation at the commencement, and v_1 that at end of the motion. If, therefore, $\frac{c_1^2}{2g}$ be the height due to the velocity at the entrance A, and $\frac{c_2^2}{2g}$ that at the exit B, we have

Fig. 246.



$\frac{c_2^2}{2g} = \frac{c_1^2}{2g} - \left(\frac{v^2}{2g} - \frac{v_1^2}{2g}\right)$, and, therefore, $c_2^2 = c_1^2 - v^2 + v_1^2$; or as $c_1^2 = c^2 + v^2 - 2\,c\,v\,cos.\,\delta$, $c_2^2 = c^2 + v_1^2 - 2\,c\,v\,cos.\,\delta$, and $c_2 = \sqrt{c^2 + v_1^2 - 2\,c\,v\,cos.\,\delta}$, it being constantly borne in mind that v is the velocity of rotation at the outer periphery, and v_1 that at the inner. In order to rob the water of all its *vis viva*, the end B of the bucket should be laid tangentially to the inner periphery of the wheel, and also c_2 should be made

$$= v_1, \text{ or } c^2 + v_1^2 - 2\,c\,v\,cos.\,\delta = v_1^2, \text{ i. e., } v\,cos.\,\delta = \frac{c}{2}.$$

For the sake of an unimpeded discharge of the water to the interior, the angle δ_1 at which the inner end of the bucket cuts the wheel, must be made 15° to 30°, and, hence, the absolute velocity of the water discharged $w = \sqrt{c_2^2 + v_1^2 - 2\,c_2\,v_1\,cos.\,\delta_1}$, or, if we assume

$v\ cos.\ \delta = \frac{c}{2}$, or, $v_1 = c_2$, $w = 2\, v_1\, sin.\ \frac{\delta_1}{2}$, and the loss of mechanical effect corresponding is:

$$\frac{w^2}{2g}\, Q\gamma = \left(2\, v_1\, sin.\ \frac{\delta_1}{2}\right)^2 \frac{Q\gamma}{2g},$$

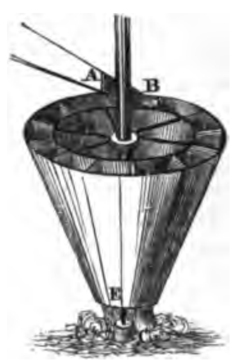
lastly, the remaining useful effect of the wheel:

$$L = \left[c^2 - \left(2\, v_1\, sin.\ \frac{\delta_1}{2}\right)^2\right] \frac{Q\gamma}{2g}.$$

According to Poncelet, these wheels should give an efficiency of 0,65 to 0,75.

§ 136. *Danaïdes.*—We shall next treat of horizontal wheels which have more or less the form of an inverted cone, and which are termed in France *roues à poires*, or *Danaïdes*. Belidor describes them in the "Architecture hydraulique." Fig. 247 represents the general arrangement of these wheels. They consist essentially of a vertical axis, with a double conical casing attached. The space between the casing is intersected by division plates, forming conduits running from top to bottom. The water is laid on by a trough A at top, and flows off through the bottom of the cone at E, near to the axis, after having passed through the conduits above mentioned. In the simplest form of wheels, the division plates are plane surfaces, running vertically; in other cases they are spiral or *screw-formed.* Belidor describes the wheel without the outer casing, but the wheel is placed in a conical vessel fitting pretty closely to the blades, or division plates.

Fig. 247.



In these wheels, gravity and centrifugal force act simultaneously on the water. If the water enters the wheel with the relative velocity c_1 above, at the point B, the velocity of rotation of which is v, and flows, in the wheel, through a height h_1, the velocity at the bottom of the wheel near the axis will be c_2, determined by the formula $c_2{}^2 = c_1{}^2 + 2gh_1 - v^2$. In order that this may be 0, we must have $v^2 = c_1{}^2 + 2gh_1$. Further, that the water may enter the wheel without shock, the horizontal component of its velocity must equal the velocity of rotation, that is, $c\ cos.\ \delta = v$, δ being the inclination of the stream of water to the horizon.

The relative velocity of entrance is $c_1 = c\ sin.\ \delta$, and, therefore, the above equation of condition becomes,

$$c^2\ cos.\ \delta^2 = c^2\ sin.\ \delta^2 + 2gh_1, \text{ i. e., } c^2\ cos.\ 2\,\delta = 2gh_1.$$

The fall necessary for the velocity is, therefore, $h_2 = \frac{c^2}{2g} = \frac{h_1}{cos.\ 2\,\delta}$.

If, now, the whole fall $h_1 + \frac{h_1}{cos.\ 2\,\delta} = h$, then the depth of the

wheel $h_1 = \frac{h \cos. 2 \delta}{1 + \cos. 2 \delta}$, and the height due to the velocity:

$$h_2 = \frac{h}{1 + \cos. 2 \delta}.$$

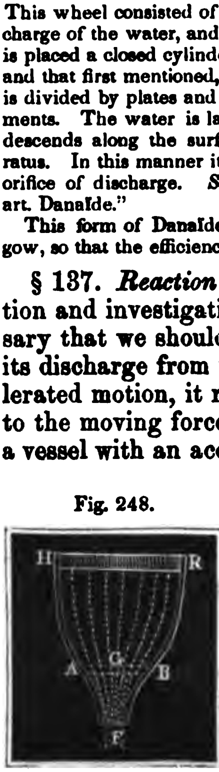
By this arrangement there is no loss of mechanical effect, but as the axis has a certain sectional area, and the water too requires a certain area of orifice for discharge, and thus the water can only be brought to within a certain distance of the axis, its *vis viva* cannot be entirely taken up, so that the efficiency is not nearly 1,0.

Remark. The wheel just described is known as Burdin's Danaïde. The older Danaïde of Manouri d'Ectot was differently constructed, though in principle it was the same. This wheel consisted of a sheet iron cylinder, with an orifice in the bottom for the discharge of the water, and through which the axis passed. In this hollow cylinder, there is placed a closed cylinder in such a position as to leave an annular space between it and that first mentioned, and also a space between the bottoms of the two. This latter is divided by plates and buckets placed vertically and radially into a series of compartments. The water is laid on tangentially into the space between the two cylinders, descends along the surface to the bottom, inducing a rotary motion of the whole apparatus. In this manner it flowed gradually to the bottom, and from thence reached the orifice of discharge. *See* "Dictionnaire des Sciences mathématiques par Montferrier, art. Danaïde."

This form of Danaïde has been recently perfected by Mr. James Thomson, of Glasgow, so that the efficiency of a model has been proved to be 0,85.

§ 137. *Reaction of Water.*—Before proceeding with the description and investigation of the theory of *reaction wheels*, it is necessary that we should illustrate the nature of the reaction of water in its discharge from vessels. As a solid body endowed with an accelerated motion, it reacts in the opposite direction with a force equal to the moving force, so it is in the case of water when it issues from a vessel with an accelerated motion from the orifice. This acceleration always takes place when the area of the orifice is less than the area of the vessel, or the velocity of discharge greater than the velocity of the water through the vessel. On these grounds the vertical pressure of the water in the vessel HRF, Fig. 248, from which the water flows downwards at F, is *less* than the weight of the mass of water in the vessel. The decrease of this force, or *the reaction of the water* flowing away, may be determined as follows. If the horizontal layer AB of the water flowing out, has a variable section G, a variable thickness x, and a variable acceleration p, its weight is $G x \gamma$ and its mass $= \frac{G x \gamma}{g}$, and, therefore, its reaction $K = \frac{G x \gamma}{g} \cdot p$. If, now, w represent the variable velocity of the layer of water, and $\varkappa$ its increase in passing through the elementary distance x, we have (according to Vol. I. § 19) $p x = w \varkappa$, and, therefore, $K = \frac{G \gamma}{g} w \varkappa$.

Fig. 248.



If F be the area of the orifice, and v the velocity of discharge, then $Gw = Fv$, and, therefore, $K = \frac{F\gamma}{g} v\,x$. To obtain the reaction of all the layers of water, we must substitute in the last expression for x, the increments of velocity $x_1, x_2, x_3, \ldots x_n$ of all the layers of water, and sum the results. The reaction of the whole mass is thus $P = \frac{Fv}{g}\gamma(x_1 + x_2 + \ldots + x_n)$. If c be the velocity of entrance of the water, the sum of all the increments of velocity $= v - c$, and, therefore, the reaction required:

$$P = \frac{Fv\gamma}{g}(v - c) = \frac{(v - c)}{g} Fv \,.\, \gamma = \frac{v - c}{g} \,.\, Q\gamma,$$

Q being the quantity discharged per second. If, however, the orifice F be very small compared with the surface HR, then c may be neglected, compared with v, and

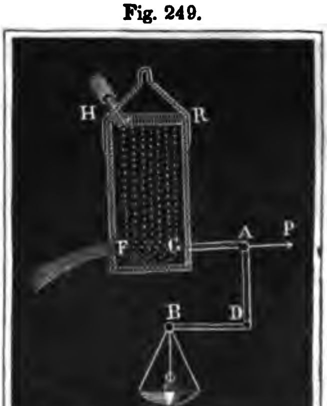
$$P = \frac{v^2}{g} F\gamma = 2 \,.\, \frac{v}{2g} F\gamma = 2h \,.\, F\gamma.$$

So that the reaction is as great as the vertical impact of the water on a plane surface (Vol. I. § 385), that is, *equal to the weight of a column of water, the basis of which is the area F of the orifice, or of the stream, and whose height is double the height due to the velocity* (2h) *of the water discharged.*

If the water flow out by the side of the vessel, as shown in Fig. 249, the direction of the reaction is then horizontal, and the amount is in like manner $= \frac{c^2}{g} F\gamma$. If the water vein be contracted, and if a be the co-efficient of contraction, then, instead of F, we must put Fa, or $P = \frac{c^2}{g} \,.\, a\,F\gamma$.

Remark. Mr. Peter Ewart, of Manchester, made experiments to test this result, ("Memoirs of the Manchester Phil. Soc.," Vol. II.) The vessel HRF (Fig. 249) was hung on a horizontal axis C, and the reaction measured by a bent lever balance ADB, upon which the vessel acted by means of a rod GA, bearing on the point directly opposite to the orifice F. In the discharge through an orifice in the *thin plate*, it was found that

Fig. 249.



$$P = 1{,}14 \frac{v^2}{2g} F\gamma.$$

If we take the section of the stream: $F_1 = 0{,}64 \,.\, F$, and the effective velocity of discharge $v_1 = 0{,}960$ (Vol. I. § 315), we have, according to the theoretical formula:

$$P = 2 \,.\, \frac{v_1^2}{2g} F_1\gamma = 2 \,.\, 0{,}96^2 \,.\, 0{,}64 \,.\, \frac{v^2}{2g} F\gamma$$
$$= 1{,}18 \frac{v^2}{2g} F\gamma,$$

nearly the same as the experimental result. When the orifice was provided with a mouth-piece formed like the *vena contracta*, it was found that:

$P = 1,73 \cdot \frac{v^2}{2g} F \gamma$, the co-efficient of discharge being 0,94. As in this case $F_1 = F$, and $v_1 = 0,94\, v$, the theoretical result is:

$$P = 2 \cdot 0,94^2 \frac{v^2}{2g} F \gamma = 1,77 \cdot \frac{v^2}{2g} F \gamma,$$

or a very close agreement with the experimental result.

§ 138. *Reaction Wheels.*—If a vessel, as *HRF*, Fig. 250, be placed on a wheeled carriage, the reaction moves the carriage in the opposite direction from that in which the discharge takes place, and if a vessel *AF*, Fig. 251, be connected with a vertical axis *C*, it will

Fig. 250.

Fig. 251.

cause it to revolve in the direction opposite to that in which discharge takes place. If a constant supply of water be maintained, a continuous rotary motion results. This contrivance is the *reaction* wheel (Fr. *roue à réaction;* Ger. *Reactionsrad*), commonly called *Barker's* mill in Britain, and *Segner's* water wheel in Germany. The simplest form of this wheel is shown in Fig. 252. It consists of a pipe *BC*, firmly connected with a vertical axle *AX*, of two pipes *CF* and *CG* at right angles to the first, having orifices in the sides at *F* and *G*. The water discharged from these orifices is continually supplied by a trough leading into the top of the upright

Fig. 252.

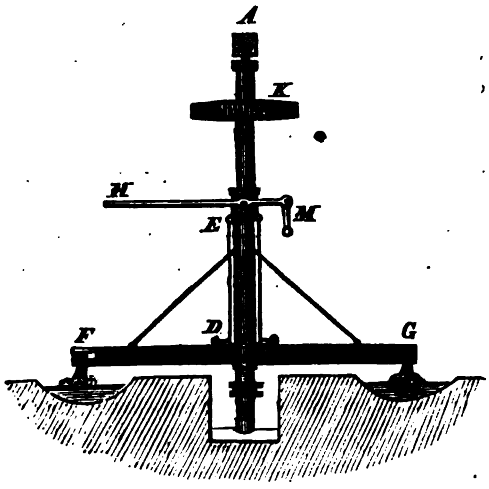
Fig. 253.

pipe. In applying this arrangement, the upper millstone is generally hung immediately on the axle *AX*; but for other applications the motion might be transmitted by any suitable gear.

Reaction wheels are also made with more discharge pipes or conduits than two, as shown in Fig. 253. The vessel HR is made either cylindrical or conical. In order to bring in the water at the top without shock, the great Euler adapted a cylindrical end to the pentrough, immediately above the wheel, putting a series of inclined guide-buckets into it, analogous to the arrangement introduced by Burdin for his turbines (Vol. II. § 133).

There is a simple reaction wheel erected by M. Althans, of Vallender, in the neighborhood of Ehrenbreitstein, for driving two pair of grindstones, which we have seen and admired. The arrangement of this machine is shown in the accompanying sketch, Fig. 254. The water is laid on by a pipe B descending beneath the wheel, and turning vertically upwards. The upright axle AC, with its two arms CF and CG, is hollow, and fits on to the end B of the supply pipe. There is a stuffing box at B allowing of the free rotary motion of the wheel, and at the same time preventing loss of water at the joint. The rectangular orifices F and G are opened or shut by means of vanes or slide valves moved by rods attached to a collar E on the axle, movable by means of the lever HM. The water supplied by the 9 inch pipe B flows through the arms of the wheel, and through the apertures F and G. This arrangement has the advantage of supporting the whole, or great part of the weight of the machine upon the water, so that there is little or no friction on the base. If G be the weight of the machine, h the head of water, $2r$ the diameter of the pipe at B, then $\pi r^2 h \gamma = G$, and, therefore, in order to support the machine, the radius of the pipe should be $r = \sqrt{\frac{G}{\pi h \gamma}}$. The quantity of water expended by this machine is 18 cubic feet per minute, the fall is 94 feet, and, therefore, the mechanical effect at disposition is 1755 feet lbs. per second. The length of the arms is $12\frac{1}{2}$ feet, and the number of revolutions 30 per minute, or the velocity at the periphery 39,3 feet per second.

Fig. 254.



Remark 1. The first account of a reaction wheel, as an invention of Barker, is given in Desagulier's "Course of Experimental Philosophy, vol. ii. London, 1745." Euler treats

in detail of the theory and best construction of these wheels in the "Memoirs of the Berlin Academy, 1750—1754."

Remark 2. The efficiency of reaction wheels is reputed as extremely small. Nordwall makes it only half of that of an overshot wheel, and Schitko's experiments on such a wheel gave the efficiency only 0,15.

§ 189. *Theory of the Reaction Wheel.*—The effects of reaction wheels may be determined theoretically as follows. If h be the fall, or the depth of the centre of the orifices below the surface in the feed-pipes, we have the height measuring the pressure of water on the orifices $h_1 = h + \frac{v^2}{2g}$, and, therefore, the theoretical velocity of discharge $c = \phi \sqrt{2gh + v^2}$. This is not, however, the absolute velocity of the water at efflux from the wheel, for it partakes of the velocity of rotation v in common with the wheel, in the opposite direction. Hence the absolute velocity of the water leaving the wheel: $w = c - v = \phi \sqrt{2gh + v^2} - v$, and the loss of mechanical effect corresponding:

$$L_\phi = \frac{w^2}{2g} Q \gamma = \frac{(\phi \sqrt{2gh + v^2} - v)^2}{2g} Q \gamma.$$

The co-efficient of velocity ϕ being assumed $= 1$, then

$$L_1 = \frac{(\sqrt{2gh + v^2} - v)^2}{2g} Q \gamma = \left(h - \frac{v(\sqrt{2gh + v^2} - v)}{g}\right) Q \gamma,$$

and deducting this from the effect at disposition, the useful effect remaining is:

$$L = \left(h - \frac{w^2}{2g}\right) Q \gamma = \frac{v(\sqrt{2gh + v^2} - v)}{g} Q \gamma.$$

This increases as v increases, for if we put:

$$\sqrt{v^2 + 2gh} = v + \frac{gh}{v} - \frac{g^2 h^2}{2 v^3} + \ldots \text{ we have:}$$

$$L = v\left(\frac{gh}{v} - \frac{g^2 h^2}{2 v^3} + \ldots\right) \cdot \frac{Q\gamma}{g},$$

and for $v = \infty$, $L = Q h \gamma$, the whole effect available.

This circumstance of the maximum effect depending on the wheels acquiring an infinite velocity, is very unfavorable; because, as the velocity increases, the prejudicial resistances increase, and even when the wheel runs without any useful resistance, the velocity it acquires is far from being infinite, proving the absorption of effect at these great velocities.

The question of course is, as to whether the effect for mean velocities of rotation be very much less than the maximum effect, or $Q h \gamma$. If we load the machine to such an extent that the height due to velocity, corresponding to the velocity of rotation, is equal to the fall, or $\frac{v^2}{2g} = h$, or $v = \sqrt{2gh}$, then, according to the above formula:

$$L = \frac{\sqrt{2gh}\,(\sqrt{4gh} - \sqrt{2gh})}{g} Q\gamma = 2(\sqrt{2} - 1) Q h \gamma = 0{,}828\, Q h \gamma,$$

but if $\frac{v^2}{2g} = 2h$, then:

$$L = \frac{\sqrt{4gh}\,(\sqrt{6gh} - \sqrt{4gh})}{g} Q\gamma = 4\,(\sqrt{1,5} - 1)\, Qh\gamma = 0,899\; Qh\gamma.$$

and, lastly, if $\frac{v^2}{2g} = 4h$, then:

$$L = \frac{\sqrt{8gh}\,(\sqrt{10gh} - \sqrt{8gh})}{g} Q\gamma = 8\,(\sqrt{1,25} - 1)\, Qh\gamma = 0,944\; Qh\gamma.$$

It thus appears that, in the first case, we lose 17, in the second 10, and in the third only 6 per cent. of the available effect, and, therefore, for moderate falls, and when a velocity of rotation exceeding the velocity due to the height of fall may be adopted, there is a great effect to be expected from these wheels. Considering the great simplicity of these wheels, the balance must often be much in their favor when compared with other wheels.

Remark. The *force* of rotation or of reaction is:

$$P = \frac{L}{v} = \frac{\sqrt{2gh + v^2} - v}{g} Q\gamma,$$

and for $v = 0$, $P = \frac{\sqrt{2gh}}{g} Q\gamma = \frac{c}{g} Q\gamma = 2 \cdot \frac{c^2}{2g} F\gamma$, as we showed, Vol. II. § 146, although by a different method.

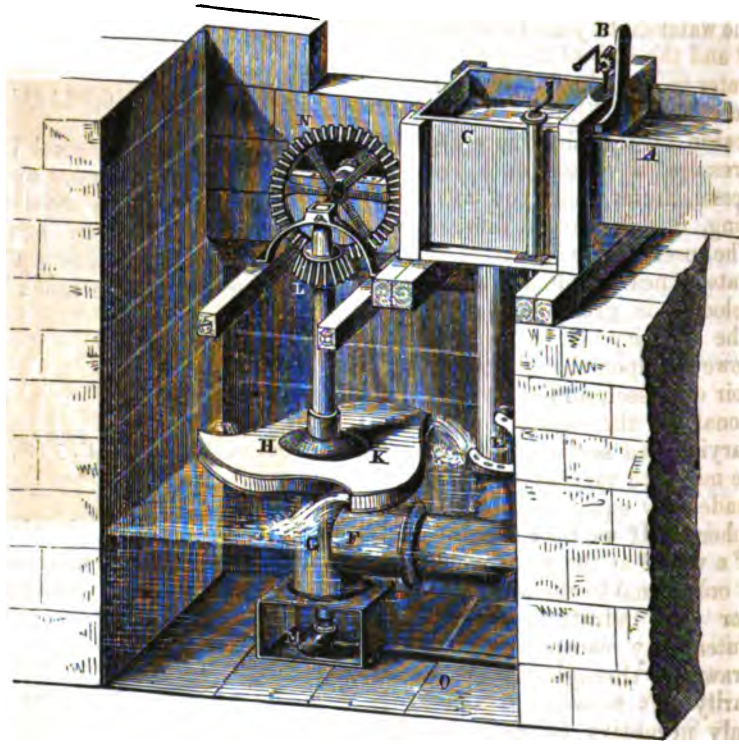
§ 140. *Whitelaw's Turbine.*—Within the last few years, the pipes or conduits of reaction wheels have been made curved, and such wheels are known as *Whitelaw's*, or *Scottish* turbines. Manouri d'Ectot constructed wheels on nearly the same plan as long ago as 1813, (*see* "Journal des Mines, t. xxxiii.") The Scottish turbines, constructed by Messrs. Whitelaw and Stirrat, of Paisley, are described in the "Description of Whitelaw and Stirrat's Patent Water Mill, 2d edition, London and Birmingham, 1843." One peculiarity of Whitelaw's wheel consists in the introduction of a movable piece at the *outer orifice* of the conduits by which its section is regulated. Fig. 255 is a horizontal section of one of Whitelaw's turbines. There are, in this case, three arms. The water enters at E, and is discharged at A. OA is a valve movable round O, by which the orifice of discharge is regulated. The adjustment of these regulators has been made self-acting by a peculiar arrangement, but is better regulated by the hand, by an apparatus analogous to that shown in Fig. 254.

Fig. 255.

The general arrangement of Whitelaw's turbines is clearly shown by Fig. 256. A is the lead for the water. B the sluice. C a reservoir immediately above the pressure pipe. E is a valve for regulating the expenditure of water. At F the water enters the

cylinder *G*, and goes from thence into the wheel *HK* placed above it, and fixed on the vertical axis *LM*. The reaction of the water

Fig. 256.



streaming from the three orifices, drives the wheel round in the opposite direction, and this motion is transmitted by the bevelled gear *LN*. The wheel, the axle, and the pressure pipe are of cast iron. The footstep for the pivot at *M* is of brass. Oil is introduced by a pipe *O* from the wheel room.

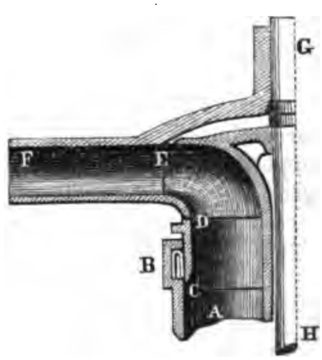
We shall hereafter recur to the theory and geometrical construction of this wheel.

§ 141. *Combe's Reaction Wheels.*—As being analogous to Whitelaw's, we may next consider Combe's reaction wheel. The water flows from below upwards into these wheels, and the wheel differs essentially from Whitelaw's in having so great a number of conduits or orifices of discharge, that it may be said to discharge at every point of the circumference, as the plan of the wheel in Fig. 257 shows. *AA* is a plate connected with the axis, and forming the upper shrouding or cover of the wheel. *BB* is the under shrouding, and upon it, between this and the upper plate, the buckets *EE* are fastened. *DD* is a cylinder surrounding the lower part of the axis,

through which the water is laid on to the wheel, entering the wheel by all the apertures between the buckets on the inside, and streaming through the conduits formed by the buckets to be discharged at every point of the outer circumference. Another essential difference between this and Whitelaw's wheel arises from the absence of the water-tight joint between the wheel *B* and the end of the pipe leading the water to the wheel, and which is quite necessary to Whitelaw's wheels. The reason of this difference is, that the pressure of water in a reservoir or vessel, from which the water is running, is different at different points. The pressure is greatest where the water is nearly still, and least where the velocity is greatest (Vol. I. § 307). The velocity of the water depends, however, upon the section of the reservoir or vessel, and is inversely proportional to the section, and thus, by varying the section, the pressure may be made to vary at will, or it may be made only equal to that of the atmosphere. If we bore a hole in the side of a vessel at the point where the pressure of the water flowing past is only equal to the atmospheric pressure, there will be no discharge, nor any indraft. Thus, that no water may escape, and no air be drawn in through the space necessarily left between *B* and *D*, it is only necessary to give the section certain dimensions at the point of passage.

Fig. 257.

Fig. 258.



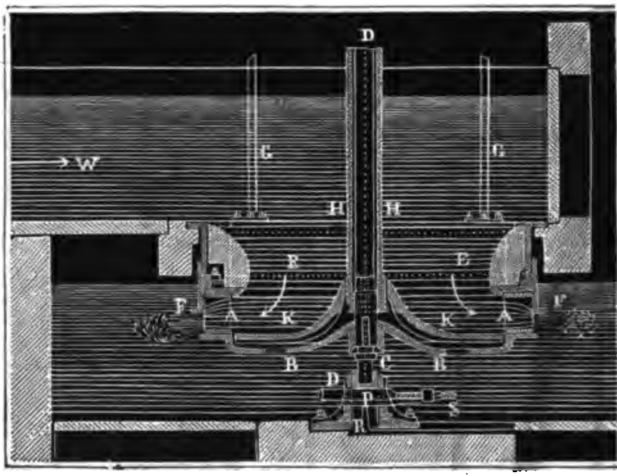
Remark. Combe's wheels are sometimes provided with guide-buckets for laying the water on to the wheel in a definite direction.

Redtenbacher forms the water-tight joint between the main pipe *AB* and the wheel *DEF*, by means of a movable brass ring *CD*, which is pressed so tightly up against the lower ring-surface *D* of the wheel, that the water does not escape. The ring *CD* must slide in a well-constructed, water-tight collar.

§ 142. *Cadiat's Turbine.*—The next wheel we shall describe is Cadiat's turbine. These have no guide-curves, like Whitelaw's and Combe's, but as in Fourneyron's turbine, the water is brought in from above. The peculiarity of this wheel is the introduction of a cylindrical sluice, which shuts the wheel *on the outside.* Fig. 259 is a vertical section of this wheel. *AA* is the wheel, *BB* being a saucer-formed plate connecting the wheel with the axle *CD*. The

pivot C of this axle rests in a footstep, which we shall hereafter have occasion to allude to more particularly. EE is the reservoir with circular section communicating with the lead W, and coming

Fig. 259.



in immediate contact with the upper plate or shrouding of the wheel. That the water coming into the wheel may not be unnecessarily disturbed or contracted, the reservoir gradually widens both upwards and downwards, as the figure shows. The discharge of the water is regulated by the cylindrical sluice FF on the outside. This sluice is raised or lowered by 3 or 4 rods, connected with mechanism for the purpose. That there may be no escape of water between the sluice and the side of the reservoir, the joint is made with leather.

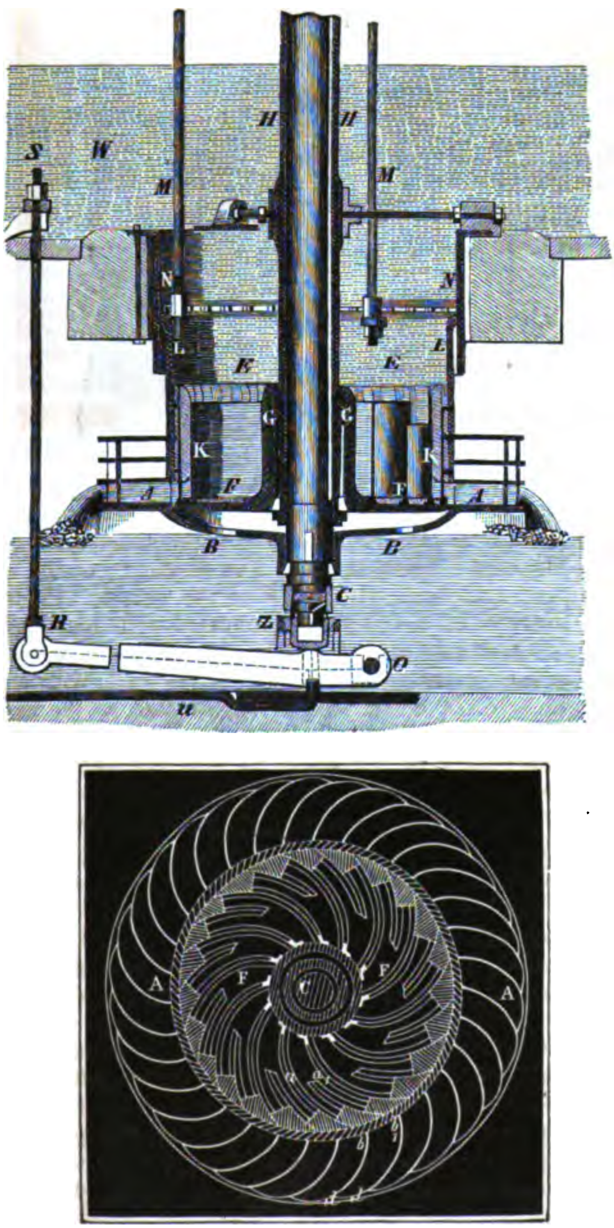
The upright axle CD is enclosed in a pipe HH, to the bottom of which is attached a plate KK, reaching to the inner circumference of the lower shrouding of the wheel, so that the water is shut off from the disc or plate BB of the wheel. This arrangement is adopted from that first introduced by Fourneyron in his turbine.

Remark. A complete and accurate description of one of Cadiat's turbines, as originally constructed, is given by Armengaud, sen., in the second volume of his "Publication Industrielle."

§ 143. *Fourneyron's Turbine.*—Fourneyron's turbines, as they have been recently made, may be considered as among the most perfect horizontal wheels. They work either in or out of back-water, are applicable to high and to low falls—are either high pressure and or low pressure turbines. In the low pressure turbine, the water flows into the reservoir, open above, as shown in Fig. 260. In high pressure turbines, the reservoir is shut in at top, and the water is laid on by a pipe at one side, as represented in Fig. 261. The wheel consists essentially of two shroudings, between which are the buckets of the connecting plate or arms, and the upright axle, as in

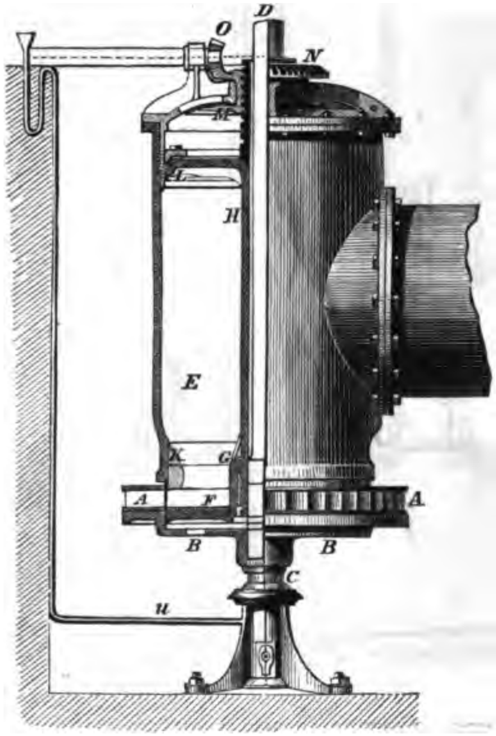
Cadiat's turbine. The water from the lead *W* flows into the reservoir *EE*. In order that the water may not rest directly on the wheel disc *BB*, which would greatly increase the pivot friction, a pipe encloses the upright axle, to which there is attached a disc *FF*,

Fig. 260.



upon which the mass of water presses as it flows to the wheel. On this disc the so-called *guide-curves*, ab, a_1b_1, &c. (Fig. 260 or 262), are fastened. These give the water a certain direction of motion on to the wheel, which surrounds them, and through it along the buckets, to be discharged at the outer circumference. The reaction is such

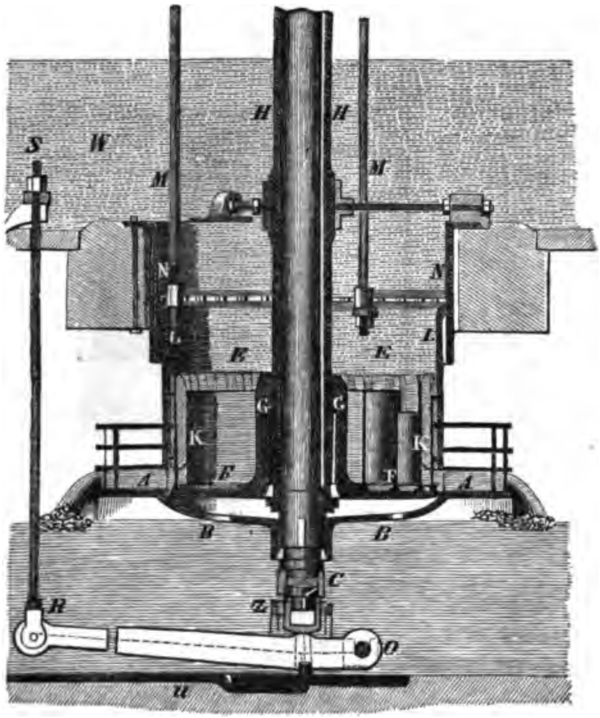
Fig. 261.

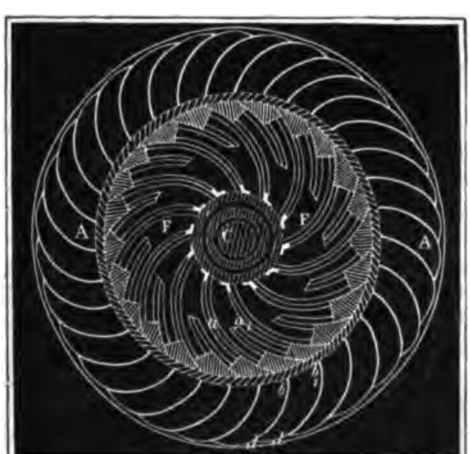


that the wheel revolves in the opposite direction, the guide-curves and their support remaining at rest. To regulate the discharge of water, there is a cylindrical sluice *KLLK*, Fig. 262, in the inside, which is raised and depressed by three rods *M*, *M* . . . , connected with mechanism suited to the object. The sluice *KL* is a hollow cast iron cylinder, the outer surface of which nearly touches the inside of the upper shrouding of the wheel, and they must, therefore, be both accurately turned. The sluice is made to slide watertight in the reservoir by a leather or other fitting above *LL*. The sluice is generally lined with wood, rounded at top and bottom, so as to prevent contraction as much as possible, that is, to prevent all losses of *vis viva*. In high pressure turbines, the rods for working the sluice pass through the cover of the reservoir through stuffing boxes. Sometimes the regulation is effected by raising or de-

pressing the bottom plate Fig. 263. For this purpose, the top of the pipe *GH* is screwed, and the female screw *M* is attached to a conical wheel, moved by gear, as *O*. The female screw is fixed so

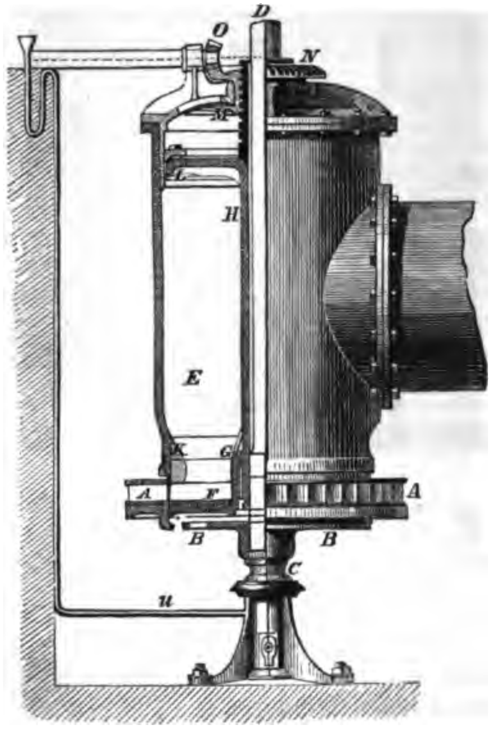
Fig. 262.





that its motion raises or lowers the pipe *GH*. There is also attached to *GH* a plate or piston *HL*, having a water-tight packing.

Fig. 263.



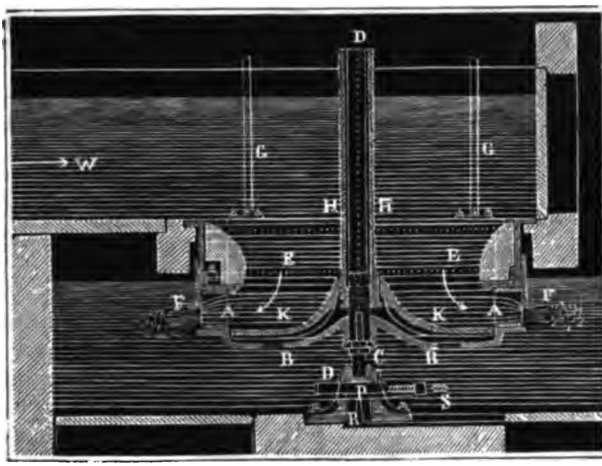
§ 144. *The Pivot and Footstep.*—The pivot and footstep are very important parts of the turbine. The weight of the turbine, often considerable, and the velocity of rotation, give rise to a great moment of friction on the pivot, which would wear very rapidly, unless it were well greased. It has been frequently observed that the pivot and brass of turbine axles wear much more rapidly than the pivots of other upright axles. This is attributable partly to the heating of the pivot from great velocity of rotation, but chiefly to the difficulty of lubricating the bearing-points which are under water. In order to meet this evil, turbine makers have endeavored to diminish the weight as much as possible, to increase the rubbing surface, to prevent the contact of the water with the rubbing surfaces, and also, to keep up a continuous supply of olive oil or nut oil, between the surfaces in contact.

At the upper end of the axle, there must of course be a collar or other support, in which it can revolve.

A very simple footstep, applicable chiefly when the weight is inconsiderable, is shown in Fig. 264. The pivot *C* rests in a *brass*

D, which is supported on a block, movable up or down, as may be required, by means of the *folding key PS*. The oil is supplied by

Fig. 264.



a pipe *R* passing by the side of the key, and through the bottom of the block and brass.

Fig. 265 is the arrangement of footstep adopted by Cadiat. *A* the foot of the upright shaft. *B* a hardened steel pivot attached to *A* by screw or welding. *C* is a hardened steel *step* for the pivot. *DEED* is a cast iron block or case for the step. *EE* being a brass casing. *F* is a pipe taking oil to the space between *B* and *E*. *G* is a lever for raising or depressing the turbine.

Fig. 265.

Fourneyron has very much complicated the construction of the pivot and footstep, to attain permanence. The general arrangement is shown in Fig. 262, and its details are shown in Fig. 266. Fig. 262 shows that the footstep *Z* is in a block which rests on a lever *OR*, turning round *O*, when elevated or depressed by the rod *RS*. *U* is a pipe for bringing oil to the footstep, the head of which should be as high as possible. That the circulation of the oil may be active, it should, at all events, be considerably above the surface of the water in the lead in low pressure turbines, and there should be a means of *forcing* in a supply for high pressure turbines. The parts *A* and *B* immediately in contact with each other are of hard steel. The upper part *A* is fixed in the shaft *G*, and the lower part *B* fits into a hollow piece *DD*, and is movable upwards and downwards by a lever supporting the whole, and passing through *O* (*OR*, Fig. 262). The surface of *A* is hollowed,

Fig. 266.

and the head of B rounded, and both are surrounded by a collar EE, which keeps the oil between the rubbing surfaces. The oil, brought down in a pipe, enters at a into the hollow space b, and from thence, through c, passes into the space d. Out of this it flows through three channels $e\,f$, on the periphery of the steel bearing, rising perpendicularly from the bottom and running inclined to the top, where three radial furrows serve to distribute it sufficiently. Lastly, there goes from the centre of A, a hole gh into the axis, through which the oil escapes outwards, so that a circulation is maintained.

§ 145. *Strength or Dimensions.*—In designing a turbine for a certain fall of water, there is, besides the chief dimensions of the wheel itself, the strength of certain parts to be calculated. The strength of pipes, &c., is to be calculated by the formulas given in Vol. I. § 283; and at Vol. II. § 84, we have treated of the dimensions necessary for shafts. If L be the useful effect of a turbine in horse's power, and u the number of revolutions per minute, we have for the requisite diameter of the upright shaft

$$d = 6{,}12 \sqrt[3]{\frac{L}{u}} \text{ inches.}$$

The strength of the bottom plate, &c., is easily determined by reference to the theory of the strength of materials; but the nature of the casting fixes the dimensions, so that there can very rarely occur any necessity for calculation.

Remark 1. In the erection of turbines, not only the weight of the parts of the machine, but the water pressure, has to be considered. The latter has especially to be considered in high pressure turbines. It must not, for example, be lost sight of, that the water presses against the reservoir with a force equal to the weight of a column of water having the section of the pressure pipes as base, and the *head of water* as height; and that the knee piece on the pressure pipes tends to move with the same force, but in the opposite direction.

Remark 2. For determining the dimensions of the pivot, the rule which limits the pressure on every square inch of the brass to 1500 lbs., and that for a steel pivot on a steel bearing to 7000 lbs., might be used; but we know from what has been said above in reference to the wear of these points, that it is preferable to make the pivot only very little less than the diameter of the shaft. The above numbers refer to shafts having a moderate velocity of rotation, and as the wear increases as the weight, and as the velocity of rotation besides, it is evident that turbines, revolving with great speed, should have proportionally large pivots and bearings.

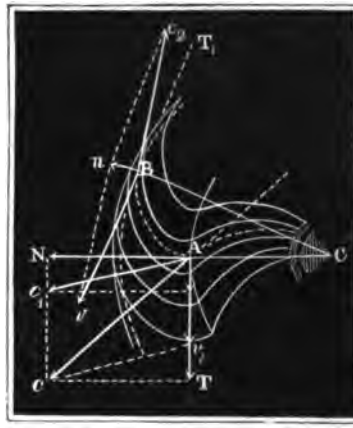
§ 146. *Theory of Reaction Turbines.*—For the investigation of the mechanical proportions and effects of Fourneyron's turbines, we shall use the following notation:

$r_1 = CA$, Fig. 267, the radius of the inside of the wheel, or approximately, that of the periphery of the bottom plate.

$r = CB$, the radius of the outside of the wheel.

$v_1 =$ the velocity of the interior periphery.

Fig. 267.



v = the velocity of the exterior periphery.

c = the velocity with which the water flows from the reservoir or guide-curves.

c_1 = the velocity with which the water enters the wheel.

c_2 = the velocity with which the water leaves the wheel.

α = the angle cAT which the direction of the water leaving the reservoir makes with the inner circumference of the wheel.

β = the angle c_1AT made by the water entering the wheel buckets with the inner periphery of the wheel.

δ = the angle c_2AT made by the water stream leaving the wheel with the outer periphery.

F = the area of the orifices of discharge from the guide-curves.

F_1 = the sum of the areas of the orifices by which the water enters the wheel.

F_2 = the sum of the areas of the orifices at the outside of the buckets.

h = the entire fall of water.

h_1 = the height from surface of lead to centre of wheel, or centre of orifice of discharge from reservoir.

$h_2 = h_1 - h$ the depth of the entrance orifices to the wheel, below the orifices of discharge, or, if the wheel works under water, below the surface of the tail-race; and, lastly:

x = the height, measuring the pressure of the water at the point where it passes from the reservoir or guide-curves into the wheel.

In the first place, for the velocity c due to the difference of pressures $h_1 - x$, we have $\frac{c^2}{2g} = h_1 - x$, or, more accurately, if the water, in flowing from the guide-curves, loses an amount of hydraulic pressure $= \zeta \cdot \frac{c^2}{2g}$, then $(1 + \zeta)\frac{c^2}{2g} = h_1 - x$, therefore,

$$c = \sqrt{\frac{2g\,(h_1 - x)}{1 + \zeta}}, \text{ and, inversely, } x = h_1 - (1 + \zeta)\frac{c^2}{2g}.$$

In order that the water may enter the wheel without shock, it is necessary that the velocity of discharge should resolve itself into two components, the one of which must coincide in magnitude and direction with the velocity of the inner circumference of the wheel, and the other must coincide in direction with the stream entering the wheel conduits or channels. This being taken for granted: if the velocity with which the water begins to flow through the channels $Ac_1 = c_1$, we have it from the formula:

$$c_1^2 = c^2 + v_1^2 - 2\,c\,v_1 \cos.\,\alpha.$$

The velocity of discharge c_2 may be deduced from the pressure height x and h_2 at entrance and exit, from the height $\frac{c_1^2}{2g}$, being that corresponding to the velocity of entrance, and from the increase of pressure height corresponding to centrifugal force of the water in the wheel $\frac{v^2 - v_1^2}{2g}$ (Vol. II. § 143):

$\frac{c_2^2}{2g} = x - h_2 + \frac{c_1^2}{2g} + \frac{v^2 - v_1^2}{2g}$, or, substituting the above values of x and c_1:

$$\frac{c_2^2}{2g} = h_1 - h_2 - (1 + \zeta)\frac{c^2}{2g} + \frac{c^2}{2g} + \frac{v^2}{2g} - \frac{2\,c\,v_1\,cos.\,\alpha}{2g};$$

or as $h_1 - h_2 = h$, the whole fall:

$$c_2^2 = 2gh + v^2 - 2\,c\,v_1\,cos.\,\alpha - \zeta\,.\,c_2.$$

If we further assume that, by friction and curvilinear motion in the wheel channels, there is a loss of hydraulic head $= \frac{\varkappa\,c_2^2}{2g}$, then, more accurately: $(1 + \varkappa)\,c_2^2 = 2gh + v^2 - 2\,c\,v_1\,cos.\,\alpha - \zeta\,.\,c^2$.

The quantity of water $Q = Fc = F_1c_1 = F_2c_2 \therefore c = \frac{F_2c_2}{F}$, and $v_1 = \frac{r_1}{r}v$, and, therefore, we have for the velocity of the water leaving the wheel:

$$\left[1 + \zeta\left(\frac{F_2}{F}\right)^2 + \varkappa\right]c_2^2 + 2\frac{F_2}{F}\,.\,\frac{r_1}{r}\,c_2\,v\,cos.\,\alpha - v^2 = 2gh.$$

§ 147. *Best Velocity.*—In order to get the maximum effect from the water, the absolute velocity of the water leaving the wheel must be the least possible. But this velocity is the diagonal Bw of a parallelogram constructed from the velocity of discharge c_2 and the velocity of rotation v:

$$w = \sqrt{c_2^2 + v^2 - 2\,c_2\,v\,cos.\,\delta} = \sqrt{(c_2 - v)^2 + 4\,c_2\,v\left(sin.\,\frac{\delta}{2}\right)^2};$$

and δ is to be made as small as possible, and $c_2 = v$. But in order that there may be free passage for the necessary quantity of water, it is not possible to make $\delta = 0$, but this has to be made 10° to 20°; whenever, therefore, we make $c_2 = v$, there remains the absolute velocity:

$$w = \sqrt{4\,c_2\,v\left(sin.\,\frac{\delta}{2}\right)^2} = 2\,v\,sin.\,\frac{\delta}{2},$$

and the loss of effect corresponding is:

$$\frac{w^2}{2g}\,Q\gamma = \frac{\left(2\,v\,sin.\,\frac{\delta}{2}\right)^2}{2g}\,Q\gamma.$$

We now perceive that the maximum effect is not got when $v = c_2$, but when v is something less than c_2; but it is also manifest that for $v = c_2$, and for a small value of δ, the deficiency below the

maximum effect can only be very small. As, besides, in assuming $v = c_2$ we get very simple relations, we shall do so in the sequel, and introduce this into the last equation of the preceding paragraph. We then have:

$\left[1 + \zeta\left(\frac{F_2}{F}\right)^2 + x\right] v^2 + 2\frac{F^2}{F} \cdot \frac{r_1}{r} v^2 \cos. \alpha - v^2 = 2gh$, or,

$\left[2\frac{F_2}{F} \cdot \frac{r_1}{r} \cos. \alpha + \zeta\left(\frac{F^2}{F}\right)^2 + x\right] v^2 = 2gh$; and, therefore, the velocity of the wheel corresponding to the maximum effect required, is:

$$v = \sqrt{\frac{2gh}{2\frac{F_2}{F} \cdot \frac{r_1}{r} \cos. \alpha + \zeta\left(\frac{F_2}{F}\right)^2 + x}}.$$

Instead of the section ratio $\frac{F_2}{F}$, we may introduce the angle β. The unimpeded entrance of the water into the wheel requires that c should not be altered in entering into it, or that the *radial component* of c, $AN = c \sin. \alpha$, should be equal to the radial component of c_1, i. e. $c_1 \sin. \beta$, and also to the tangential component $c \cos. \alpha$ of c the tangential velocity $AT = c_1 \cos. \beta + v_1$ of the water already within the wheel. According to this:

$\frac{c_1}{c} = \frac{\sin. \alpha}{\sin. \beta}$, $c \cos. \alpha - c_1 \cos. \beta = v_1$, and $\frac{c}{v_1} = \frac{\sin. \beta}{\sin. (\beta - \alpha)}$.

Besides this, as $Fc = F_2 c_2 = F_2 v = \frac{r}{r_1} F_2 v_1$; we have

$$\frac{F_2}{F} = \frac{r_1}{r} \cdot \frac{c}{v_1} = \frac{r_1}{r} \cdot \frac{\sin. \beta}{\sin. (\beta - \alpha)},$$

and the velocity of the outer periphery of the wheel:

$$v = \sqrt{\frac{2gh}{2\left(\frac{r_1}{r}\right)^2 \frac{\sin. \beta \cos. \alpha}{\sin. (\beta - \alpha)} + \zeta\left(\frac{r_1 \sin. \beta}{r \sin. (\beta - \alpha)}\right)^2 + x}}.$$

and hence the velocity of the inner periphery:

$$v_1 = \frac{r_1}{r} v = \sqrt{\frac{2gh}{2\frac{\sin. \beta \cos. \alpha}{\sin. (\beta - \alpha)} + \zeta\left(\frac{\sin. \beta}{r \sin. (\beta - \alpha)}\right)^2 + x\left(\frac{r}{r_1}\right)^2}}.$$

Neglecting the prejudicial resistances, we should have:

$$v_1 = \sqrt{\frac{gh \sin. (\beta - \alpha)}{\sin. \beta \cos. \alpha}} = \sqrt{gh\,(1 - tang. \alpha \, cotang. \beta)}.$$

§ 148. *Pressure of the Water.*—With the aid of this formula for v, we can determine the pressure exerted on that part of the reservoir where the water passes from it on to the wheel; we have:

$$x = h_1 - (1 + \zeta)\frac{c^2}{2g} = h_1 - (1 + \zeta)\frac{v_1^2}{2g}\left(\frac{\sin. \beta}{\sin. (\beta - \alpha)}\right)^2$$

$$= h_1 - \frac{(1+\zeta)\, h \sin.\beta^2}{2 \sin.\beta \cos.\alpha \sin.(\beta-\alpha) + \zeta \sin.\beta^2 + \varkappa \left(\frac{r}{r_1}\right)^2 [\sin.(\beta-\alpha)]^2}$$

$$= h_1 - \frac{(1+\zeta)\, h}{1 + \cos. 2\alpha - cotg.\beta \sin. 2\alpha + \zeta + \varkappa \left(\frac{r}{r_1}\right)^2 \left(\frac{\sin.(\beta-\alpha)}{\sin.\beta}\right)^2}.$$

Neglecting prejudicial resistances, we have:

$$x = h_1 - \frac{h}{1 + \cos. 2\alpha - cotg.\beta \sin. 2\alpha}.$$

If the turbine work free of back water, we have, in the case of the turbines of Fourneyron, Cadiat, and Whitelaw, $h_1 = h$, and, therefore,

$$x = \frac{\cos. 2\alpha - cotg.\beta \sin. 2\alpha}{1 + \cos. 2\alpha - cotg.\beta \sin. 2\alpha} \cdot h;$$

if, however, the turbine works in back water, then $h_1 = h + h_2$, and, therefore,

$$x = \frac{\cos. 2\alpha - cotg.\beta \sin. 2\alpha}{1 + \cos. 2\alpha + cotg.\beta \sin. 2\alpha} \cdot h + h_2.$$

If, in the first case, the pressure is to be $= 0$, *i. e.*, equal to the atmospheric pressure, then $x = 0$; but if in the second case, it must be equal to the pressure of the back water against the orifices of the wheel, then $x = h_2$, but in both cases we should have: $\cos. 2\alpha - cotg.\beta \sin. 2\alpha = 0$, i. e., $tang.\beta = tang. 2\alpha$, or $\beta = 2\alpha$.

If, therefore, the angle of the water's entrance β be twicet he angle of exit α, the pressure at the point where the water passes from the reservoir to the wheel, is equal to the external pressure of the atmosphere, or of the back water.

On the other hand, it is easy to perceive that this internal pressure is greater than the external pressure, if $\beta > 2\alpha$, and it is less than this when $\beta < 2\alpha$. These relations are somewhat different when the prejudicial resistances are taken into account, as it is very proper to do. The equation between the external and internal pressure then stands thus:

$$1 + \cos. 2\alpha - cotg.\beta \sin. 2\alpha + \zeta + \varkappa \left(\frac{r}{r_1}\right)^2 \left(\frac{\sin.\beta - \alpha)}{\sin.\beta}\right)^2 = 1 + \zeta,$$

or $cotg.\beta \sin. 2\alpha = \cos. 2\alpha + \varkappa \left(\frac{r}{r_1}\right)^2 (\cos.\alpha - cotg.\beta \sin.\alpha)^2$.

If, in the last member, we introduce:

$$cotg.\beta = cotg. 2\alpha = \frac{\cos. 2\alpha}{\sin. 2\alpha},$$

that is:

$$cotg.\beta \sin. 2\alpha = \cos. 2\alpha + \varkappa \left(\frac{r}{r_1}\right)^2 \left(\frac{\sin.\alpha}{\sin. 2\alpha}\right)^2$$

$$= \cos. 2\alpha + \varkappa \left(\frac{r}{r_1}\right)^2 \cdot \frac{1}{4(\cos.\alpha)^2},$$

it follows that:

$$tang.\,\beta = \frac{sin.\,2\,\alpha}{cos.\,2\,\alpha + x\left(\frac{r}{r_1}\right)^2 \cdot \frac{1}{4\,(cos.\,\alpha)^2}},$$

consequently, β is somewhat smaller than $2\,\alpha$.

If we neglect again ζ and x, we have, by introducing the value $\beta = 2\,\alpha$.

$$v_1 = \sqrt{gh\,(1 - tang.\,\alpha\,cotg.\,2\,\alpha)} = \sqrt{\frac{gh\,(1 + tang.\,\alpha^2)}{2}} = \frac{\sqrt{\frac{1}{2}gh}}{cos.\,\alpha},$$

and $c = \sqrt{2gh}$. If the internal pressure be greater than the external, then $v_1 > \frac{\sqrt{\frac{1}{2}gh}}{cos.\,\alpha}$, and $c < \sqrt{2gh}$, and if it be less than this, then $v_1 < \frac{\sqrt{\frac{1}{2}gh}}{cos.\,\alpha}$, and $c > \sqrt{2gh}$.

§ 149. The discussion as to pressures in the last paragraph is of great importance in the question of the construction of turbines; for the point of transit from the reservoir to the wheel cannot be made water-tight, and, therefore, water may escape, or water and air get in, by the annular aperture. That neither of these circumstances may occur, turbines must be so constructed, that the internal pressure of the water passing the slit may equal the external pressure of the atmosphere, or of the back water, if the wheel be submerged; we must, in short, have $\beta = 2\,\alpha$, or, better still, satisfy the equation:

$$tang.\,\beta = \frac{sin.\,2\,\alpha}{cos.\,2\,\alpha + x\left(\frac{r}{r_1}\right)^2 \cdot \frac{1}{(2\,cos.\,\alpha)^2}}.$$

Turbines are constructed so that, in the normal state of the sluice being fully opened, the above equation is satisfied, or, so that a slight excess of pressure x exists, at the risk of losing some water through the space left between the bottom plate and the wheel, thus providing for variations of velocity of discharge, from variations in the area of the section F of the orifices by the different positions of the turbine-sluice, regulating the quantity discharged.

§ 150. *Choice of the Angles α and β.*—If we do not take the internal pressure into consideration, the angles α and β may have very arbitrary values. The formula

$$v_1 = \sqrt{gh\,(1 - tang.\,\alpha\,cotg.\,\beta)} = \sqrt{gh\left(1 - \frac{tang.\,\alpha}{tang.\,\beta}\right)}$$

gives an impossible value for v_1, when $\frac{tang.\,\alpha}{tang.\,\beta} > 1$, that is, when $\alpha < 90°$, and $\beta < \alpha$, or when $\alpha > 90°$, and $\beta > \alpha$. These values of α and β are, therefore, not to be admitted. If $\alpha = \beta$, then $v_1 = 0$, and hence we see that the best velocity of rotation becomes so much the less, the nearer the angles α and β approach equality. The formulas $c = \frac{v_1\,sin.\,\beta}{sin.\,(\beta - \alpha)}$, and $F_2 = \frac{r_1}{r} \cdot \frac{sin.\,\beta}{sin.\,\beta - \alpha)}\,F$, given negative

values, involving impossible conditions, when $\beta > \alpha$. It is, therefore, necessary, in the construction of turbines, that $\beta > \alpha$, and that $\alpha < 90°$.

Between these limits, we may choose various values for α and β, although they do not all lead to an equally convenient or advantageous construction. Fourneyron makes $\beta = 90°$, and $\alpha = 30°$ to 33°, some constructors make β less, and others greater than 90°. When β is made 90°, or less, the curvature of the buckets is greater than when β is made more obtuse. Great curvature involves greater resistance in the efflux, and hence it is advisable to make β rather obtuse than acute, that is, to make $\beta = 100°$ to 110°. The angle α must then be 50° to 55°, if the internal pressure is to balance the external. In order, however, that the channels formed by the guide-curves may not diverge too much, and that the equilibrium of pressure may not be disturbed by depressions of the sluice, this angle is made from 30° to 40°, and if the turbine revolves free of back water, then α should not be more than 25° to 30°; α is, however, never made very acute, because with the angle α, the area of the orifices of discharge varies, and, therefore, the quantity of water discharged would diminish, or the diameter of the wheel must be greater for a given quantity of water expended. On the other hand, we have to bear in mind, that the losses of effect increase as v^2, and that, therefore, *cæteris paribus*, a turbine will yield a greater effect —will be more efficient—when revolving slowly, than when it has a great velocity of rotation. According to this, the construction should be so arranged that α and β do not differ widely from one another, from which would follow an internal pressure less than the external. If a be the height of a column of water balancing the atmospheric pressure, the absolute pressure of the water as it passes over the space between the wheel and bottom plate is $a + x$, and if this pressure $= 0$, then the water flows with a maximum velocity $c = \sqrt{2g(h_1 - x)} = \sqrt{2g(h_1 + a)}$ from the reservoir. If $a + x$ were negative, or $x < -a$, there would arise a vacuum at the point of passage of the water into the wheel, for the water would flow even faster through the wheel than it flowed on to it from the reservoir, and so air would rush in from the exterior, greatly disturbing the flow of the water. If, therefore, we introduce into the formula, instead of

$$x = h - \frac{h}{1 + \cos. 2\alpha - \operatorname{cotg}. \beta \sin. 2\alpha}.$$

$x = -a$, we then have:

$$1 + \cos. 2\alpha - \operatorname{cotg}. \beta \sin. 2\alpha = \frac{h}{h + a}, \text{ hence:}$$

$$\operatorname{tang}. \beta = \frac{\sin. 2\alpha}{1 + \cos. 2\alpha - \frac{h}{h + a}} = \frac{(h + a) \sin. 2\alpha}{(h + a) \cos. 2\alpha + a},$$

and, therefore, the corresponding best velocity of rotation:

$$v_1 = \sqrt{gh\left(1 - tang.\,\alpha\,.\,\frac{(h+a)\,cos.\,2\,\alpha + a}{(h+a)\,sin.\,2\,\alpha}\right)} = \frac{h}{cos.\,\alpha}\sqrt{\frac{g}{2\,(h+a)}}.$$

§ 151. *Turbines without Guide-Curves.* — For turbines without guide-curves we may make $\alpha = 90°$, because the water flows by the shortest way, or radially, out of the reservoir. In this light we have to consider the turbines of Combe, Cadiat, and Whitelaw. If we introduce into the formula for the best velocity $\alpha = 90°$, we get:

$$v_1 = \sqrt{\frac{2gh}{\frac{2\,sin.\,\beta\,cos.\,90°}{cos.\,\beta} + \zeta\left(\frac{sin.\,\beta}{cos.\,\beta}\right)^2 + \varkappa\left(\frac{r}{r_1}\right)^2}}.$$

$$= \sqrt{\frac{2gh}{\zeta\,(tang.\,\beta)^2 + \varkappa\left(\frac{r}{r_1}\right)^2}};$$ neglecting prejudicial resistances,

however, $v_1 = \sqrt{\frac{2gh}{0}} = \infty$. But, for two reasons, the wheel cannot acquire an infinite velocity. There is a limit, in the first place, when the mechanical effect at disposition is absorbed in overcoming prejudicial resistances, that is, when

$$Q\,h\,\gamma = \left(\frac{w^2}{2g} + \zeta\,\frac{c^2}{2g} + \varkappa\,.\,\frac{c_2^{\,2}}{2g}\right)Q\,\gamma, \text{ i. e.,}$$

$$h = \left[\left(2\,sin.\,\frac{\delta}{2}\right)^2 + \zeta\left(\frac{r_1}{r}\,tang.\,\beta\right)^2 + \varkappa\,\right]\frac{v^2}{2g}, \text{ or,}$$

$$v = \sqrt{\frac{2gh}{\left(2\,sin.\,\frac{\delta}{2}\right)^2 + \zeta\left(\frac{r_1}{r}\,tang.\,\beta\right)^2 + \varkappa}},$$

and in the second place when

$$x = -a, \text{ i. e. } h - \frac{c^2}{2g} = -a, \text{ or } \frac{c^2}{2g} = a + h, \text{ or,}$$

$$\frac{1}{2g}\left(\frac{r_1}{r}\,.\,\frac{v\,sin.\,\beta}{sin.\,(\beta - 90°)}\right)^2 = a + h, \text{ that is, when}$$

$v = \frac{r}{r_1}\,cotg.\,\beta\,\sqrt{2g\,(a+h)}$, because then the full discharge by *full flow* ceases, and the circumstances are quite changed, seeing that the water cannot flow from the reservoir in quantities sufficient to supply the discharge of the wheel channels, when their section is filled.

If we introduce into the above formula:

$$v_1 = \sqrt{\frac{2gh}{\zeta\,(tang.\,\beta)^2 + \varkappa\left(\frac{r}{r_1}\right)^2}},$$

the experimental co-efficients ζ and $\varkappa$, it is still far from giving us $v = \infty$. Now for the most accurate construction of the guide-curve apparatus, the co-efficient of velocity ϕ is not greater than

0,93, and, therefore, the co-efficient of resistance ζ corresponding $= \frac{1}{\phi^2} - 1 =$ not less than $\frac{1}{0,93^2} - 1 = 0,16$, or about 16 per cent. In the case of turbines without this apparatus, this resistance does not exist; but still there remains always a certain loss for the entrance into the wheel channels, which, in Combe's and Cadiat's wheel, does not probably exceed 5 per cent. though in Whitelaw's it may be taken as 10 per cent. at least, for the channels are too wide to admit of the supposition that the whole stream of water has the definite direction (β). The co-efficient $\varkappa$ corresponding to the resistance from the curvature and friction in the channels may be set at from 0,5 to 0,15, as we shall see in the sequel, and hence, for turbines without guide-curves, we have, putting $\varkappa = 0,1$, the best velocity

$$v_1 = \sqrt{\frac{2gh}{0,05\,(tang.\,\beta)^2 + 0,1\left(\frac{r}{r_1}\right)^2}},$$

and for Whitelaw's reaction wheel:

$$v_2 = \sqrt{\frac{2gh}{0,1\,(tang.\,\beta)^2 + 0,1\left(\frac{r}{r_1}\right)^2}}.$$

If, again, we put $\beta = 60°$, and $\frac{r}{r_1} = \frac{4}{3}$, we have, in the first case:

$$v_1 = \sqrt{\frac{2gh}{0,148 + 0,178}} = 1,75\sqrt{2gh},$$

and in the second case:

$$v_1 = \sqrt{\frac{2gh}{0,296 + 0,178}} = 1,45\sqrt{2gh}.$$

For other reasons, we shall hereafter learn that the most advantageous velocity of rotation is not even equal to the velocity due to the height h.

In order that the water may enter the wheel with the least shock possible, in the case of wheels without guide-curves, the equation $\frac{F_2}{F} = \frac{r_1}{r}\,tang.\,\beta$ must be satisfied. But as F_2 is determined by the relative position of the sluice, it follows that the maximum effect is got for a certain position of the sluice.

§ 152. *Influence of the Position of the Sluice.*—In one point of view, turbines are inferior to overshot and breast wheels. When in these wheels there is only a small quantity of water available, or it is only required to produce a portion of the power of the *fall*, and for this purpose we partially close the sluice, we know that the efficiency of such wheels is rather increased than diminished from the cells being proportionally less filled. In the turbine the contrary is the case, for as the sluice is lowered, the water enters the wheel under circumstances involving greater loss of effect. This is

a circumstance so much the more unfavorable, inasmuch as it is generally requisite to economize power the more, as the water supply fails. The loss, however, by lowering the sluice, is never very great, as the following investigation proves.

If we decompose the velocities c and c_1 into their radial and tangential components $c\ sin.\ \alpha$, $c\ cos.\ \alpha$, $c_1\ sin.\ \beta$ and $c_1\ cos.\ \beta$, and subtract the two from each other, there remain the relative velocities:

$$c\ sin.\ \alpha - c_1\ sin.\ \beta, \text{ and } c\ cos.\ \alpha - c_1\ cos.\ \beta;$$

as, however, the water has the velocity v_1 in common with the wheel, the latter relative velocity is in fact $= c\ cos.\ \alpha - c_1\ cos.\ \beta - v_1$. According to a known law, the loss of pressure height corresponding to a sudden cessation of this velocity is:

$$y = \frac{1}{2g}[(c\ sin.\ \alpha - c_1\ sin.\ \beta)^2 + (c\ cos.\ \alpha - c_1\ cos.\ \beta - v_1)^2],$$

or, in mechanical effect:

$$Y = y\ Q\gamma = [(c\ sin.\ \alpha - c_1\ sin.\ \beta)^2 + (c\ cos.\ \alpha - c_1\ cos.\ \beta - v_1)^2]\ \frac{Q\gamma}{2g}.$$

If we introduce into this formula $c_2 = v$ and $v_1 = \frac{r_1}{r}\ v$, further $c = \frac{F_2}{F} v$ and $c_1 = \frac{F_2}{F_1} v$, we have, as the loss of mechanical effect:

$$Y = \left[\left(\frac{F_2\ sin.\ \alpha}{F} - \frac{F_2\ sin.\ \beta}{F_1}\right)^2 = \left(\frac{F_2\ cos.\ \alpha}{F} - \frac{F_2\ cos.\ \beta}{F_1} - \frac{r_2}{r}\right)^2\right] \frac{v^2}{2g} Q\gamma.$$

From this we may judge as to the loss of effect in turbines that do not fulfil the conditions expressed in the equation:

$$F_1\ sin.\ \alpha = F\ sin.\ \beta \text{ and } F_1\ cos.\ \alpha = F\ cos.\ \beta + \frac{FF_1}{F_2} \cdot \frac{r_1}{r}.$$

However, even if these conditions be fulfilled in the normal state of the turbines' working, *i. e.*, when the sluice is fully drawn, they cannot be so when the sluice is depressed, and F becomes F_x. The loss of mechanical effect then, even when the effect is a maximum, viz.: $c_2 = v$, is:

$$Y = \left[\left(\frac{F_2\ sin.\ \alpha}{F_x} - \frac{F_2\ sin.\ \beta}{F_1}\right)^2 + \left(\frac{F_2\ cos.\ \alpha}{F_x} - \frac{F_2\ cos.\ \alpha}{F_1} - \frac{r_1}{r}\right)^2\right] \frac{v^2}{2g} Q\gamma,$$

or substituting $F\ sin.\ \beta = F_1\ sin.\ \alpha$ and $F\ cos.\ \beta + \frac{FF_1}{F_2} \cdot \frac{r_1}{r} = F_1\ cos.\ \alpha$,

$$Y = \left[\left(\frac{1}{F_x} - \frac{1}{F}\right)^2 (F_2\ sin.\ \alpha)^2 + \left(\frac{1}{F_x} - \frac{1}{F}\right)^2 (F_2\ cos.\ \alpha)^2\right] \frac{v^2}{2g} Q\gamma$$

$$= \left(\frac{F_2}{F_x} - \frac{F_2}{F}\right)^2 \frac{v^2}{2g} Q\gamma.$$

If, as an example, we put $\frac{v_1^2}{2g} = \frac{1}{2} h$, which is allowable in Fourneyron's turbines, we have:

$$Y = \left(\frac{F_2}{F_x} - \frac{F_2}{F}\right)^2 \cdot \left(\frac{r}{r_1}\right)^2 \cdot \frac{1}{2} Q h \gamma;$$

or, for the sluice half open, in which case:

$$F_x = \tfrac{1}{2} F,\ Y = \tfrac{1}{2}\left(\frac{F_2 r}{F r_1}\right)^2 Q h \gamma.$$

We see from this, that this loss may be diminished by making the ratios $\frac{F_2}{F}$ and $\frac{r}{r_1}$ small, that is to say, by making the orifice of discharge of the wheel and the external radius small, but keeping the orifices and radius of the reservoir large. As:

$$\frac{F_2}{F} = \frac{r_1 \sin. \beta}{r \sin. (\beta - \alpha)},$$

we have in the last case:

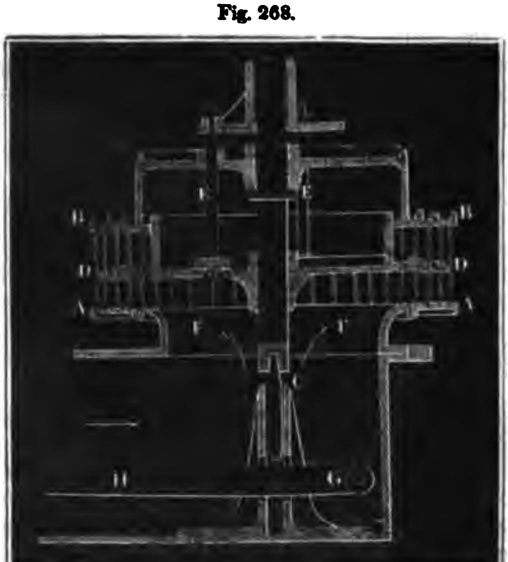
$$Y = \tfrac{1}{2}\left(\frac{\sin. \beta}{\sin. (\beta - \alpha)}\right)^2 Q h \gamma,$$

and, therefore, for $\beta = 90^\circ$, and $\alpha = 40^\circ$, $Y = 0{,}57\ Q h \gamma$, or there is in this case a loss of 57 per cent. of the effect.

Generally, when the sluice is much depressed, when $F_x < \tfrac{1}{2} F$, the *full discharge* ceases, that is, the water no longer fills up the wheel channels, the wheel becomes a pressure turbine only.

§ 158. *Sluice Adjustment.*—To avoid, or at least to diminish the loss of mechanical effect which results from lowering the sluice, and in order to retain the *full flow* of water through the wheel, many devices have been recently introduced by Fourneyron and others. Fourneyron divides the whole depth of the wheel into stages, by introducing horizontal annular division plates, dividing the total depth into two or three separate spaces, so that, when the sluice is lowered, one or two of the spaces are completely shut off, and the water flows through the other subdivision. This arrangement does

Fig. 268.



not entirely fulfil its object; but the apparatus shown in Fig. 268, invented by Combes, does. This contrivance consists in a plate or disc DD, between the two shroudings of the wheel, which, by means of rods EE, can be raised or depressed by means of a simple mechanism, so that the water flowing through the wheel always fills up the channels open to it. This apparatus fulfils the required conditions, but it is difficult, and very costly in construction.

The turbines of Callon, and also those of Gentilhomme, are likewise constructed so that the water may fill up the channels, how small soever the quantity supplied.

Fig. 269 represents a part of Callon's wheel in elevation and section. This shows that the guide-curves are covered in on top, and in the inside by sluices $E, E \ldots$, each of which closes two apertures. To regulate the discharge of water by this arrangement, it is only necessary to keep a certain number of apertures closed. Although this arrangement certainly provides against impact on the wheel, yet it is imperfect, inasmuch as the water can work little, if at all, by reaction, as it does not run through the wheel channels in an unbroken stream. In this alternate filling and emptying of the wheel-channels, the velocities c, c_1 and c_2 undergo continual variations unless $x = 0$, that is $\beta = 2\alpha$. If, for example, the wheel-channels not being filled, $c = \sqrt{2gh_1}$, we should have $c = \sqrt{2g(h - x)}$, when the water stream filled up the channels. Thus for each filling and emptying, or while the wheel passes from one open aperture to another, the velocity c continually oscillates between the limits $\sqrt{2gh}$, and $\sqrt{2g(h - x)}$. As the maximum effect can only be obtained for determinate values of v and $c_2 = \frac{Fc}{F_2}$, it is quite evident that, as c_2 varies, we fail in this.

Fig. 269.

For Gentilhomme's turbine, the same object is attained by circular sectors, which are so placed by means of mechanism, that they close a part of the guide-curve apparatus; an arrangement evidently even more imperfect than Callon's. Hänel, a German engineer, describes an arrangement of sluice for effecting the objects now in question, very similar to that of Combe's. (*See* "Deutsche Gewerbzeitung, 1846.)

§ 154. *Pressure Turbines.*—This is the place to compare the reaction turbines hitherto under discussion with the *impact* and pressure turbines, into which they always become converted when the sluice C, Fig. 270, closes the greater part of the depth of the wheel

AB. As the water *W* only partially fills the section of the wheel channels, the remainder is filled with air, unless the wheel works free of back water, and, therefore, the pressure on the outside of the wheel is that of the atmosphere; the velocity is $c = \sqrt{2gh}$, and is independent of the motion of the wheel. But for the velocity of discharge we have $c_2^2 = 2gh + v^2 - 2\,c\,v_1\,cos.\,\alpha$, and for the maximum effect $c_2 = v$, and then for these wheels, the expression

Fig. 270.

$2\,c\,v_1\,cos.\,\alpha = 2gh$, or substituting $c = \sqrt{2gh}$, $v_1 = \frac{\sqrt{2gh}}{2\,cos.\,\alpha}$, obtains.

In the case of reaction wheels, we found:

$$v_1 = \sqrt{gh\,(1 - tang.\,\alpha\,cotg.\,\beta)};$$

and hence we perceive that the conditions of maximum effect are identical in both cases, if $\frac{1}{2\,cos.\,\alpha^2} = 1 - tang.\,\alpha\,cotg.\,\beta$, or if $tang.\,\beta = tang.\,2\,\alpha$, that is $\beta = 2\,\alpha$; which result we have already ascertained in the form of the condition that $x = 0$. There is, therefore, an essential difference in the turbines of the two classes, in as far as the velocity for the maximum of effect does not in the one depend upon β, whilst it does in the other; and it is only when $\beta = 2\,\alpha$ that this velocity is the same for both. While, therefore, for reaction wheels the velocity v_1 may be made to vary within wide limits by selection of the angle β, we have no such joice in the case of impact turbines.

In reference to the effect of both wheels, we adduce the following facts. When, in a reaction wheel, the sluice is gradually lowered, the efficiency diminishes, and when it is so far lowered that the water does not fill up the wheel channels, the turbine then passing into a pressure-turbine, the efficiency suddenly rises, because the loss of mechanical effect by the sudden change of velocity ceases. By lowering the sluice still further, the further loss of effect is inconsiderable. According to this, the pressure turbine seems to be a better wheel than the reaction turbine; but from other circumstances the advantages do not preponderate, and can only be accorded when the supply of water is liable to much fluctuation.

As the water entering the wheel finds a much greater sectional area than at its velocity it can fill, it gets into an irregular oscillatory motion, and not only does not discharge with the velocity c_2 above calculated, but also loses a part of its *vis viva* absorbed in creating the eddying irregular motion alluded to (and which no doubt is *consumed in raising the temperature* of the water, Tr.). Numerous experiments have proved this, and these may be repeated with any turbine, if it be made to revolve with the best velocity, first as a reaction wheel, and then as a pressure wheel. Turbines always give a greater effect for an open sluice and full discharge than when the sluice is

lowered and the water does not fill the section of the wheel's channels.

When turbines work under water, the flow is always *full* through them, and these wheels are, therefore, always *reaction* wheels. A greater efficiency is naturally to be expected from these when the sluice is fully opened, than from the turbines revolving free of back water; on the other hand, we may safely assume that, when the sluice is lowered so that only $\frac{3}{4}$ or less of the depth of wheel is open, the efficiency of the reaction wheel will be less than that of the pressure turbine. From this we can easily understand the great advantage of introducing the horizontal dividing plates.

Remark. All the older turbines of Fourneyron were pressure turbines; but, as experience pointed out the greater efficiency of the reaction turbines, almost all turbines are now reduced to this principle.

§ 155. *Mechanical Effect of Turbines.*—We can now calculate the mechanical effect of turbines. The effect, which is not taken from the water if it flow from the wheel with an absolute velocity:

$$w = \sqrt{c_2^2 + v^2 - 2\, c_2 v \cos.\, \delta}, \text{ or if } c_2 = v,\ w = 2\, v \sin.\, \frac{\delta}{2},$$

$$\text{is, } L_1 = \frac{w^2}{2g} Q \gamma = \frac{4\, v^2 \left(\sin.\, \frac{\delta}{2}\right)^2}{2g} Q \gamma.$$

The effect which the water loses in the guide-curve apparatus, or in getting into the wheel, is:

$$L_2 = \zeta \frac{c^2}{2g} Q \gamma = \zeta \left(\frac{F_2}{F}\right)^2 \cdot \frac{v^2}{2g} Q \gamma,$$

in which $\zeta = 0{,}10$ to $0{,}20$, according as the wheel is provided with guide-curves or not.

The third loss of effect is $= \varkappa \cdot \frac{c_2^2}{2g} Q \gamma$, and consists of the friction and curve resistance. The resistance in passing round the curve may be found by the rules in Vol. I. § 334. The corresponding loss of head $= \zeta_1 \frac{\phi}{\pi} \cdot \frac{1}{2g} \cdot \left(\frac{Q}{F}\right)^2$ in which ζ is a co-efficient dependent on the ratio $\frac{d}{2\, a}$ of the half of the mean width of the channel to the mean radius, ϕ the central angle, F the sum of the mean section of the channels. If n be the number of channels or of buckets, and if e be the mean height of a channel, then $F = n\, d\, e$, and, therefore, the height of head lost by the resistance in the curves:

$$y = \zeta_1 \frac{\phi}{\pi} \cdot \frac{1}{2g} \cdot \left(\frac{Q}{n\, d\, e}\right)^2, \text{ or if we put:}$$

$$\zeta_1 = 0{,}124 + 3{,}104 \left(\frac{d}{2a}\right)^{\frac{7}{2}} \text{ (as in Vol. I. § 334), then:}$$

$$y = \left[0{,}124 + 3{,}104 \left(\frac{d}{2a}\right)^{\frac{7}{2}}\right) \cdot \frac{\phi}{\pi} \cdot \frac{1}{2g} \left(\frac{Q}{n\, d\, e}\right)^2,$$

and the loss of mechanical effect corresponding, is:

$$L_3 = \left[0,124 + 3,104 \left(\frac{d}{2a}\right)^{\frac{7}{2}}\right] \cdot \frac{\phi}{\pi} \cdot \frac{1}{2g} \frac{Q^3 \gamma}{(n\, d\, e)^2},$$

or putting $Q^2 = (F_2\, v)^2$,

$$L_3 = \left[0,124 + 3,104 \left(\frac{d}{2a}\right)^{\frac{7}{2}}\right] \frac{\phi}{\pi} \cdot \left(\frac{F_2}{n\, d\, e}\right)^2 \cdot \frac{v^2}{2g} \cdot Q\,\gamma.$$

The wheel buckets consist usually of two parts of different curvatures, and, hence, L_3 would be made up of two items. It is evident from the above, that this source of resistance increases the wider the channels are, and the less the radius of curvature a. Hence it is advisable to make β *obtuse*, so as to diminish the curvature of the buckets, which is also advisable in respect of a *full flow* through them.

The resistance from friction is to be calculated according to Vol. I. § 330. If ζ_2 be the co-efficient of friction, p the mean periphery, and l the length of the wheel channel, the height due to the resistance from friction:

$$z = \zeta_2 \cdot \frac{p\, l}{d e} \cdot \frac{1}{2g} \cdot \left(\frac{Q}{n\, d\, e}\right)^2 = \zeta_2 \cdot \frac{p\, l}{d e} \cdot \left(\frac{F_2}{n\, d\, e}\right)^2 \cdot \frac{v_2}{2g},$$

and the loss of effect corresponding:

$$L_4 = \zeta_2 \cdot \frac{p\, l}{d e} \cdot \left(\frac{F_2}{n\, d\, e}\right)^2 \cdot \frac{v^2}{2g} Q\,\gamma;$$

or, if $p = 2\,(d + e)$, and $\zeta_2 = 0,0144 + 0,0169 \sqrt{\frac{n\, d\, e}{Q}}$ be introduced:

$$L_4 = \left(0,0144 + 0,0169 \sqrt{\frac{n\, d\, e}{Q}}\right) \cdot \frac{(d + e)\, l}{2\, d\, e} \cdot \left(\frac{F_2}{n\, d\, e}\right)^2 \cdot \frac{v^2}{2g} Q\,\gamma.$$

If, lastly, G be the weight of the turbine in revolution, and r_2 the radius of the pivot, the loss of effect by the friction, then, is:

$L_5 = \frac{2}{3} f\, G \cdot \frac{r_2}{r} v$ (Vol. I. § 171).

If, now, we deduct these five losses of effect from the power at disposition, there remains of useful effect:

$$Pv = \left(h - \left[\left(2 \sin. \frac{\delta}{2}\right)^2 + \zeta \left(\frac{F_2}{F}\right)^2 + \zeta_1 \cdot \frac{\phi}{\pi} \left(\frac{F^2}{n\, d\, e}\right)^2 \right.\right.$$
$$\left.\left. + \tfrac{1}{2} \zeta_2\, l \left(\frac{1}{d} + \frac{1}{e}\right) \left(\frac{F_1}{n\, d\, e}\right)^2\right] \frac{v^2}{2g}\right) Q\gamma - \tfrac{2}{3} \frac{r_2}{v} v f G.$$

In order to have this mechanical effect great, it is necessary to make the velocity of rotation v, the area of the orifice F_2, the orifice angle δ small. In how far this is possible we have above shown.

It is only in the case of turbines working under water that the height h is to be measured from water surface to water surface. For turbines working in air, h is to be measured from the upper surface to the centre of the wheel. In the latter case, the freeing the wheel of back water involves a loss of head, measured by the distance from the centre of the wheel to the surface of the race,

whilst for wheels working under water there is a loss from the *resistance of the medium.*

Remark. For high pressure turbines there is an additional source of loss in the resistance of the flow of water through the pressure pipe.

§ 156. *Construction of Guide-Curve Turbines.*—Let us now endeavor to deduce the *rules* necessary for planning a wheel consistently with the above principles. We may of course assume the quantity of water discharged Q, and the fall h to be given; and if, instead of Q, the useful effect L were given, we might then at least derive Q from L, and the efficiency η (about 0,75) by the formula: $Q = \frac{L}{\eta h \gamma}$. The remaining quantities $r, r_1\ \alpha, \beta, \delta, v, n, e$, &c., are determined, partly by discretion, partly by experience, and partly by theory.

The angle α is generally assumed. For wheels without guide-curves it is taken as 90°, but for wheels with guide-curves it must be made from 25° to 40°; the former for high falls, the latter for small falls, in order that, in the former case, the orifices may not be too large, and in the latter not too small, or, in order that, in the former case, the wheels may not be too small in diameter, and in the latter not too great. The angle β is, in a certain degree, fixed by the value of α. That the water may enter the wheel without pressure on the free space, we must have $\beta = 2\alpha$; but as this pressure diminishes as the sluice is depressed, in order to prevent *negative* pressure, β is made greater than 2α, and probably $\beta = 2\alpha + 30°$ to $2\alpha + 50°$ are good limits; the former in high falls, the latter in low falls.

The ratio $v = \frac{r}{r_1}$ of the internal and external radii of the wheel falls within the limits of 1,25 and 1,5. For reasons easily understood, the smaller ratio is to be chosen for large values of β, and for wheels of considerable diameter, and *vice versâ.*

In order further to determine the radius of the wheel, and of the reservoir, we shall, as is the case in the best turbines hitherto made, require fulfillment of the condition that the velocity of the water in the reservoir shall not exceed 3 feet per second. If we adopt this velocity as ground work of our calculation, and leave out of the question the section of the upright pipe encasing the axle, and that of the sluice, then $Q = 3\pi r_1^2$, and, therefore, inversely, the radius of the reservoir, or the internal radius of the wheel:

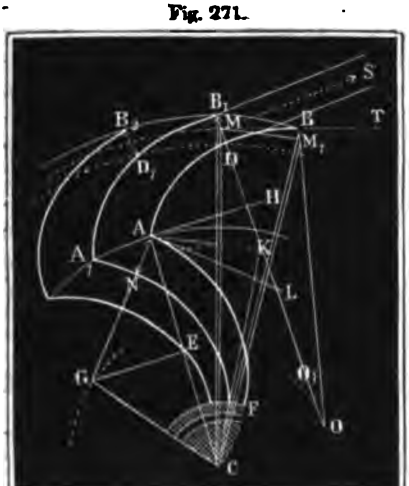
$r_1 = \sqrt{\frac{Q}{3\pi}} = 0,326\sqrt{Q}$, when r_1 is in feet, and Q in cubic feet.

From this radius we get the external radius $r = v r_1$. The velocity at the inner periphery of the wheel is determined by the formula:

$$v_1 = \sqrt{\frac{2gh}{\frac{2\sin.\beta\cos.\alpha}{\sin.(\beta-\alpha)} + \zeta\left(\frac{\sin.\beta}{\sin.(\beta-\alpha)}\right)^2 + \kappa\left(\frac{r}{r_1}\right)}}$$

into which we must, in the first place, introduce an approximate value of x. From this, however, we get the velocity of discharge: $c = \frac{v_1 \sin. \beta}{\sin. (\beta - \alpha)}$, and the section $F = \frac{Q}{c} = \frac{Q \sin. (\beta - \alpha)}{v_1 \sin. \beta}$; further, the velocity of entrance $c_1 = \frac{c \sin. \alpha}{\sin. \beta} = \frac{v_1 \sin. \alpha}{\sin. (\beta - \alpha)}$, and the section $F_1 = \frac{Q}{c_1} = \frac{Q \sin. (\beta - \alpha)}{v_1 \sin. \alpha}$. Lastly, the velocity of the external periphery of the wheel, and of the exit from it: $v = c_2 = \frac{r}{r_1} v_1$, and the contents of the united orifices of discharge from the wheel: $F_2 = \frac{Q}{c_2} = \frac{r_1}{r} \cdot \frac{Q}{v_1} = \frac{r_1}{r} \cdot \frac{Fc}{v_1}$. Besides this, we ascertain the number of revolutions of the wheel per min. to be: $u = \frac{30\, v}{\pi r} = 9{,}55 \frac{v}{r}$.

Fig. 271.



In order to find the height of the wheel, or of the orifice e, we pursue the following method. If n_1 be the number of guide-curves, and d_1 the least distance AN (Fig. 271), between any two of the guide-curves at the entrance on to the wheel, then $n_1 d_1 e = F$. If, further, d_1 and e be in a determinate proportion to each other: $\psi_1 = \frac{e}{d_1}$, then $n_1 \psi_1 d_1^2 = F$; and if s be the thickness of one of the guide-curves, we may put with tolerable accuracy:

$$d_1 = AA_1 \sin. \alpha = \frac{2 \pi r_1 \sin. \alpha}{n_1} - s_1,$$

and, hence,

$$n_1 \psi_1 \left(\frac{2 \pi r_1 \sin. \alpha}{n_1} - s_1\right)^2 = F,$$

so that, by inversion, the number of buckets required:

$$n_1 = \frac{\psi_1 (2 \pi r_1 \sin. \alpha - n_1 s_1)^2}{F},$$

or, approximately: $= \frac{\psi_1 (2 \pi r_1 \sin. \alpha)^2}{F}$, for which a whole number would be taken. If n_1 be once fixed, then:

$$d_1 = \frac{2 \pi r_1 \sin. \alpha}{n_1} - s_1 = \frac{F}{\psi_1 (2 \pi r_1 \sin. \alpha - n_1 s_1)},$$

and the height of wheel: $e = \frac{F}{2 \pi r_1 \sin. \alpha - n_1 s_1}$.

§ 157. We have still to deduce rules by which to calculate the number of wheel-buckets, and the dimensions of the orifices of the wheel. The orifices of discharge, the united area of which is $F_2 = \frac{Q}{c_2}$, is *not* the outer periphery of the wheel, but the section B_1D, B_2D_1, &c., through the outer end of the buckets B_1, B_2, &c. (Fig. 271). Again, for r in the above formulas we are not to understand the radius of the outer periphery, but the distance CM of the centre of the orifice B_1D from the axis of rotation, and, in like manner, v is not the velocity of rotation of B, but of M. If, now, δ be the angle SMT, which the axis of the stream flowing through BD makes with the tangent MT, or the normal to the radius $CM = r$; and, further, if n be the number of wheel-buckets, s their thickness, d the width B_1D of the orifices of discharge, and ψ the ratio $\frac{e}{d}$, we may put: $n\,d\,e = n\,\psi\,d^2 = \frac{n\,e^2}{\psi} = F_2$, therefore, inversely, the number of wheel-buckets $n = \frac{\psi F_2}{e^2}$. Again, as

$$2\,\pi\,r\,\sin.\,\delta - n\,s = n\,d = \frac{ne}{\psi} = \frac{F_2}{e},$$

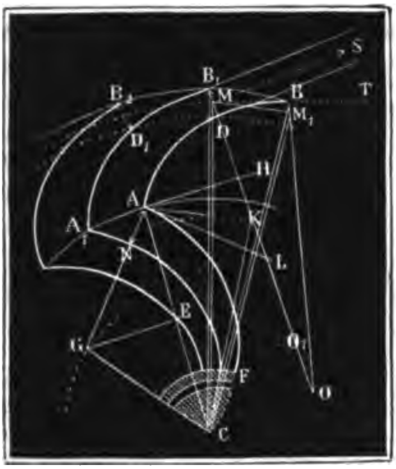
we have for the angle of discharge $\sin.\,\delta = \frac{F_2(e + \psi\,s)}{2\,\pi\,r\,e^2}$.

This angle δ should not in any case be more than 20°, and, therefore, if it comes out more than this, by the latter formula, some one or more of the elements composing it must be changed. Thus, for example, for this purpose $F_2 = \frac{r_1}{r} \cdot \frac{Q}{v_1}$ may be made less, *i. e.*, v_1, or which amounts to the same, the difference between α and β may be made greater. Some engineers have endeavored to keep δ small, by making the wheel deeper at the outside than in the inside, giving two values to e (Fig. 269). This, however, has the disadvantage, that *full* discharge is thereby interfered with—at least in wheels revolving free of back water, and that the water follows the wheel shroudings when these diverge from each other to any considerable extent. As to the ratio $\psi = \frac{e}{d}$, its influence on e and δ is but trifling. It is within the limits 2 and 5 in wheels giving good results. The small value refers to small wheels, and *vice versâ;* for, otherwise, the channels fall out too wide, and the *full flow* is liable to be lost.

§ 158. *Construction of the Bucket.*—The buckets are generally circular arcs. For the guide-curves one arc is sufficient; but for the wheel-curves, or buckets, two arcs, tangential to each other, are usually required. How to fix the radius of these arcs, and how to combine them, may be explained as follows: With $CM = r$, Fig. 272, describe a circle, draw the tangent MT, and upon it set off the angle of discharge $SMT = \delta$, as determined above. Draw MO at right angles to MS, and set off on each side of M, $MD = MB_1 = \frac{1}{2}\,d$.

Now draw the radius CB_1, and from C lay off the angle $B_1CB = \phi^\circ$, as determined by the formula $\phi = \frac{2\pi}{n} - \frac{s}{r\ sin.\ \delta}$. Also, from C, as centre, draw circles through B_1 and D. The first of these circles gives the external periphery of the wheel, and the points B, B_1, &c., are the outer ends of the buckets. If we draw BO, so that $BOD = BCB_1 = \phi$, we have in O the centre, and in $BO = DO$, the radius of the arc DB forming the outer portion of the bucket. If we make $B_1O_1 = DO$, we have the centre O_1 of the outer piece B_1D_1 of the next following bucket, &c. &c. The radius $OB = OD = a$ of the arc BD may be also determined by solution of the triangle MOM_1. We have: $\frac{MO}{MM_1} = \frac{sin.\ MM_1O}{sin.\ MOM_1}$, but the cord $MM_1 = 2\ r\ sin.\ \frac{\phi}{2}$, $MOM_1 = \phi$, and $MM_1O = 90 + \delta - \frac{\phi}{2}$, therefore,

Fig. 272.



$$OM = \frac{2\ r\ sin.\ \frac{\phi}{2}\ cos.\left(\delta - \frac{\phi}{2}\right)}{sin.\ \phi} = \frac{r\ cos.\left(\delta - \frac{\phi}{2}\right)}{cos.\ \frac{\phi}{2}},$$

and the radius a required

$$= \frac{r\ cos.\left(\delta - \frac{\phi}{2}\right)}{cos.\ \frac{\phi}{2}} - \tfrac{1}{2}\ d.$$

By this method of construction, the end B_1 of the bucket is quite parallel to the element D opposite, and, therefore, the stream flows out without contraction. If this parallelism be not effected, it is always disadvantageous; if the tangents to B and D diverge outwards, there is danger of losing the *full flow*, and if they converge, there arises a partial contraction, and the stream then strikes upon the outer surface of BD (Vol. I. § 319).

The inner piece DA of a wheel-bucket may generally be formed of one arc of a circle. The radius $KD = KA = a_1$ of this circle is found as follows: In the triangle

CMK, $CM = r$, $MK = a_1 + \frac{d}{2}$, and $\angle\ CMK = SMT = \delta$,

$$\therefore \overline{CK}^2 = r^2 + \left(a_1 + \frac{d}{2}\right)^2 - 2\,r\left(a_1 + \frac{d}{2}\right) cos.\ \delta.$$

In the triangle CAK, on the other hand, $CA = r_1$, $AK = a_1$, and $CAK = 180^\circ - \beta$, therefore, $\overline{CK}^2 = r_1^2 + a_1^2 + 2\,r_1\,a_1\,cos.\ \beta$. By equating the two expressions, we have:

$$r^2 + a_1\,d + \frac{d^2}{4} - 2\,r\,a_1\,cos.\ \delta - r\,d\,cos.\ \delta = r_1^2 + 2\,r\,a\,cos.\ \beta,$$

and hence the radius required:

$$a_1 = \frac{r^2 - r_1^2 - r\,d\,cos.\ \delta + \frac{d^2}{4}}{2\,(r\,cos.\ \delta + r_1\,cos.\ \beta) - d}.$$

As to the arc to be adopted as the curvature of the guide-curves, we get its radius and centre by drawing AL at the known angle α to the tangent AH of the inner circumference of the wheel. Raise a perpendicular AG to it, and cut this in G by another normal, raised from the middle point E of the radius CA. This point G is the centre of the guide-curve AF, which may be drawn either quite up to the case pipe of the axle, or to within any convenient distance of it. The radius $GA = GC = a_2$ of this bucket is: $a_2 = \frac{r_1}{2\,cos.\ \alpha}$.

The centres of the arcs forming the outer arcs are in circles described with the radii CO, CK, and CG.

Example. It is required to determine all the proportions and lines of construction of a Fourneyron's turbine for a fall of 5 feet, with 30 cubic feet of water per second. We shall take $\alpha = 30^\circ$, and $\beta = 110^\circ$, and adopt the ratio $\frac{r}{r_1} = \nu = 1{,}35$. This being assumed, we have, from the rules above given, the internal radius $r_1 = 0{,}326\sqrt{Q} = 1{,}785$ feet, for which we take 1,8 feet. Hence $r_1 = CM$ (Fig. 272), the external radius $= 1{,}8 \times 1{,}35 = 2{,}43$ feet, for which we shall put 2,45. The width of the shrouding, therefore, measured to the centre of the orifice of discharge, $= 2{,}45 - 1{,}8 = 0{,}65$ feet. Neglecting prejudicial resistance, the best velocity of the wheel is:

$$v_1 = \sqrt{gh\,(1 - tang.\ \alpha\ cotg.\ \beta)} = \sqrt{5\,.\,31{,}25\,(1 + tang.\ 30^\circ\ cotg.\ 70^\circ)}$$
$$= \sqrt{156{,}25\,.\,1{,}21014} = 13{,}75 \text{ feet},$$

but, taking these resistances into account, if $\zeta = 0{,}18$ and $\kappa = 0{,}06$:

$$v_1 = \sqrt{\frac{2gh}{\frac{2\ sin.\ \beta\ cos.\ \alpha}{sin.\ (\beta - \alpha)} + 0{,}18\left(\frac{sin.\ \beta}{(sin.\ \beta - \alpha)}\right)^2 + 0{,}06\left(\frac{r}{r_1}\right)^2}}$$

$$= \sqrt{\frac{2\,.\,31{,}25\,.\,5}{\frac{2\ sin.\ 110^\circ\ cos.\ 30^\circ}{sin.\ 80^\circ} + 0{,}18\left(\frac{sin.\ 110^\circ}{sin.\ 80^\circ}\right)^2 + 0{,}06\,.\,1{,}4^2}}$$

$$= \sqrt{\frac{312{,}5}{1{,}6527 + 0{,}1639 + 0{,}1176}} = \sqrt{\frac{312{,}5}{1{,}9342}} = 12{,}71 \text{ feet},$$

and then $v = c_0 = \nu\,v_1 = 1{,}35\,.\,12{,}71 = 17{,}15$ feet. The velocity of discharge

$$c = \frac{v_1\ sin.\ \beta}{sin.\ (\beta - \alpha)} = \frac{12{,}71\ sin.\ 70^\circ}{sin.\ 80^\circ} = 12{,}13 \text{ feet}.$$

The number of revolutions per minute is $u = 9{,}55\,.\,\frac{v_1}{r_1} = \frac{9{,}55\,.\,12{,}71}{18} = 67{,}4$.

From this we have the areas of the orifices of discharge:

$F = \frac{Q}{c} = \frac{30}{12,15} = 2,473$ square feet, and $F_2 = \frac{Q}{c_2} = \frac{Q}{v} = \frac{30}{17,15} = 1,748$ square feet. Assuming the thickness of the buckets to be $2\frac{1}{2}$ lines $= 0,017$ feet, and supposing that the ratio of the depth to the width of the orifices $\psi_1 = \frac{e}{d_1} = \frac{3}{1}$, we have as the requisite number of buckets:

$$n_1 = \frac{\psi_1 (2 \pi r_1 \sin. \alpha - n_1 s_1)^2}{F} = \frac{3 . (5,655 - 0,017 n_1)^2}{2,473} = 30,$$

and, hence, we have as the height of the wheel, or of the orifices:

$$e = \frac{F}{2 \pi r_1 \sin. \alpha - n_1 s_1} = \frac{2,473}{5,655 - 0,51} = \frac{2,473}{5,145} = 0,4808 \text{ feet.}$$

Supposing $\psi = \frac{5}{1}$, the number of buckets would be:

$n = \frac{\psi F_2}{e^2} = \frac{5 . 1,748}{0,4808^2} = \frac{8,74}{0,2311} = 37,8$, for which we may adopt 36. From this we get the required angle of discharge:

$$\sin. \delta = \frac{F_2 (e + \psi s)}{2 \pi r e^2} = \frac{1,748 \, (0,4808 + 0,017 . 5)}{2 \pi . 2,45 . 0,4808^2}$$

$$= \frac{1,748 . 0,5658}{4,9 . \pi . 0,4808^2} = 0,2780, \text{ consequently } \delta = 16^\circ, 8', \text{ and}$$

$$d = \frac{F_2}{n e} = \frac{1,748}{36 . 0,4808} = 0,1010 \text{ feet.}$$

If the turbine is to be clear of the back water, it must be raised a certain height above the surface of the tail-race; and as the half height of the wheel $\frac{e}{2} = 0,2404$ feet, this distance may be estimated at 0,5 feet. If this excess of fall does not exist, then the calculations must be based on a fall of $4\frac{1}{2}$ feet, instead of on that of 5 feet. In order to judge of the loss of water, we have to find the amount of x, the excess of pressure of the water passing under the sluice. We have:

$x = h - (1 + \zeta) \frac{c^2}{2g} = 5 - 1,18 . 0,016 . 12,13^2 = 5 - 2,778 = 2,222$ feet, and the velocity corresponding $= 7,906 \sqrt{2,222} = 11,78$ feet. If, therefore, the space between the wheel and the bottom plate be $\frac{1}{12}$ inch, its area is: $2 . 1,8 . \pi . \frac{1}{144} \frac{\pi}{8} = 0,0393$ square feet, and, therefore, the quantity of water escaping: $Q_1 = 11,78 . 0,0393 = 0,46$ cubic feet. To diminish this loss, which will be the less the lower the sluice, the *fitting-up* of the wheel must be very accurately done, so that the space between the wheel and bottom plate may be as small as possible; or, by increasing c and making β less, x must be reduced as low as possible.

The dividing angle of the wheel is $\frac{360^\circ}{36} = 10^\circ$; but the thickness of the buckets takes up an angle $= \frac{s}{r \sin. \delta} = \frac{0,017}{2,45 \sin. 16^\circ, 8'} = 0,02497$, or an angle $= 1^\circ, 26'$, hence $\phi = 8^\circ, 34'$, the angle of curvature of one part of the bucket. The radius corresponding is:

$$a = \frac{r \cos. (\delta - \frac{1}{2} \phi)}{\cos. \frac{1}{2} \phi} - \frac{1}{2} d = \frac{2,45 \cos. 11^\circ, 51'}{\cos. 4^\circ, 17'} - 0,0505 = 2,3541 \text{ feet.}$$

The radius of the second part of the wheel-bucket:

$$a_1 = \frac{r^2 - r_1^2 - r d \cos. \delta + \frac{1}{4} d^2}{2 (r \cos \delta + r_1 \cos. \beta) - d} = \frac{2,785 - 0,2377}{3,476 - 0,101} = \frac{2,5473}{3,375} = 0,755 \text{ feet.}$$

The corresponding angle of curvature is: $\phi_1 = 180^\circ - \beta - \delta + \sigma - \tau$, in which $tang. \sigma = \frac{a_1 \sin. \beta}{r_1 - a_1 \cos. \beta}$, and $tang. \tau = \frac{a_1 \sin. \delta}{r - a_1 \cos. \delta}$; expressed numerically:

$$\phi_1 = 70^\circ - 16^\circ, 8' + 24^\circ, 42' - 6^\circ, 56' = 71^\circ, 38'.$$

These investigations afford us the necessary elements for the construction of a turbine for the fall in question, and we have now only to calculate the useful effect that such a machine will yield. The absolute velocity of the water discharged is $w = 2 c_2 \sin. \frac{1}{2} \delta = 2 . 17,15 \sin. 8^\circ, 4' = 4,813$ feet, and, hence, the loss of fall corresponding $= \frac{w^2}{2g} = 0,016 . 4,813^2 = 0,371$ feet. Again, the loss of fall occasioned by the resist-

ance in the guide-curves $= 0{,}18\ \frac{c^2}{2g} = 0{,}18 \cdot 0{,}16 \cdot 12{,}13^2 = 0{,}423$ feet. The loss of fall arising from the hydraulic resistances, may be estimated as follows: From an accurate drawing of the wheel, and the results of the calculations given above, it will be found that each wheel channel consists of two parts, of which the one is 0,11 feet wide, and 0,2 long; the radius 2,35 feet, the central angle $4\frac{1}{2}^\circ$, and the other is 0,21 feet wide, and 0,95 feet long, the radius of curvature 0,755 feet, and central angle of 71°, 38′. From this we deduce the co-efficient of resistance for the smaller part:

$$\zeta_1 = 0{,}124 + 3{,}104 \left(\frac{d}{2\,a}\right)^{\frac{7}{2}} = 0{,}124 + 3{,}104 \left(\frac{0{,}11}{4{,}71}\right)^{\frac{7}{2}} = 0{,}124$$

and for the larger: $\zeta_1 = 0{,}124 + 3{,}104 \cdot \left(\frac{0{,}21}{1{,}51}\right)^{\frac{7}{2}} = 0{,}127$. Again, the angle ratio for the first part is $\frac{\phi}{\pi} = \frac{4{,}5}{180} = 0{,}025$, and for the second $\frac{71{,}63}{180} = 0{,}398$. Again, the section of the first is $\frac{F_2}{n\,d\,e} = \frac{1{,}748}{36 \cdot 0{,}11 \cdot 0{,}4808} = 0{,}918$, and for the second part $= \frac{1{,}748}{36 \cdot 0{,}21 \cdot 0{,}4808} = 0{,}481$, and hence we have for the co-efficient of the whole resistance arising from curvature:

$\varkappa_1 = 0{,}124 \cdot 0{,}025 \cdot 0{,}918^2 + 0.127 \cdot 0.398 \cdot 0{,}481^2 = 0{,}0026 + 0{,}0117 = 0{,}0143.$

Further, the co-efficient of friction in the first part:

$$\zeta_2 = 0{,}0144 + 0{,}0169 \sqrt{\frac{n\,d\,e}{Q}} = 0{,}0144 + 0{,}0169 \sqrt{\frac{36 \cdot 0{,}11 \cdot 0{,}4808}{30}}$$

$= 0{,}0187$, and for the second $= 0{,}0144 + 0{,}0169 \sqrt{\frac{36 \cdot 0{,}21 \cdot 0{,}4808}{30}} = 0{,}0203$. The ratio $\left(\frac{d+e}{2\,d\,e}\right) l$ for the first part $= \frac{0{,}5908 \cdot 0{,}2}{2 \cdot 0{,}11 \cdot 0{,}4808} = 1{,}117$, and for the second $= \frac{0{,}6908 \cdot 0{,}95}{2 \cdot 0{,}21 \cdot 0{,}4808} = 3{,}250$, and from this we have the co efficient of friction for the whole channel:

$\varkappa_2 = 0{,}0187 \cdot 1{,}117 \cdot 0{,}918^2 + 0{,}0203 \cdot 3{,}250 \cdot 0{,}481^2 = 0{,}0176 + 0{,}0152 = 0{,}0328,$

and hence, lastly, the co-efficient for all the resistances in a wheel-channel is:

$\varkappa = \varkappa_1 + \varkappa_2 = 0{,}0143 + 0{,}0328 = 0{,}0471$, and the loss of fall corresponding to this $= \varkappa \cdot \frac{v^2}{2g} = 0{,}0471 \cdot 0{,}016 \cdot 17{,}15^2 = 0{,}222$ feet. The three losses of fall just estimated $= 0{,}371 + 0{,}424 + 0{,}222 = 1{,}017$ feet, and thus there remains of the total effect at disposition, $Q\,h\,g = 30 \cdot 5 \cdot 66 = 9900$ feet lbs., only

$Pv = 30 \cdot (5 - 1{,}017)\ 66 = 7886$ lbs. as *useful* effect. There is, however, some portion of this consumed by the friction of the pivot. If the weight of the wheel, &c., be 2000 lbs., and supposing the radius of the pivot $= 1\frac{1}{2}$ inch, and the co-efficient of friction 0,075, the mechanical effect consumed by the friction of the pivot

$= \frac{r_2}{r_1} f\, G v_1 = \frac{1}{8 \cdot 1{,}8} \cdot 0{,}075 \cdot 2000 \cdot 12{,}71 = 132$ feet lbs. (Suppose the co efficient of friction 0,12, which is more likely, then the friction $= 210$ feet lbs.) The useful effect available, directly at the axle of the wheel, is then:

$L = 7886 - 210 = 7676$ feet lbs. $= 13{,}9$ horse power. If we assume 0,5 feet lost besides, by keeping the wheel free of the water in the race, then the efficiency:

$$\eta = \frac{L}{Q\,h\,\gamma} = \frac{7676}{30 \cdot 5{,}5 \cdot 66} = 0{,}705.^*$$

§ 159. *Turbines without Guide-Curves.*—The proportions of turbines without guide-curves are only partly deducible in the manner of turbines with guide-curves. The angle α is in these 90°, and the

* [It will be observed that, in working this example, we have retained the co-efficients applicable to the Prussian weights and measures, viz.: $\frac{1}{2g} = 0{,}016$, and the weight of the cubic foot of water 66 lbs., for which the student can at pleasure substitute 0,0155 and 62,5 respectively, also 8,02 for 7,906.—Am. Ed.]

angle $\beta = 150^\circ$ to 160°, in order to have x as low as possible. The ratio $\nu = \frac{r}{r_1}$ is only 1,15 to 1,30, as, otherwise, from β being so large, the *length* of bucket would be inconvenient. In order to have the loss of mechanical effect for the entrance on the wheel as low as possible, the water is laid on to the wheel with a velocity of only 2 feet, and hence the internal radius r_1 is only $0,4 \sqrt{Q}$, and the external radius $r = \nu\, r_1 = 0,4\, \nu \sqrt{Q}$.

The best velocity of rotation is also to be calculated by a different rule, as the formula:

$$v_1 = \sqrt{\frac{2gh}{\frac{2\, sin.\, \beta\, cos.\, \alpha}{sin.\, (\beta - \alpha)} + \zeta \left(\frac{sin.\, \beta}{sin.\, (\beta - \alpha)}\right)^2 + \varkappa \left(\frac{r}{r_1}\right)^2}},$$

which in this case has the form:

$$v_1 = \sqrt{\frac{2gh}{\zeta\, .\, tang.\, \beta^2 + \varkappa \left(\frac{r}{r_1}\right)^2}},$$

gives too great values. The reason of this is, that the condition of making the velocity of discharge $= 0$, does not, on account of the prejudicial resistances, lead to the maximum effect being produced; and it is only for turbines with guide-curves, that the fulfilment of this condition gives satisfactory approximations to this maximum. On the other hand, for turbines without guide-curves, and in all cases in which α is nearly 90°, the influence of the prejudicial resistances on the working of the wheel becomes too great for its being possible to assume that $w = 0$, or $v = c_2$. In order to find the least velocity for these wheels, we adopt the following method: We have already (Vol. II. § 147) found that

$$(1 + \varkappa)\, c_2^2 = 2gh + v^2 - 2\, c\, v_1\, cos.\, \alpha - \zeta\, c^2, \text{ and as}$$
$$cos.\, \alpha = cos.\, 90^\circ = 0, \text{ and}$$

$$c = \frac{v_1\, sin.\, \beta}{sin.\, (\beta - 90)} = -\, v_1\, tang.\, \beta = -\frac{r_1}{r}\, v\, tang.\, \beta, \text{ we may put:}$$

$$(1 + \varkappa)\, c_2^2 = 2gh + v^2 \left[1 - \zeta \left(\frac{r_1}{r}\right)^2 tang.\, \beta^2\right];$$

and, therefore, the velocity of discharge:

$$c_2 = \sqrt{\frac{2gh + v^2 \left[1 - \zeta \left(\frac{r_1}{r}\right)^2 tang.\, \beta^2\right]}{1 + \varkappa}}.$$

Hence, the loss of fall:

$$y = \frac{c_2^2 + v^2 - 2\, c_2\, v\, cos.\, \delta + \varkappa\, c_2^2 + \zeta\, c^2}{2g}$$

$$= \frac{(1 + \varkappa)\, c_2^2 + v^2 \left[1 + \zeta \left(\frac{r_1}{r}\right)^2 tang.\, \beta^2\right] - 2\, v\, c_2\, cos.\, \delta}{2g}$$

$$= \left(2gh + 2v^2 - 2v\cos.\delta \sqrt{\frac{2gh + v^2\left[1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\beta^2\right]}{1+\varkappa}}\right)\frac{1}{2g}$$

$$= h - \left(v\cos.\delta \sqrt{\frac{2gh + v^2\left[1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\beta^2\right]}{1+\varkappa}} - v^2\right)\cdot\frac{1}{g},$$

and, therefore, the effect to be expected from the wheel:

$$L = \left(v\cos.\delta \sqrt{\frac{2gh + v^2\left[1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\beta^2\right]}{1+\varkappa}} - v^2\right)\frac{Q\gamma}{g}.$$

If we put ψ for $1 - \zeta\left(\frac{r_1}{r}\right)^2 tang.\beta^2$, and ϕ for $\frac{\sqrt{1+\varkappa}}{\cos.\delta}$, then we have, more simply, $L = (v\sqrt{2gh + \psi v^2} - \phi v^2)\frac{Q\gamma}{\phi g}$.

In order that this value may give a maximum, we can deduce by the higher calculus that $\phi\, v = \frac{gh + \psi v^2}{\sqrt{2gh + \psi v^2}}$, or, if we represent the ratio of the height due to velocity $\frac{v^2}{2g}$ to the pressure height h, that is, $\frac{v^2}{2gh}$, by χ, then $\frac{\frac{1}{2} + \psi\chi}{\sqrt{+\psi\chi^2}} = \phi$, and hence, $\chi = \frac{\phi - \sqrt{\phi^2 - \psi}}{2\psi\sqrt{\phi^2 - \psi}}$. If from this we have got χ, we have for the velocity $v = \sqrt{\chi\,.\,2gh}$, $v_1 = \frac{r_1}{r}v$, $c = -v_1\, tang.\beta$, and $c_2 = \sqrt{\frac{2gh + \psi v^2}{1+\varkappa}}$. Hence the sections $F = \frac{Q}{c}$, and $F_2 = \frac{Q}{c_2}$, and, lastly, the height of the wheel, or of the orifice $e = \frac{F}{2\pi r_1}$.

The other proportions, as the construction of the buckets, &c. &c., do not differ from those of turbines having guide-curves.

Remark. Strictly speaking, turbines with guide-curves should also be treated in this manner, but as the expressions are very complicated, and lead to a value of $\frac{c_2}{v}$, which differs very little from unity, we have deemed the investigation unnecessary.

Example. It is required to make the necessary calculations for the design of a turbine on Cadiat's plan, for a fall of 5 feet, with 30 cubic feet of water per second. Assuming $\beta = 150°$, $\nu = 1{,}2$, and $r_1 = 0{,}4\sqrt{Q} = 0{,}4\sqrt{30} = 2{,}19$, which we shall make 2,25, and, hence, $r = 1{,}2\,.\,2{,}25 = 2{,}70$ feet. If, further, $\zeta = 0{,}15$, and $\varkappa = 0{,}10$, and $\delta = 16°$, then $\psi = 1 - \zeta\left(\frac{r_1}{r}\right)^2 .\, tang.\beta^2 = 1 - 0{,}15\,.\,\frac{(tang.\,30°)^2}{1{,}44} = 1 - 0{,}035 = 0{,}965$, and

$\phi = \frac{\sqrt{1+\varkappa}}{\cos.\delta} = \frac{\sqrt{1{,}1}}{\cos.16°} = 1{,}091$; and, therefore,

$$\chi = \frac{\phi - \sqrt{\phi^2 - \psi^2}}{2\psi\sqrt{\phi^2 - \psi^2}} = \frac{1{,}091 - 0{,}475}{1{,}93\,.\,0{,}475} = 0{,}672, \text{ and } \sqrt{\chi} = 0{,}820.$$

From this we have the most advantageous velocity of rotation:
$v = \sqrt{\chi \cdot 2gh} = 0,82 \cdot 7,906 \sqrt{5} = 14,50$ feet. Again, $v_1 = \frac{v}{\nu} = \frac{14,50}{1,2} = 12,08$ feet,
$c = - v_1 \, tang. \, \beta = 12,08 \, tang. \, 30^\circ = 6,97$ feet, and

$$c_2 = \sqrt{\frac{2gh + \psi v^2}{1 + \kappa}} = \sqrt{\frac{312,5 + 202,9}{1,1}} = 21,65 \text{ feet;}$$

and now we have the section $F = \frac{Q}{c} = \frac{30}{6,97} = 4,304$ square feet, and the section $F_2 = \frac{Q}{c_2} = \frac{30}{21,65} = 1,386$ square feet. Hence, again, we have the height of the wheel: $e = \frac{F}{2 \pi r_1} = \frac{4,304}{2 \cdot 2,25 \cdot \pi} = 0,304$ feet, and if we take for the orifices of discharge of the wheel, the proportional dimensions $\frac{e}{d} = \frac{4}{3}$, we have for the number of buckets:
$n = \frac{2 F_2}{e^2} = \frac{2 \cdot 1,386}{0,304^2} = \frac{2,772}{0,0924} = 30$. If the thickness of the bucket plates $s = 0,017$ feet, we have as the angle of discharge:

$$sin. \, \delta = \frac{F_2 - n e s}{2 \pi r e} = \frac{1,386 - 30 \cdot 0,304 \cdot 0,017}{2 \cdot 2,7 \cdot 0,304 \, \pi} = \frac{1,226}{1,6416 \, \pi} = 0,238,$$

and, hence, $\delta = 13\frac{3}{4}^\circ$. As we assumed above that for $\psi = \frac{\sqrt{1+\kappa}}{cos. \, \delta}$, $\delta = 16^\circ$, the velocities, sectional areas, &c., just found, will be slightly varied by the introduction of $\delta = 13\frac{3}{4}^\circ$. The efficiency of this wheel is:

$$\eta = \left(1 - (\phi^2 - \psi) \frac{v^2}{gh}\right) \frac{1}{\phi \sqrt{1+\kappa}} = \left(\frac{1 - 0,281 \cdot 178,5}{156,25}\right) \frac{1}{1,116 \cdot 1,049}$$

$= \frac{1 - 0,321}{1,17} = \frac{0,679}{1,17} = 0,578$, the co-efficient 7,906 being taken for Prussian measures, as in last example.

For the same fall, a Fourneyron's turbine gave an efficiency $\eta = 0,705$. (*See* example to last paragraph.)

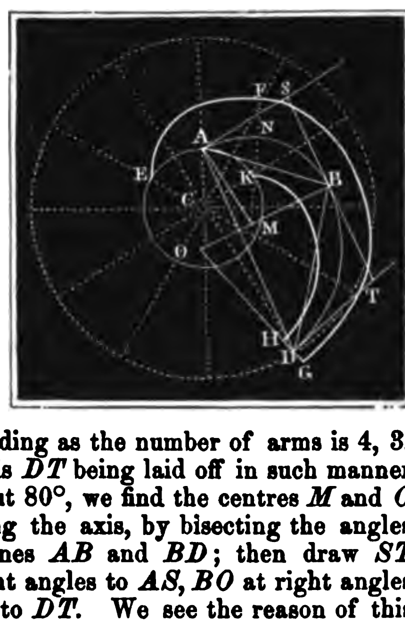
§ 160. *Whitelaw's Turbines.*—The Scottish turbine has to be treated differently from that of Cadiat, inasmuch as the water enters the wheel, in great measure, in a manner involving *shock*, and because in these turbines the dimensions and form of the wheel-channels are much more arbitrary than for the other. The angle δ may be made much less in these than in the other forms of turbine. They are peculiarly adapted for falls of great height, with small supply of water.

The width of the pressure pipe may be determined by the condition that there shall be a maximum velocity of 6 feet per second through it. So that the internal radius of the wheel, or the radius of the pressure pipe $r_1 = \sqrt{\frac{Q}{6 \pi}} = 0,23 \sqrt{Q}$. The external radius is made 2, 3, or 4 times this, according as the number of *arms* or discharge channels is 4, 3, or 2. The velocities v, v_1, and c, and, therefore, the sections F_1 and F_2, may be determined as in the case of turbines without guide-curves (last paragraph). The depth or height of the wheel $e = \frac{F}{2 \pi r_1}$, and the width of the orifices of discharge $= d = \frac{F_2}{n e}$.
In determining v or $\chi = \frac{v^2}{2gh}$, it will be necessary to take ζ higher than 0,15, as shock cannot be avoided where the stream of water

divides itself to run in so many different directions. It may be assumed, without much risk of error, that $\zeta = 0,20$. As the arms or wheel channels are considerably longer than in any other turbine, we must take a higher value for κ, and assume it at least 0,15.

The arms are generally curved according to the Archimedian spiral; they may be curved to the form ABD, Fig. 272, composed of two circular arcs, AB and BD. For this, the orifice of the inlet pipe, or internal periphery of the wheel, is divided into as many equal parts as there are to be arms—three in the case, Fig. 273, and from each of these draw the line AS, making the angle β with the tangent at that point; or, for example, make $SAC = 270^\circ - \beta^\circ = 90^\circ + \beta^\circ$, then with the external radius r, describe a circle, and divide it into as many equal parts as there are to be arms, but so that between the two points A and D of the two peripheries, a central angle of about 135°, 150°, or 180° is included, according as the number of arms is 4, 3, or 2. The direction of the axis DT being laid off in such manner that the angle CDT equals about 80°, we find the centres M and O of the arcs AB and BD forming the axis, by bisecting the angles SAD, TDA by the straight lines AB and BD; then draw ST parallel to AD, and AM at right angles to AS, BO at right angles to ST, and DO at right angles to DT. We see the reason of this at once, if we consider that, by division of the angles SAD and TDA, and by drawing the parallel ST, the angles MBA and MAB, and also the straight lines MA and MB, are made equal to each other, that in like manner the angles ODB and OBD, as also the lines OB and OD, are made equal to each other.

Fig. 273.



To find the outsides of the pipes, DG is made $= DH =$ the half width of orifice $\frac{d}{2}$, and FN is made $= KN$, and the arcs HK and GF are drawn, so that the width GH gradually passes into FK, &c.

Example. Required to design a Scottish turbine for a fall of 150 feet, with a supply of water of 1½ cubic feet per second. In the first place, the internal radius $r_1 = 0,23\sqrt{Q} = 0,23\sqrt{1,5} = 0,282$ feet; but we shall put it $= 0,3$ feet, and the diameter of the pressure pipe 9 inches, or 0,75 feet; we shall have only two arms, and make the external radius $r = 4 \cdot r_1 = 1,2$ feet. We shall put $\beta = 150^\circ$, and $\delta = 10^\circ$, and assume $\kappa = 0,15$, and $\zeta = 0,25$, hence:

$\psi = 1 - 0,25 \cdot \left(\frac{r_1}{r}\right)^2 tang.\, \beta^2 = 1 - 0,25 \cdot \frac{1}{16} (tang.\, 30^\circ)^2 = 1 - 0,0052 = 0,9948$, and

$$\phi = \frac{\sqrt{1+\kappa}}{cos.\, \delta} = \frac{\sqrt{1,15}}{cos.\, 10^\circ} = 1,0890.$$

Of the fall $h = 150$, the friction of the water in the 9 inch pipe, which may be presumed to be 200 feet long, consumes, according to Vol. I. §§ 331 and 332, an amount:

$$z = 0,0213 \,.\, 0,016 \,.\, \left(\frac{4}{\pi}\right)^2 . \frac{lQ^2}{d^5} = 0,0003408 \,.\, \left(\frac{4}{\pi}\right)^2 . \frac{200 \,.\, 1,5^2}{0,75^5}$$

$= 0,0003408 \,.\, 1,621 \,.\, \frac{200 \,.\, 256}{27} = 0,03408 \,.\, 1,621 \,.\, \frac{512}{27} = 1,05$ feet; therefore, we must only introduce $h = 148,95$ feet in our calculations. For the most advantageous velocity:

$$\chi = \frac{\phi - \sqrt{\phi^2 - \psi}}{2\,\psi\,\sqrt{\phi^2 - \psi}} = \frac{1,089 - \sqrt{1,1858 - 0,9948}}{1,9896\,\sqrt{0,191}} = \frac{1,089 - 0,437}{0,8695} = 0,750.$$

And hence:

$v = \sqrt{\chi \,.\, 2gh} = \sqrt{0,75 \,.\, 62,5 \,.\, 148,95} = 83,56$ feet, $v_1 = \frac{v}{4} = 20,89$ feet;

$c = -v_1 \, tang. \, \beta = 20,89 \, tang. \, 30° = 12,06$ feet; and

$$c_2 = \sqrt{\frac{2gh + \psi v^2}{1 + z}} = \sqrt{\frac{9309,4 + 6945,8}{1,15}} = 118,89 \text{ feet. (Prussian.)}$$

From this we have the sections: $F = \frac{Q}{c} = \frac{1,5}{12,06} = 0,1244$ square feet, and

$F_2 = \frac{Q}{c_2} = \frac{1,5}{118,82} = 0,01262$ square feet. From this again we have the height of wheel $e = \frac{F}{2\pi r_1} = \frac{0,1244}{0,6\,\pi} = 0,066$, and, lastly, the width of orifice:

$d = \frac{F_2}{n\,e} = \frac{0,01262}{2 \,.\, 0,066} = \frac{0,01262}{0,132} = 0,0956$ feet $= 1,15$ inches. In order to have a greater ratio $\frac{e}{d}$ between the sides of the orifices, we should have to introduce more arms or wheel channels; but as the channels are very long, even the above proportion would be found to insure a *full flow* through them. The efficiency of the wheel, neglecting the friction at the joint and losses in the pressure pipe, is:

$$\eta = \left[1 - (\phi^2 - \psi)\, 2\,\chi\right] \frac{1}{\phi\,\sqrt{1 + z}}$$

$$= (1 - 0,191 \,.\, 1,5)\,\frac{1}{1,089 \,.\, 1,07} = \frac{0,7135}{1,167} = 0,611.$$

§ 161. *Comparison of Turbines.*—Let us now draw a comparison between the three turbines of Fourneyron, Cadiat, and Whitelaw. The turbine with guide-curves is unquestionably the more perfect construction, mechanically considered, as by this arrangement (when $c_2 = v$) the entire *vis viva* of the water may be taken from it, which cannot be done without this apparatus. All things considered, the velocity of rotation for all the wheels is nearly the same, viz.: $r_3\, v = 0,7\,\sqrt{2gh}$ to $\sqrt{2gh}$ for the maximum effect. This maximum effect is nearly the same for each of them, the advantage being on the side of Fourneyron's wheels, when working in its normal state, and on the side of Whitelaw's, when the supply of water is very variable. The Scottish turbine may be constructed at less cost than Fourneyron's turbines with guide-curves.

In general terms, we believe that the turbines of Fourneyron and Cadiat are better adapted for very low falls and those of moderate height (up to 30 feet) with large supplies of water, whilst for high falls and small supplies of water, Whitelaw's wheels are to be preferred.

Remark. In the case of turbines without guide-curves, especially when the fall is high, the water leaving the wheel retains a considerable absolute velocity $w = c_2 - v$, and hence a notable amount of *vis viva* is lost. This loss may be avoided, or at least much diminished, if the *vis viva* of the water leaving the turbine be applied to a second wheel. M. Althans, of the Sain Iron Works, has put this into practice at a mill near Ehrenbreitstein. The essential part of the construction of this wheel is represented by Fig. 274. *AEA* is a reaction wheel with four curved discharge pipes, the fall being 120 feet (compare § 147), *BB* is a larger wheel with curved buckets, set in motion by the water discharged at *A*, *A*. As the wheels revolve in opposite directions, they have to be connected with each other by reversing gearing. The outer wheel has this further advantage, that it adds to the fly, or regulating power of the machine.* (*See* "Inner-österreichisches Gewerbeblatt," Jahrgang 5, 1843.)

Fig. 274.

§ 162. *Experiments on Turbines.*—Numberless experiments on turbines of the different forms we have now been discussing are extant, but the reported results are not all trustworthy. These recipients of water power are in many respects admirable machines, but to suppose that an efficiency = 0,85 to 0,90 has been obtained from them, arises from some mistake. As the discharge of water through the most perfectly formed orifice has a velocity co-efficient $\phi = 0,97$ (Vol. I. § 312), there must be a loss of mechanical effect at entering the wheel, represented by $\left(\frac{1}{\phi^2} - 1\right)\frac{c^2}{2g} Q\gamma = 0,06 \frac{c^2}{2g} Q\gamma$. As, again, the friction of water in a pipe six times as long as it is wide, consumes (Vol. I. § 331) $0,019 \,.\, 6 \,.\, \frac{v^2}{2g} Q\gamma = 0,114 \frac{v^2}{2g} Q\gamma$, or 11,4 per cent. of the available fall $\left(\text{as } \frac{v^2}{2g} = \frac{c_2}{2g} \text{ nearly } = h_2\right)$, we see that, deducting these resistances, there remain only 88 per cent. of effect over. If we allow only 2 per cent. for the resistance in the curved conduits, 2 per cent. for shock on the ends of the buckets, and 3 per cent. for the mechanical effect retained by the water discharged, and neglecting all other sources of loss, such as is involved in the guide-curves, &c., there remain only 76 per cent. of useful effect, and, therefore, a turbine that gives us an efficiency $\eta = 0,75$, may be considered as a very excellent one. The experiments of Morin and other impartial persons give results as to efficiency as high as 0,75, but never above this.

Morin's experiments were published about ten years ago, under the title "Expériences sur les roues hydrauliques à axe vertical,

* [A second wheel to receive the water from a common Barker's mill was used in model by the Editor, to illustrate his lectures before the Franklin Institute, about the year 1830–31. The model is probably now at Carlisle, Pa.—Am. Ed.]

appelées Turbines, Metz et Paris, 1838." The first experiments were made on one of Fourneyron's turbines at Moussay, external diameter of wheel = 2,8 feet, depth = 0,36 feet, fall = 24,6 feet, and the quantity of water laid on = 26 cubic feet per second. Thus there was a fall of upwards of 70 horse power at disposition. The result of these experiments, stated in general terms, was that, whether the wheel worked more or less in back-water, it gave for 180 to 190 revolutions per minute, a maximum effect of 69 per cent. of the whole power. When the number of revolutions was greater or less by from 40 to 50 per cent. of the above, the efficiency was from 7 to 8 per cent. less. These were the results when the cylindrical sluice was quite drawn up; but when the sluice was lowered to half the height of the wheel, the efficiency was reduced about 8 per cent. Had the wheel been entirely free of back-water, this falling off in efficiency must have been greater.

Experiments on a turbine at Mühlbach for a fall of 120 horse power, gave the following results: diameter of wheel 2 metres, height ½ metre, fall 12 feet, with 86 cubic feet of water per second. With the sluice quite drawn, the wheel made 50 to 60 revolutions, and the efficiency was 0,78 according to Morin; but he has adopted too low a co-efficient of discharge in calculating the quantity of water, and, therefore, 0,75 is probably the true efficiency. For variations of from 30 to 80 revolutions, the efficiency did not vary more than 4 per cent. from the above. The efficiency was the same whether the wheel was only a few inches, or 3 feet under water. The efficiency was nearly constant for great variations in the quantity of water laid on. As the sluice was depressed, the efficiency fell off rapidly. Morin directed experiments to ascertaining the ratio $\frac{v}{\sqrt{2gh}}$, and found, as theory indicates, that this ratio increases as v increases (owing to the influence of centrifugal force), and decreases as the sluice is raised.

§ 163. Redtenbacher gives the result of some experiments on turbines in Switzerland, in his work "Ueber die Theorie und den Bau der Turbinen und Ventilatoren." These were ill constructed, and gave low results.

Among other interesting results which Redtenbacher deduces from the recorded experiments on Fourneyron's turbines, we may particularly mention that these wheels, when working with their maximum effect, and with sluice fully drawn, make half the number of revolutions that they do when working free of all load but their own inherent resistances.

Combes' experiments, with models of his wheels, give less efficiency than those above mentioned, viz: 0,51 to 0,56.

Mr. Ellwood Morris, of Philadelphia, has recorded a very complete set of experiments on two turbines of Fourneyron, (*see* "Journal of the Franklin Institute," Dec. 1843.)

One wheel was 4¾ feet diameter, 8 inches high, 6 feet fall, 1700

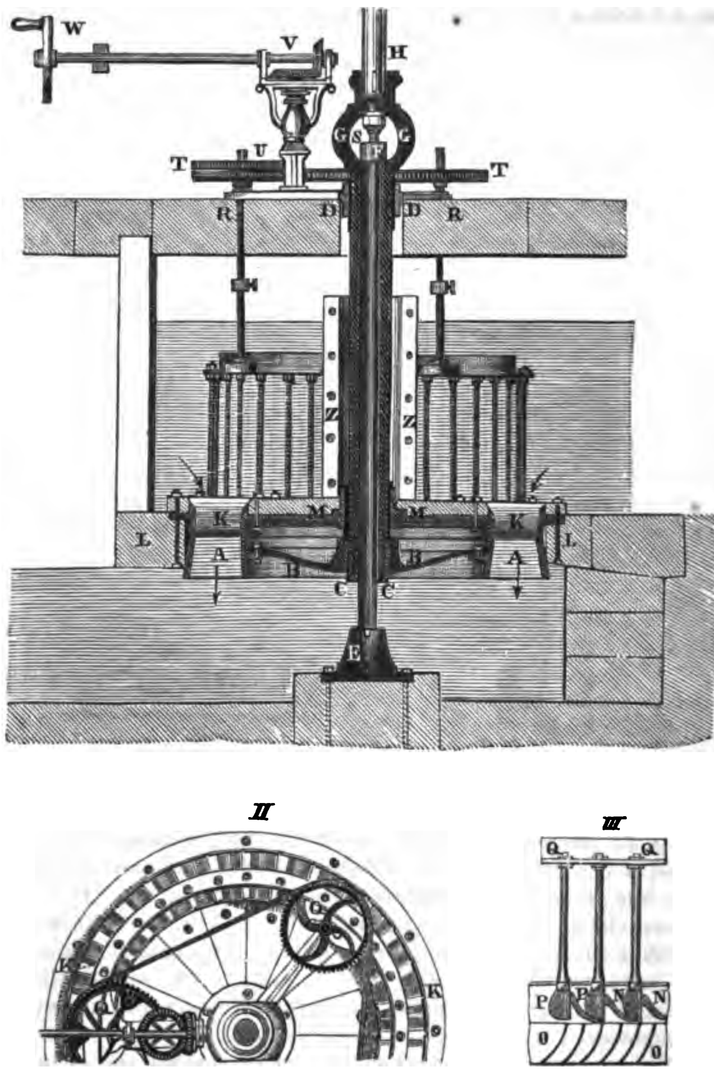
cubic feet of water per minute. Sluice drawn 6 inches, 52 revolutions per minute, efficiency found to be 0,7. The velocity v_1 of the inner periphery of the wheel was then $= 0,46 \sqrt{2gh}$. For variations between $v_1 = 0,5 \sqrt{2gh}$ to $0,9 \sqrt{2gh}$, the value of η varied from 0,64 to 0,70. The other wheel was 4′ — 5″ diameter, 6 inches deep, 4½ feet fall, 14 cubic feet per second. It revolved *under* water, and when the sluice was drawn 4½ inches, the effects were as follows: For $v_1 = 25$ to 30 per cent. of $\sqrt{2gh}$, then $\eta = 0,71$. For $\frac{v_1}{\sqrt{2gh}} = 0,45$, that is $u = 49$, the maximum effect was obtained, or $\eta = 0,75$. For $\frac{v_1}{\sqrt{2gh}} = 0,5$ to 0,7, the value of $\eta = 0,60$.

Remark. The results of experiments on Cadiat's wheels are probably overstated. Experiments on Whitelaw's wheels, as made by Messrs. Randolph and Co., of Glasgow, give results varying from 0,60 to 0,75 for the efficiency.

§ 164. *Fontaine's Turbine.*—The turbines recently introduced by Fontaine and Jonval, differ from those of Fourneyron, inasmuch as the guide-curves, instead of being in one place with, are placed *above* the wheel, and thus the water does not flow *outwards* on to the wheel, but from above downwards, and is discharged from the bottom of the wheel. Centrifugal force plays only a very subordinate part in the motion of the water through these wheels, gravity taking its place. The difference between the turbine of Fontaine and that of Jonval consists in the former being placed immediately on or under the surface of the water in the race; while, in Jonval's arrangement, the water flowing through the wheel forms a column of water *under the wheel*, but acts upon the wheel just as if it pressed upon it. The arrangement of Fontaine's turbine is shown in Fig. 275, in vertical section and in plan. *AA* is the wheel, *BB* is the wheel plate, uniting the wheel with the hollow axle *CCDD*. In order that the pivot may be out of the water, the axle *CD* ends in an eye *GG*, in which there is a steel plug *FS* (which can be raised or depressed by the screw *S*) resting on the solid axle *EF* at *F*.

The motion of the wheel is transmitted by an axle *H*, firmly connected with the head of the hollow shaft. To keep the upright shaft from the water, it is surrounded by a casing, as in Fourneyron's turbines. The guide-curve apparatus *KK* is screwed on to the beams *LL*, and to it there is a plate *KMMK* united, having a cylindrical metallic bed *MM*, in which there is a collar similar to that at *DD*, for maintaining the perpendicularity and steadiness of the shaft. The form of the guide-curves *N*, and of the wheel-buckets *O*, is represented at III. For regulating the quantity of water laid on, there is a compound sluice, having as many separate valves as there are guide-curves. These valves are covered by round pieces of wood, and are fastened by screws and nuts to the cylindrical casing of the guide-curve apparatus. The sluice-rods *PQ*, *PQ* . . . are firmly united to each other by an iron ring *QQ*, which can be

Fig. 275.

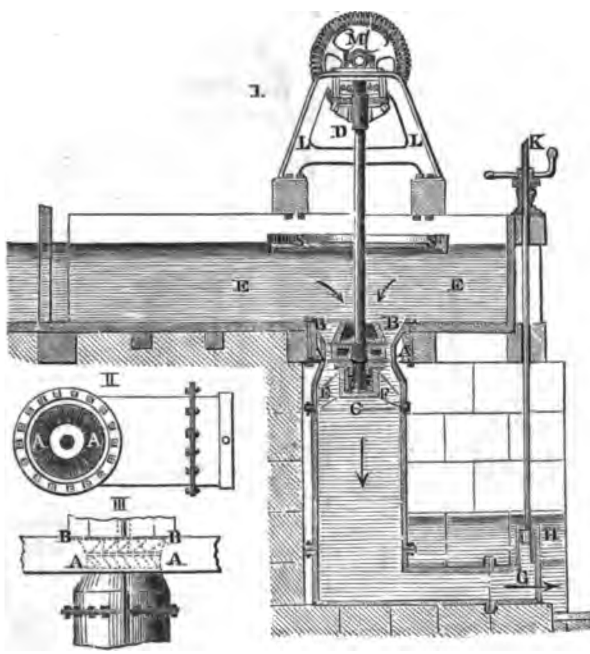


raised and depressed by three lifting-rods *QR*, *QR* . . . For this purpose the ends *R*, *R* . . . of these rods are cut as screws, and toothed wheels *T*, *T* . . . put on them, the nave or box of these having female screws, and the peripheries of the whole being encompassed by an endless chain.

When one of these wheels is moved by means of a winch, or otherwise, it is evident that the rest must be so too, so that the three rods are moved simultaneously.

§ 165. *Jonval's Turbine.*—Figs. 276 and 277, I, II, and III, represent Jonval's turbine. Here, again, *AA* is the wheel, united to

Fig. 276.



the upright shaft *CD* by a disc or plate; *BB* is the guide-curve apparatus opening as a diverging cone into the lead. The pivot rests on a footstep *C*, supported by *EE*. The relative position of the wheel and guide-curves, as also a part of the outside of the pipe in which the wheel is enclosed, is represented at II and III.

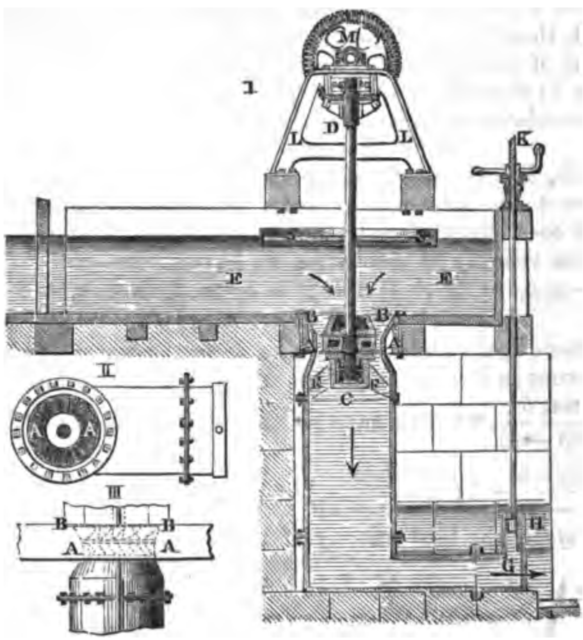
To keep the surface of the water in the lead free from agitation, a float *SS* is placed on it, and for regulating the wheel's motion, a sluice *G* is introduced, worked by a handle at *K*. According as this sluice is raised or depressed, more or less water flows away, and thus the power is regulated.

The framing *LL* supports the plumber-blocks for the upper end of the shaft, and for a horizontal shaft, through which the motion is first transmitted by a pair of mitre wheels. When the wheels are small, the reservoir or well in which the wheel is enclosed may be of cast iron; for large wheels it should be built of solid masonry.

It is evident, from what we have now detailed, that the turbines of Fontaine and Jonval are essentially alike in their main proportions, and that their *theory* is the same. In both, the water in the lead stands at a certain height h_1 above the point of entrance on the wheel. The water in the race, however, stands in Jonval's turbine

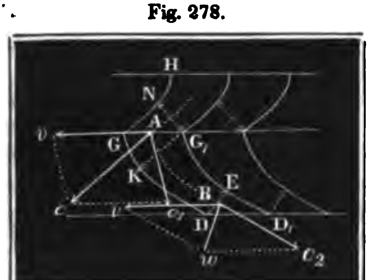
at a certain depth h_2 *below* the wheel, while, in Fontaine's arrangement, the race-water is *in immediate contact with* the wheel.* The regulation of the wheel's motion is managed in Fontaine's by an

Fig. 277.



internal, and in Jonval's by an external sluice — the one being analogous in this respect to Fourneyron's turbine, the other to that of Cadiat.

Fig. 278.



§ 166. ***Theory of Fontaine's and Jonval's Turbines.***—In developing the theory of the turbines of Fontaine and Jonval, we shall adopt the following symbols.

Let the angle of inclination of a guide-curve NG to the horizon, or the angle of entrance of the water $NGG_1 = cAv$, Fig 278, $= \alpha$. The angle $c_1 Av$ which the upper end of the wheel bucket A makes with the motion of the wheel $= \beta$, and the angle DD_1E, at which the bottom of the wheel bucket meets the horizontal $= \delta$. Let the absolute velocity of entrance of the water on the wheel $Ac = c$, the mean radius of the wheel $r = \frac{r_1 + r_2}{2}$, corresponding to the velocity of the wheel $Av = v$. The relative velocity of entrance $Ac_1 = c_1$, and the velo-

city of discharge $Bc_2 = c_2$. Again, let $F =$ the sum of the areas of all the sections NG_1 of the water flowing out of the guide-curve apparatus, F_1 the sum of the upper sections G_1K, and F_2 the sum of the lower sections DE of the wheel channels.

If, again, ζ be the co-efficient of resistance in the guide-curve canals, and x the head measuring the pressure of the water entering the wheel, then $(1 + \zeta)\, c^2 = 2g\,(h_1 - x)$; and reckoning the height a (34 feet) of a column of water equal to the atmospheric pressure, then $(1 + \zeta)\, c^2 = 2g\,(a + h_1 - x)$.

For the relative velocity, we have:

$$c_1^{\,2} = c^2 + v^2 - 2\,c\,v\,cos.\,\alpha.$$

If, again, $b =$ the depth of the wheel, $y =$ the height of a column of water $=$ to the pressure of water immediately under the wheel, and κ the co-efficient of resistance in the wheel channels, then, for the relative velocity of discharge, we have:

$$(1 + \kappa)\, c_2^{\,2} = 2g\,(b + x - y) + c_1^{\,2} = 2g\,(a + h_1 + b - y) + v^2 - 2\,c\,v\,cos.\,\alpha - \zeta\, c^2$$

If we here again endeavor to take from the water as much effect as is inherent in it, and, therefore, make $c_2 = v$, and also $c = \frac{v\, sin.\,\beta}{sin.\,(\beta - \kappa)}$, we then have for the relative velocity of discharge:

$$\left[2\,\frac{sin.\,\beta\; cos.\,\alpha}{sin.\,(\beta - \alpha)} + \zeta\left(\frac{sin.\,\beta}{sin.\,(\beta - \alpha)}\right)^2 + \kappa\right] v^2 = 2g\,(a + h_1 + b - y),$$

and, therefore, the best velocity of the wheel:

$$v = \sqrt{\frac{2g\,(a + h_1 + b - y)}{2\,\frac{sin.\,\beta\; cos.\,\alpha}{sin.\,(\beta - \alpha)} + \zeta\left(\frac{sin.\,(\beta - \alpha)}{(sin.\,\beta - \alpha)}\right)^2 + \kappa}}.$$

The pressure-height y, when the turbine revolves in free air, is equal to the atmospheric pressure a; but when the turbine is in back water, it $= a + h_2$, where h_2 is the height of the surface of the water above the bottom of the wheel; and lastly, when the wheel is above the race water, as in Jonval's arrangement, $y = a - h_2 + z$, where $h_2 =$ the depth of the race surface underneath the bottom of the wheel, and z is the height due to the velocity of the water flowing through the sluice from the reservoir to the tail-race. The total fall for the case of the wheel revolving free of back water is $h = h_1 + b$; when the wheel is in back water, $h = h_1 + b - h_2$; and when the wheel is above the tail water, $h = h_1 + b + h_2$. Hence, for the two first cases:

$$v = \sqrt{\frac{2gh}{2\,\frac{sin.\,\beta\; cos.\,\alpha}{sin.\,(\beta - \alpha)} + \zeta\left(\frac{sin.\,\alpha}{sin.\,(\beta - \alpha)}\right)^2 + \kappa}}.$$

and, for the latter:

$$v = \sqrt{\frac{2g\,(h - z)}{2\,\frac{sin.\,\beta\; cos.\,\alpha}{sin.\,(\beta - \alpha)} + \zeta\left(\frac{sin.\,\beta}{sin.\,(\beta - \alpha)}\right)^2 + \kappa}},$$

and, when the orifice G by which the vessel communicates with the tail-race is large, or when the water flows away very slowly:

$$z = \frac{1}{2g}\left(\frac{Q}{G}\right)^2 = 0.$$

§ 167. From the velocity $v = c_2$, the absolute velocity of entrance $c = \frac{v \sin. \beta}{\sin. (\beta - \alpha)}$, and the pressure height:

$$x = a + h_1 - (1 + \zeta)\frac{c^2}{2g} = a + h_1 - (1 + \zeta)\frac{v^2 \sin. \beta^2}{2g \sin. (\beta - \alpha)^2}$$

may be calculated. Neglecting prejudicial resistances:

$$x = a + h_1 - \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)},$$

and neglecting the atmospheric pressure:

$$x = h_1 - \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)}.$$

$x = 0$, or more correctly $x =$ the external pressure of the atmosphere when $h_1 = \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)}$. The loss of water involved in the free space necessarily left, depends on the difference between the internal pressure (x), and the external pressure at this point, and is different in the two turbines now under consideration. That the water may flow on in a connected stream, x must never descend to 0, that is, we must have $a + h_1 > \frac{h \sin. \beta}{2 \cos. \alpha \sin. (\beta - \alpha)}$. Again, that the water may not recede from the bottom of the wheel, we must never have $y = 0$, that is, we must have:

$$a - h_2 + z > 0, \text{ or } h_2 < a + z, \text{ or } h_2 < a + \frac{1}{2g}\left(\frac{Q}{G}\right)^2.$$

Hence, when the area of the orifice G is large, we must have $h_2 < a$. From this we see that the height of the wheel above the surface of the tail-race must never reach to the water-barometric height of 34 feet.

If, in Jonval's turbine, the reservoir be high and narrow, so that the velocity of the water in it is considerable, there arise losses of effect at this point from friction, resistance in curves, impact, &c., &c. On this account, it is advisable to make the reservoir wide in proportion to the wheel's diameter.

§ 168. *The Mechanical Effect of Fontaine's and Jonval's Turbines.*—The effect of these turbines may be deduced exactly as that of Fourneyron's has been, by subtracting from the total mechanical effect of the fall $Q h \gamma$, the effect consumed by different prejudicial resistances, &c. The loss in the guide-curve apparatus $L_1 = \zeta \cdot \frac{c^2}{2g} Q \gamma$, and that in the wheel-channel $L_2 = \varkappa \frac{c^2}{2g} Q \gamma$. Again, the loss of the *vis viva* retained by the water at its exit from the wheel, is

$$= \frac{w^2}{2g} Q\gamma = \frac{\left(2\, v\, sin. \frac{\delta}{2}\right)^2}{2g} Q\gamma.$$

In Jonval's turbines, there has to be added to these the loss of effect involved in the velocity of discharge (w_1) through the sluice

$$= \frac{w_1^2}{2g} Q\gamma = \frac{1}{2g} \cdot \frac{Q^3}{G^2}\gamma.$$

Hence the total effect of the wheel:

$$L = \left(h - \left[\zeta\, c^2 + \varkappa\, c_2^2 + \left(2\, v\, sin. \frac{\delta}{2}\right)^2 + w_1^2\right] \cdot \frac{1}{2g}\right) Q\gamma.$$

We see from this that the loss of effect increases as the angle δ increases, and as the velocity w_1 is greater, or as the velocity of discharge and sluice-opening G are less.

When the sluice is fully drawn, and the reservoir is wide, w_1 may be assumed $= 0$. Hence, in Jonval's turbine, the efficiency decreases as the quantity of water diminishes, or as the sluice is lowered. In Fontaine's turbines, the same relative effects are produced for different positions of the sluice as in Fourneyron's turbines. It appears, therefore, that the efficiency of the turbines now under discussion, cannot be much more or less than that of Fourneyron's in the same circumstances. Experiments, hereafter cited, confirm this.

§ 169. *Construction of Fontaine's and Jonval's Turbines.*—We have now to determine the general rules for the proportions and construction of these wheels.

The angles β and δ of the wheel-buckets are taken arbitrarily—the latter, however, as small as possible, *i. e.*, 15° to 20°, and $\beta = 100°$ to 110°. From these we have the guide-curve angle α, if, for the sake of preventing all impact at entrance of the water, we put

$$c_1\, sin.\, \beta = c_2\, sin.\, \delta = v\, sin.\, \delta, \text{ and } \frac{c_1}{v} = \frac{sin.\, \alpha}{sin.\, (\beta - \alpha)}.$$

Hence, by combination: $\frac{sin.\, \alpha}{sin.\, (\beta - \alpha)} = \frac{sin.\, \delta}{sin.\, \beta}$; and we have

$$\frac{sin.\, (\beta - \alpha)}{sin.\, \alpha\, sin.\, \beta} = \frac{1}{sin.\, \delta}, \text{ or } cotg.\, \alpha = cotg.\, \beta + \frac{1}{sin.\, \delta}.$$

From the angles α and β, we have the velocity of the wheel

$$v = \sqrt{\frac{2gh}{2\frac{sin.\, \beta\, cos.\, \alpha}{sin.\, (\beta - \alpha)} + \zeta\left(\frac{sin.\, \beta}{sin.\, (\alpha - \beta)}\right)^2 + \varkappa}},$$

and the velocity of entrance of the water $c = \frac{v\, sin.\, \beta}{sin.\, (\beta - \alpha)}$; and from this we have the sectional areas $F = \frac{Q}{c}$, and $F_2 = \frac{Q}{c}$.

The width of the wheel, or length of the buckets measured radially, must be made in suitable proportion (as small as possible),

$\nu = \frac{d}{r}$ to the mean radius of the wheel. In the turbines hitherto made $\nu = 0{,}3$ to $0{,}4$. This being done, we have:

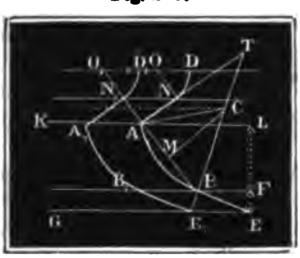
$$F = 2 \pi r d \sin. \alpha = 2 \pi . \nu r^2 \sin. \alpha,$$

and, therefore, $r = \sqrt{\frac{F}{2 \pi \nu \sin. \alpha}}$, and $d = \nu r$.

If we further assume a proportion $\psi = \frac{e}{d}$ of the width of the orifice measured at the mean circumference of the wheel to the length of the buckets (in existing turbines this is $\frac{1}{2}$), we have $e = \psi d$, and, hence, the number of buckets $n = \frac{2 \pi r \sin. \alpha}{e} = \frac{F}{de}$. The height of the wheel b is made about the same as the width d.

§ 170. *Construction of the Buckets.*—The buckets are surfaces of double curvature, the generatrix of which passes, on the one hand, through the axis at right angles, and on the other through a leading line which we may suppose drawn on a cylinder of the mean radius r. As by developing a cylinder as a plane, a rectangular surface is produced, lines may be drawn on this surface, which, when the right angle is round the cylinder, will serve as the leading line for the bucket surfaces. These developed leading lines may, however, be constructed of arcs and their tangents. If KL, Fig. 279, be the developed circle, in which the wheel and guide-curve apparatus meet, the line AND of the guide-curve, may be found by making $AA_1 = \frac{2 \pi r}{n}$, and by drawing AN, $A_1N_1 \ldots$, so that the angle of inclination, $NAL = N_1A_1L \ldots = \alpha$. Again, let fall AO perpendicular to A_1N_1. Draw a line parallel to KL at a distance equal to the height of the guide-curves. From the point O where the perpendicular AO intersects this line, describe the arc N_1D_1, and similarly, from another point O, the arc ND, &c. &c., then AND, $A_1N_1D_1$ are the *developed* guide lines of the guide-curves. To find the guide lines of the wheel buckets, draw at the distance $EL = b$ the height of the wheel, the line EG parallel to KL, make $EE_1 = \frac{2 \pi r}{n}$, and draw the straight lines EB, E_1B_1, &c., so that the angle $BEG = B_1E_1G$ becomes equal to the angle of discharge δ. Again, let fall E_1B perpendicular to BE, and lay off AB, so that the angle $ABC = \frac{\beta + \delta}{2}$. If, lastly, from the centre M of the line AB, there be raised the perpendicular MC, this cuts BT at the centre C of the arc AB, forming the upper part of the developed

Fig. 279.



guide line of a wheel bucket, whilst the straight lines BE, B_1E_1, &c., form the lower part.

It is evident that this construction of the guide and wheel curves, insures that water leaves them with the sections AN_1 and BE_1 respectively.

Example. It is required to give the leading dimensions and proportions of a Jonval's turbine for a fall of 12 feet in height, with 8 cubic feet of water per second. Assuming $\delta = 20^\circ$, and $\beta = 105^\circ$, we have for the angle of the guide-curves:

$$cotg.\ \alpha = cotg.\ \beta + \frac{1}{sin.\ \delta} = cotg.\ 105^\circ + \frac{1}{sin.\ 20^\circ} = -\ 0{,}26795 + 2{,}92380 = 2{,}65585,$$

and, therefore, $\alpha = 20^\circ, 38'$. Assuming $\zeta = 0{,}15$, and $\varkappa = 0{,}10$, the best velocity for the wheel:

$$v = \sqrt{\frac{2gh}{2\frac{sin.\ \beta\ cos.\ \alpha}{sin.\ (\beta - \alpha)} + \zeta\left(\frac{sin.\ \beta}{sin.\ (\beta - \alpha)}\right)^2 + \varkappa}} = \frac{8.02\sqrt{12}}{\sqrt{1{,}813 + 0{,}1407 + 0{,}1000}}$$

$= \frac{27{,}78}{\sqrt{2{,}0537}} = 19{,}38$ feet, and from this we have the velocity of entrance:

$$c = \frac{v\ sin.\ \beta}{sin.\ (\beta - \alpha)} = 18{,}77 \text{ feet.}$$

The sections $F = \frac{Q}{c} = \frac{8}{18{,}77} = 0{,}4262$ square feet, and $F_2 = \frac{Q}{v} = \frac{8}{19{,}38} = 0{,}4127$ square feet, and if we take the ratio $\nu = \frac{d}{r} = \frac{1}{3}$, the mean radius:

$r = \sqrt{\frac{F}{2\pi\nu\ sin.\ \alpha}} = \sqrt{\frac{0{,}4262}{\frac{2}{3}\pi\ sin.\ 20^\circ, 38'}} = 0{,}7598$ feet, and the width of wheel

$d = \nu\ .\ r = \frac{0{,}7598}{3} = 0{,}2532$ feet. From the space occupied by the buckets, each of these calculated dimensions should be somewhat increased.

The width of the channels $e = \frac{1}{2}\ d = 0{,}1266$ feet, and $\therefore n$ the number of buckets $= \frac{F}{d\,e} = \frac{0{,}4262}{0{,}2532\ .\ 0{,}1266} = \frac{0{,}4262}{0{,}032} = 13{,}29$, for which we may, however, adopt 16.

The height of the wheel b is made $= d = 0{,}2532$. The radius of the reservoir may be made somewhat greater than $r + \frac{d}{2} = 0{,}7598 + 0{,}1266 = 0{,}8864$, or about 1 foot, and hence the area of it will be $\pi = 3{,}1416$ square feet. The velocity:

$$w_1 = \frac{Q}{\pi} = \frac{8}{3{,}1416} = 2{,}546 \text{ feet, and the height due to this velocity:}$$

$z = 0{,}0155\ .\ 2{,}546^2 = 0{,}1005$ feet. The effect of this wheel, when the sluice is completely drawn, would be:

$$L = \left(h - \left[\zeta\ c^2 + \varkappa\ c_1^2 + \left(2\,v\,sin.\,\frac{\delta}{2}\right)^2 + w_1^2\right] . \frac{1}{2g}\right) Q\gamma$$

$= (12 - [0{,}15\ .\ 18{,}77^2 + 0{,}10\ .\ 19{,}38^2 + (2\ .\ 19{,}38\ sin.\ 10^\circ)^2 + 2{,}546^2]\ .\ 0{,}0155)\ 8\ .\ 62{,}5$ $= (12 - 103{,}66\ .\ 0{,}0155)\ .\ 500 = 10{,}39 \times 500 = 5195$ ft. lbs., $= 9{,}4$ horse power. The losses of effect in the reservoir would reduce this to 4800 ft. lbs., so that the efficiency would be something near 0,80; for as the power expended is $62{,}5 \times 8 \times 12 = 6000$ ft. lbs., $4800 \div 6000 = {,}80$. These calculations are for English measures.

§ 171. *Experiments on Fontaine's and Jonval's Turbines.*—Very trustworthy experiments on these wheels are detailed in the "Comptes Rendues de l'Académie des Sciences à Paris, 1846." There are also some earlier experiments by MM. Alcan and Grouvelle. (*See* Bulletin de la Société d'Encouragement, tome xliv.")

These experiments show that in Fontaine's turbines, as in Fourneyron's, the efficiency is greatest when the sluice is quite drawn up, and that the efficiency is less affected by variations of *head*,

than by variations in the quantity of water supplied. The turbine at Vadeney, near Chalons-sur-Marne, the efficiency of which was determined by Alcan and Grouvelle, has 1,6 metres (5,24 feet) external diameter, 0,12 metres (nearly 5 inches) in height, the fall was $5\frac{1}{4}$ feet, the quantity of water about 98 gallons per second. The principal result of this experiment was that for $u = 30$ to 50 per minute, the efficiency was 0,67. One of Fourneyron's wheels of an early date, made for the same fall, gave $\eta = 0,60$. Morin's experiments were made on a turbine for a powder mill, at Bouchet. The diameter was 1,2 metres, the width 0,25 metres. There were 24 guide-curves and 58 wheel buckets. It had a fall of about 12 metres, and 6 cubic feet per second supply. Experiments were made with 2, 3, and 4 inches of the sluice drawn, and the following results obtained. Sluice quite open $u = 45$, the efficiency a maximum, and $= 0,69$ to 0,70.

When the sluice was shut so as to reduce the expenditure by $\frac{1}{4}$, η was reduced to 0,57. The efficiency varied little with the velocity of the wheel, for when making 35 revolutions per minute, η was still $= 0,64$; and for 55 revolutions, $\eta = 66$. It appears, too, that the greatest power exerted, and at which the wheel moved irregularly, was about $1\frac{1}{2}$ times that with which the wheel produced its maximum effect. The wheel was a few inches in back water during the experiments. We see from these experiments, that Fontaine's turbine may be considered among the first-class of hydraulic wheels. The circumstance of the pivot being out of water is an advantage (though obtained at considerable expense, and by a method inapplicable to large machines). The "graissage atmosphérique" of Decker and Laurent accomplishes the same end, the lower end of the upright shaft being surrounded by a bell, analogous to a diving-bell, which revolves with it. The air in the bell is kept of the necessary density by a small air-pump.

§ 172. *Jonval's Turbines.* — The experiments on Jonval's turbines gave equally favorable results as those on Fontaine's. Messrs. Köchlin and Co. have detailed experiments on one constructed by them at Mühlhausen, in the "Bulletin de la Société Industr. de Mülhause, 1844." This turbine was 3,1 feet in diameter, 8 inches high. It was placed 2′ — 8″ under the surface of the water in the lead, the fall being, however, $5\frac{1}{4}$ feet, and the supply being 125 gallons per second. The efficiency for $u = 73$ to 95 per minute was 0,75 to 0,90. Morin considers, however, that the quantity of supply was reckoned too low, and that, therefore, this high efficiency must be reduced from 0,63 to 0,71.

Colonel Morin made experiments with a turbine of 0,81 metres external diameter, 0,12 metres internal width, 18 buckets, fall $5\frac{1}{4}$ feet; supply 45 to 65 gallons per second. Morin comes to the following conclusions from all his experiments. In the normal state, the water having impeded entrance and exit, the number of revolutions was 90 per minute and $\eta = 0,72$. By putting contracting

pieces on the wheel, the efficiency did not become much less (0,63) until the section was very considerably diminished.

The efficiency did not vary for variations of velocity 25 per cent. above and below that for the maximum effect. By depressing the sluice, the efficiency was diminished, so that it is evidently a very imperfect *regulator* for the wheel. When the section of the aperture for the discharge of the water was reduced to 0,4 of that for the normal condition, η was reduced to 0,625.

Redtenbacher gives some experiments on a turbine of Jonval's, the maximum efficiency for the sluice fully drawn having been $= 0,62$. As in the case of Fourneyron's turbines, these experiments indicate that the wheel working without load makes about twice as many revolutions as when furnishing its maximum effect in its normal state.

§ 173. *Comparison of different Turbines with each other.*—If we compare the turbines of Fontaine and Jonval with those of Fourneyron, we find that in Fontaine's turbines the water is less deviated from its original direction of motion than in Fourneyron's, so that for the same velocity of entrance the resistance is less in the one than in the other. Thus the velocity of entrance in Fontaine's wheel may be made greater, and, therefore, the wheel may be made less in diameter than Fourneyron's. The guide-curves of Fontaine's wheels take on the water in more nearly parallel layers than they do in Fourneyron's wheels, where a divergence of the stream entering the wheel cannot possibly be avoided.

On the other hand, Fourneyron's wheels have certain advantages. The pressure on the pivot is reduced to the *weight* of the machine in motion; whilst in Fontaine's, the *whole weight of water* is borne by the pivot, thus involving greater friction, *cæteris paribus*. Again, in Fourneyron's turbines, the particles of water move with the same velocity of rotation, which is not the case in the newer turbine, in which the velocity of the outer particles is much greater than that of the inner. This gives rise to eddying motions, consuming mechanical effect, and causing irregularities in the motion of the water through the wheel. The turbine of Fourneyron is also more easily constructed than that of Fontaine, particularly the buckets.

Remark 1. The Fontaine turbines are well adapted for *tide-mills*.

Remark 2. Jonval's turbines are considered to present advantages in respect of their being placed so that they can be easily got at. The limit at which they may be placed above the tail-race has been already pointed out to be 34 feet; but from experiments of M. Marasseau, and from certain theoretical considerations of Morin, it appears that the height of the turbine above the water in the race must not exceed even lower limits than the above, because otherwise, the water is very apt to *lose* its continuity immediately under the wheel, and thus *effect* is lost.

§ 174. *Comparison between Turbines and other Water Wheels.*—Turbines, from their nature, are applicable to falls of any height, from 1 to 500 feet. Vertical water wheels are limited in their applications to falls under 60 feet as the highest. The efficiency of turbines for very high falls is less than for smaller falls, on account of the hydraulic resistances involved, and which increase as the

square of the velocity. Vertical water wheels having from 20 to 40 feet fall, give a greater efficiency than any turbine. For falls of from 10 to 20 feet, they may be considered as being very nearly on a *par* in point of efficiency; and, for very low falls, turbines give a higher efficiency than any vertical wheel that could be substituted for them. Poncelet's wheels, for falls of from 8 *to* 6 *feet*, are on a *par* with turbines, but only within these limits. Turbines are unaffected by back-water, whilst vertical wheels lose effect in this condition. Variations in supply of water affect the efficiency of vertical water wheels less than they do that of turbines. This gives the vertical water wheel an hydraulic economical advantage, which is in some cases of great importance. When water becomes scarce, the best effect from what is available may *always* be depended upon from a good vertical wheel, whilst the turbine falls off in efficiency as its sluice is lowered, from causes which in our discussion of the theory of turbines we have fully explained.

§ 175. Variations of velocity on either side of the normal conditions, have the same result in the two kinds of wheels, but the turbines have a decided advantage, in that they make a greater number of revolutions per minute than any vertical wheels. The velocity of rotation is limited to from 4 to 8 feet per second, whilst in turbines this velocity, having a certain ratio to the height of fall, is generally much greater. The application of water to operations requiring great velocity, is, therefore, most advantageously made by turbines; whilst for operations requiring slow motions, the vertical wheel is to be preferred. It is a question of practical discretion, to decide as to whether it is better to reduce the velocity of turbines, or to raise the velocity of vertical wheels by means of the gearing that is to transmit their water-power to the work to be done.

For variable resistances, such as rolling mills, forge hammers, &c., the vertical wheel is certainly to be preferred, because its great mass serves better for regulating the motion than the smaller turbine, which for such work requires the addition of a fly-wheel.

In respect to economy of construction, turbines are at least as cheap as vertical wheels. When the fall is considerable and the quantity of water great, the turbine is the cheaper machine of the two. The turbine almost necessarily involves the use of iron in its construction, and hence cannot always be adopted. The durability, or the maintenance of a turbine, is probably less than that of a vertical wheel, *cæteris paribus*.

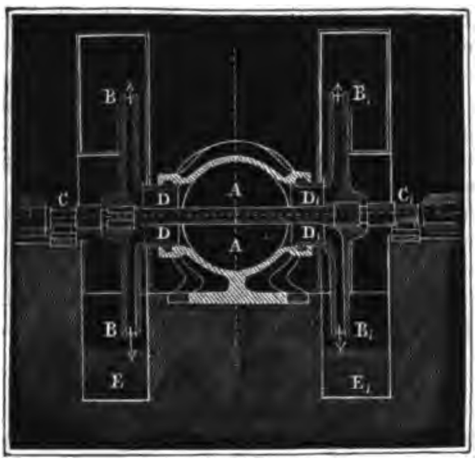
In respect to workmanship, it is manifest that the guide-curve turbines require greater *skill* than vertical water wheels do for their construction, with the same relative degree of perfection. Also, deviation from the scientific rules for their construction is of much more prejudicial consequence for turbines, than in the case of vertical wheels. This latter circumstance is the cause of the failure of many of the turbines that have been erected, and operates against their more general introduction.

Turbines, it must be borne in mind, require *clean* water to be laid on, for they would be greatly damaged by sand, mud, leaves, branches, ice, &c., passing through them, and their efficiency lessened. This is not the case with vertical wheels.

§ 176. *Turbines with Horizontal Axis.*—Examples of distorted ingenuity have been displayed in putting turbines, particularly Jonval's and Whitelaws', on horizontal axes. This mode of construction can never be advantageous, though it may have some local convenience suggesting its adoption.

Jonval and Redtenbacher have proposed the arrangement shown in Fig. 280. Where AA is the lead pipe, BB the one, and B_1B_1

Fig. 280.



the other wheel, CC_1 the horizontal axis, and DD and DD_1 the jointing-rings (Vol. II. § 151), E and E_1 being the tail-race.

A throttle valve in the main or lead pipe is the means of regulation.

Herr Schwamkrug, of Freyberg, has recently erected a vertical wheel, working on the principle of the pressure turbine. The wheel is like one of Poncelet's, but the water is introduced on the *inside* by a pipe, so that it flows through the wheel near the bottom of it. Fig. 281 shows the arrangement adopted.

Fig. 281.

The guide-curves DE, D_1E_1 are movable on centres, and serve to regulate the discharge of water. This construction has advantages in respect of the wheel being little exposed to the action of the water, and as the water acts on a very small arc, the wheel must have a greater diameter than a turbine, and hence in cases where slow motion is

required, may do away with the necessity of intermediate gear for reducing speed. But such a wheel would necessarily be more costly than a turbine, and its efficiency would certainly be less.

The same principle might be applied, as shown in elevation in Fig. 282, to a Fontaine's turbine. Such a machine is applicable for all falls, but never advantageously.

Before concluding this subject, we may add that Poncelet's turbines have been quite recently applied in Switzerland, under the name of tangential wheels.

Fig. 282.

Fig. 283.

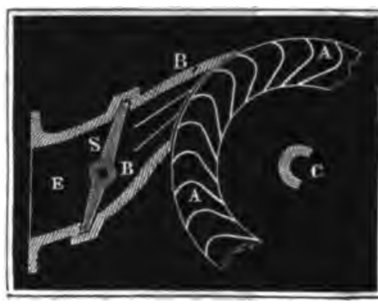


Fig. 283 represents a horizontal section of a part of one of these wheels, and the mode of laying on the water. S is the regulating sluice, in advance of which the lead is divided into three channels by guide plates. The water is discharged in the interior of the wheel in such manner that the pivot is protected from the water.

[Mr. Ellwood Morris, in the "Journal of the Franklin Institute," for November, 1842 (third series, vol. iv., p. 303), in discussing the advantages of Fourneyron's turbines, makes the following remarks: "In conclusion, the chief points of advantage promised by the use of turbines upon the mill seats of the United States, may be briefly summed up as follows:—

1. They act with perfect success in back-water.
2. They are not liable to obstruction from ice.
3. They require but little gearing to get up a high velocity at the working point.
4. They use to advantage every inch of fall.
5. They are equally applicable to very high and very low falls.
6. They are equal in power to the best overshot wheels.
7. They may vary greatly in velocity without losing power.
8. They are very compact and occupy but little room.
9. They may be very accurately regulated to an uniform speed.
10. They are perfectly simple, and not likely to get out of order.
11. They are not very expensive.
12. They are very durable.

"Upon one account or another," he adds, "the turbine is superior to all other water wheels, and consequently must be regarded as the very best hydraulic motors now known to mechanics."—Am. Ed.]

Literature. The literature on turbines has of late years become very extensive. We

have already mentioned several treatises and papers on the subject. The following are some of the more important works:—

Fourneyron's original paper appeared in the "Bulletin de la Société d'Encouragement, 1834." Morin's "Experimental Inquiry," already quoted, followed in 1838. In 1838, Poncelet published his "Théorie des Effets mécaniques de la Turbine Fourneyron," in the "Comptes Rendues," and as a separate treatise. In D'Aubuisson's "Hydraulique," the turbine is treated of, but only superficially. In 1843, Combes published, "Recherches théoretiques et experimentales sur les Roues à réaction ou à tuyaux," a tract of considerable importance, as it for the first time recognizes the necessity of taking into consideration the hydraulic resistances, which Poncelet and Redtenbacher have neglected to do. Redtenbacher's work, "Théorie und Bau der Turbinen und Ventilatoren, Manheim, 1844," is founded on Poncelet's theory, and is the best and most complete work on the subject. On the newer turbines, there appears in the "Comptes Rendues," tome xxii., 1846, "Rapport sur un Mémoire de M. M. A. Koechlin, concernant une nouvelle, Turbine (Jonval) construite dans leurs ateliers, par Poncelet, Piobert, et Morin." Also, "Note sur la Théorie de la Turbine de Koechlin, par Morin," et "Note sur l'Application de la Théorie du Mouvement des Fluides aux expériences de M. Marozeau, par Morin." In the "Comptes Rendues," &c., t. xxiii., 1846, there appears a paper "Expériences et Notes sur la Turbine de M. Fontaine-Baron, par Morin." The "Bulletin de la Société d'Encouragement, 1844–45," contains notices of the turbines of Jonval and Fontaine.

Armengaud's publication "Industrielle," contains good drawings and descriptions of the turbines of Cadiat, Callon. Fourneyron, and Gentilhomme. In the "Polytech. Centralblatt, bd. vii., 1846," Parro's turbine is described. Nagel's turbine is described in Dingler's "Journal, bd. xcv.," and Passot's turbine, in the same Journal, bd. xciv. Bourgeois' screw, is a *turbine-helice*, or with screw-formed channels. *See* "Polytechnisches Centralblatt, bd. i., 1847." In the "Proceedings of the Institution of Civil Engineers for 1842," there is a notice of turbines by Prof. Gordon. In the "Transactions of the Society of Arts of Scotland, 1805," there is a notice of a turbine erected at Mr. J. G. Stuart's flax-mill, at Balgonie, in Fifeshire. This is the first turbine erected in Britain, and is one of the largest ever made. Its efficiency is reckoned to be = 0,70.

CHAPTER VI.

WATER-PRESSURE ENGINES.

§ 177. *Water-Pressure Engines.*—Water-pressure engines, as their name indicates, are set in motion by a column of water. Their motion is a reciprocating rectilinear motion, and not rotatory as in the turbine. The leading features of a water-pressure engine are delineated in Fig. 284. *A* is a reservoir at the upper end of the pipe. *AB* is the *pressure pipe*. *C* is the working cylinder, in which the water moves the loaded *piston K*. In the pipe *BC*, by which the pressure pipe communicates with the cylinder, the regulating valve or cock is placed. It is here represented as a three-way cock, serving alternately to open and close the communication between the working cylinder and the pressure pipe. When the way is open, the water presses on the piston, and raises it, with its load, through a certain height—the *length of stroke*—when the communication between the pressure pipe and the cylinder is shut, a way is opened for the discharge of the water from the cylinder by the pipe *D*, and the piston then descends by its own gravity.

Fig. 284.

Water-pressure engines are either *single* or *double acting*. Fig. 284 shows the general arrangement of the single-acting engine, in which the piston is made to move in one direction by the pressure of the water, and to return by its own weight.

In the double-acting engine, the up stroke and down stroke, or both strokes of the piston, are made under the hydraulic pressure. Fig. 285 shows the general arrangement of a double-acting engine. The cock is in this case a four-way cock. In I. the pressure is on the upper side of the piston through *ABC*, and the discharge goes on through C_1B_1D. In II. the pressure is on the under side through AB_1C_1 of the piston, and the discharge through *CBD*.

Water-pressure engines are also made with two cylinders, each single-acting, but connected together, as in Fig. 285, so that while the one piston is ascending by the pressure of the water, the other is descending, the water being discharged therefrom. The relative

position of the passages in the four-way cock are shown in Figs. 286 and 287.

Fig. 285.

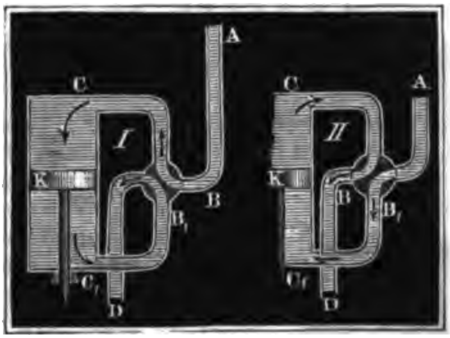


Fig. 286. I.

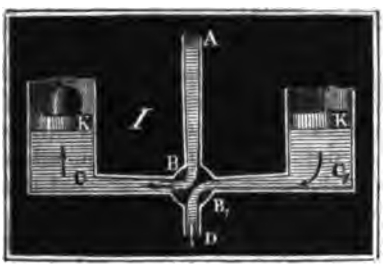
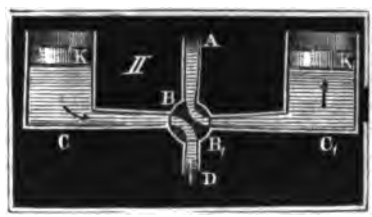


Fig. 287. II.

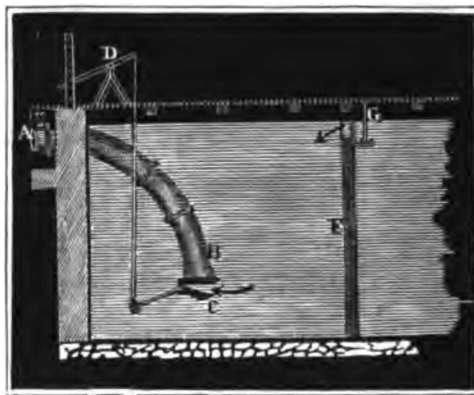


§ 178. *Pressure Pipes.* — The pressure pipes should take the water from a feeding cistern or settling reservoir, in which the water has time to deposit the foreign matters it may have carried so far along with it. In front of this a grating must be placed, to keep back leaves, ice, &c. &c.

The end of the pressure pipes should dip so as to be $1\frac{1}{2}$ foot, at least, above the bottom of the feed cistern, and 3 to 4 feet under the surface of the water in it, so as to prevent the influx of heavier particles, and to render the indraught of air impossible. For this object the end of the pipe may be conveniently curved with the mouth downwards, as shown in Fig. 288. *C* being a valve for shutting off the water from the pipe *B*, when required. *F* is a division plate in the cistern. *G* is a grating to keep back floating bodies. The pressure pipes may be either of wood or iron, but are usually of the latter material, and made from $\frac{1}{3}$ to $\frac{1}{2}$ the internal diameter of the working cylinder. The pipes for great heads are made to increase in thickness from the top downwards proportionally to the pressure. The formula: $e = 0{,}0025\, n\, d_1 + 0{,}66$ inches may be used for calculating the strength required for any given pressure n in atmospheres $= 33$ feet of water; d_1 being the internal diameter of the pipes. The formula given in Vol. I. § 283, is applicable to

ordinary water conduits, but is inapplicable to the present case, because the pressure of the water here varies frequently, and even acts with impact, when the valves are suddenly closed. The pipes

Fig. 288.

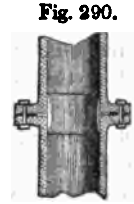


must be carefully *proved* by an hydraulic or Bramah press. The *porosity* of pipes, which at first proving is very sensible, gradually becomes insensible as oxidation goes on. In the case of the pipes for the pressure engine, at Huelgoat, described hereafter, boiled oil was used in proving the pipes, by which they become impregnated to a certain depth with the oil, and thus their porosity stopped, and even protection against corrosion insured.

The pressure pipes are usually jointed by flanches and screw-bolts; a ring of lead, or of *iron rust* being interposed, as shown in Figs. 289 and 290. A mixture of lime water, linseed oil, varnish,

Fig. 289.

Fig. 290.



and chopped flax, makes a very good pipe-joint. The spigot and faucet joint, with folding wedges of wood, make the best and cheapest joint for cast iron pipes.

§ 179. *The Working Cylinder.*—The working cylinder is made of cast iron or of gun metal. The number of strokes is limited to from 8 to 6 per minute, so that there may be the least possible loss of effect; and, therefore, the capacity of the cylinder is made to depend rather on its length than its diameter. The stroke s is made from 8 to 6 times the diameter d of the cylinder. The mean velocity v, of the piston, is usually 1 foot per second, in order that the

mean velocity v_1 of the water in the pressure pipes, and hence the hydraulic resistances may be as small as possible. It is not advisable in any case to have the latter velocity greater than 10 feet per second, and 6 feet is a better limit. If we assume $v = 1$, and $v_1 = 6$ feet, the quantity of water being: $\frac{\pi d^2 v}{4} = \frac{\pi d_1^2 v_1}{4}$, we get for the proportion of the diameter of the pressure pipe to that of the cylinder, $\frac{d_1}{d} = \sqrt{\frac{v}{v_1}} = \sqrt{\frac{1}{6}} = 0,408$, or about 0,4.

If Q be the quantity of water supplied, per second, then for a double-acting engine, or for a double-cylinder engine, $Q = \frac{\pi d^2}{4} . v$, and hence we have the diameter of the working cylinder required $d = \sqrt{\frac{4 Q}{\pi v}} = 1,13 \sqrt{\frac{Q}{v}}$, that is, for $v = 1$, $d = 1,13 \sqrt{Q}$ feet. For a single-cylinder, single-acting engine, $Q = \frac{1}{2} . \frac{\pi d^2}{4} v$ $\therefore$ $d = 1,60 \sqrt{\frac{Q}{v}}$, and if $v = 1$, $d = 1,60 \sqrt{Q}$ feet. If the stroke of the piston $= 3 d$ to $6 d$, the time for one stroke of a single-acting engine is $t = \frac{s}{v}$, or, if $v = 1$, $t = s$ in seconds, and hence the number of single strokes per minute:

$$n_1 = \frac{60''}{t} = \frac{60 . v}{s} \therefore \text{ when } v = 1,\ n_1 = \frac{60}{s},$$

and the number of double strokes:

$$n = \frac{n_1}{2} = \frac{30 v}{s}, \text{ or if } v = 1,\ n = \frac{30}{s}.$$

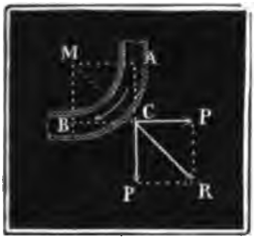
It is, however, better, in the case of a single acting, single cylinder, water-pressure engine, to begin the stroke somewhat more slowly, or to cause the descent of the piston to take place more rapidly than with the mean velocity, because the hydraulic resistances are greater for the working or up stroke, than for the return of the piston.

The working cylinder must be accurately bored. The thickness of the metal is made greater than the usual rules of calculation indicate as enough, to compensate for *wear*, and because of the shock at entrance of the water. The formula $e = 0,0025\, n\, d + 1$ will be found useful in guiding to the proper dimensions. The cylinder may be strengthened by mouldings or ribs cast round it.

The working cylinder is subject to a pressure in the direction opposite to that in which the piston moves, equal to the weight of a column of water $F h \gamma$, F being the area of the base, h the height, and γ the weight of a cubic unit; h being not unfrequently several hundred feet, this pressure of the water is very considerable, and, hence, the substructure on which the cylinder rests must be very strong. Water-pressure engines are erected in the shafts of mines

for raising water, more frequently than in any other position, and cannot, therefore, be placed on the solid rock, or foundations laid thereon, but have to be supported on cross beams or arches of stone, or of iron.

Fig. 291.



Remark. Besides this pressure, the cylinder has to withstand a horizontal pressure in the direction of the water entering it, and proportional to its section. The effect of this is less observable, because the pressure acts at a point only a little above the base of the cylinder, and because the pressure pipe, which is firmly connected with the cylinder, is equally pressed in the opposite direction. In any bend or knee piece AB, Fig. 291, there is a resultant pressure $\overline{CR} = R$, which may be put $= P\sqrt{2} = F_1 h \gamma . \sqrt{2}$, F_1 being the area of the pipe and h the pressure height.

§ 180. *The Working Piston.* — The main piston which moves under the pressure of the water, consists essentially of a cylindrical disc fitting smoothly into the cylinder. To make this piston perfectly tight, and at the same time not to cause thereby too great a resistance to motion, a *packing* (Fr. *garniture;* Ger. *Liderung*) of hemp, leather, or metal is applied, either on the piston, or in the cylinder, in which latter case the piston becomes what is termed a *plunger* or *ram.* The packing of the pistons of water-pressure engines is usually either leather or metallic rings. They are adjusted

Fig. 292.

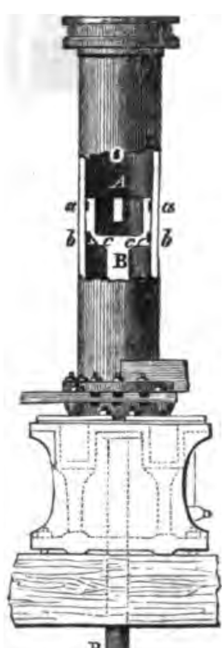
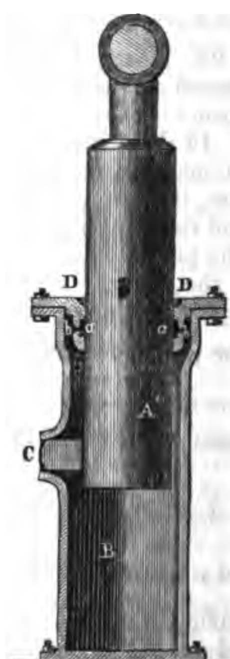


Fig. 293.



to a pressure proportional to the column of water, so that, on the one hand, no water may escape or pass, and on the other, that there may be no unnecessary friction. The best packing that can be employed, is that in which the water itself presses the leather or packing against the surface of the cylinder, or of the ram. The packing is made so that it can be gradually compressed as it wears, by means of a ring fitting upon it, and adjusted by screws. Fig. 292 is the piston of a water-pressure engine at Clausthal, in which the manner of laying in the packing is clearly represented. *A* is the piston, properly so called, and *BB* the piston rod, *a a* and *b b* are the packing rings, and *c c* two fine channels communicating with the back of the packing *b b*. Other methods of packing we shall describe hereafter.

For the plunger or ram, or *Bramah piston*, the packing may likewise be kept tight hydrostatically. *A*, Fig. 293, is the piston, *B* the cylinder, *C* the pressure pipe, *DD* the packing or stuffing box, screwed on to the piston, *a a* is the packing ring, and *b b* the five channels of communication. This manner of keeping the packing tight is more applicable to the case of a stuffing box, than to the ordinary piston.

Fig. 294.

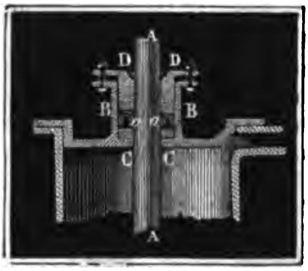
Remark. The compressed ring packing is also applied at the compensation joints, which must be introduced in the length of the pressure pipe. Fig. 294 shows such a pipe, *AA* being the enlarged end of one pipe *B*, accurately bored out, and resting on supports *CC*; *a a* are packing rings compressed by screws and nuts on to the thickened end of the upper pipe *D*.

§ 181. *The Piston Rod and Stuffing Box.*—The piston rod goes either upwards or downwards to the open end, or through the cover of the cylinder. In the first case, it requires very little special arrangement, and may be, in fact is, frequently made of wood. In the second case, it must go through a *stuffing box*, must, therefore, be turned, and can only be made of iron or gun metal. The dimensions of the piston rod is to be calculated according to the received theory of the strength of materials. If d be the diameter of the working cylinder, and p the pressure of the water, on each square inch of the piston, the force $P = \frac{\pi d^2}{4} \cdot p$; and if d_1 be the diameter of the piston rod, and K the modulus of strength of its material, then its strength $= P = \frac{\pi d_1^2}{4} K$, and by equating the two forces, we have: $d_1 = d \sqrt{\frac{p}{K}}$. K is to be taken from the table in Vol. I. § 186, and p is given by the formula $p = \frac{h \gamma}{144}$.

The stuffing box (Fr. *botte à garniture;* Ger. *Stopfbüchse*) is a box placed on the cylinder cover, so lined with leather or hempen

rings, that the piston rod, in passing through it, has freedom to move, but the passage is rendered water, or air, or steam tight, according to circumstances. For water-pressure engines, a leather packing is found to answer best. Fig. 295 shows the apparatus in question. *AA* is the piston rod, *BB* the stuffing box, *BaC* its *packing*, *DD* the cover with the screws for compressing the packing. A grease cup is sunk in the cover *D*, and kept filled with a grease composed of 6 parts hog's lard, 5 parts tallow, and 1 part palm oil, or with pure olive oil, or neat's-foot oil.

Fig. 295.



In the engine at Clausthal, oiling presses are applied, having a small piston, worked by a weight, and which forces the grease into the packing through a fine tube communicating with the channels of a brass ring, having a section of the I form, and round which the packing is lapped.

§ 182. *The Valves.*—The valves and their gear are, as it were, the very heart of the water-pressure engine, for it is by them the machine is made continuously self-acting. The valves cover and uncover apertures for the admission and discharge of the water from the cylinder, and these are worked so as to open and shut the apertures alternately, by means of *gear* connected with moving parts of the engine, so that the engine is thereby made *self-acting*. The valves are either *cocks*, or *sliding pistons*. The latter form is new generally adopted.

The manner of applying a cock as a valve has been already explained, so that we shall now only further describe the sliding piston valves. The arrangement of piston valves for a single acting, single cylinder engine is shown in Figs. 296 and 297. *E* is the pressure

Fig. 296.

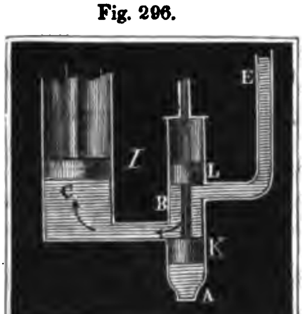
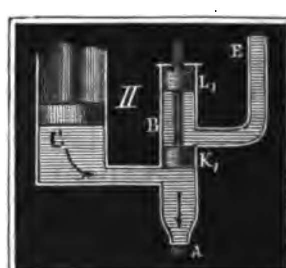


Fig. 297.



pipe, *C* the working cylinder, *B* the valve cylinder, *A* the discharge pipe, *K* the piston valve, and *L* its *counter piston*, which, by taking the equal and opposite pressure, renders the movement of the valves

more easy. When, as in Fig. 296, K is lowered, the working cylinder and pressure pipes are in communication, and when, as in Fig. 297, K is raised to the position K_1, the communication between the pressure pipes and cylinder is shut, and the passage for discharge of water from the cylinder is open. In the *double-acting* engine, or in the double-cylinder engine, the slide pistons must be arranged as in Figs. 298 and 299. E is the pressure pipe, C the pipe going

Fig. 298.

Fig. 299.

to the top, and C_1 that going to the bottom of the working cylinder (or going respectively to the bottom of the two cylinders in the double-cylinder engine). A is the discharge pipe for the water supplied by the first, and A_1 that for the water supplied by the second. From Fig. 298, we see that, when the slide valve is up, the pressure pipe is in communication with C, and the discharge made through A, and when the slide pistons are lowered, as in Fig. 299, the communication is open to C_1, and the used water discharged from C by the pipe A_1.

§ 183. *The Valve Cock.*—The cock is used for smaller engines, as shown in Fig. 300. HH is the cock, BB its cover, K is the squared end on to which a lever for turning it fits, D is a screw for raising or lowering the cock in its cover. The passages of the cock are made so as to suit the purposes to which it is applied, as we have explained above.

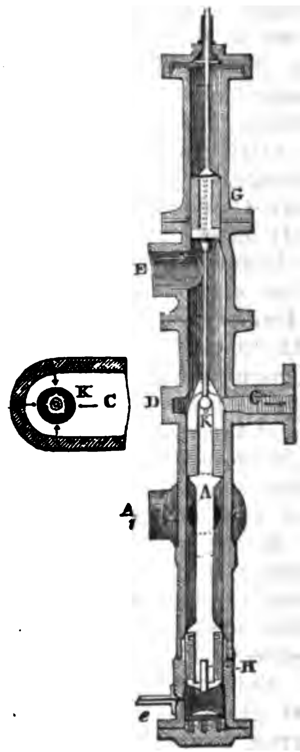
In Fig. 300, a means of counteracting the effects of greater pressure coming on one side of the cock is shown; b b_1 are two cuts on the cock, communicating with the passage a, by the openings c, c_1, so that a counter pressure is obtained, which, by proper adjustment of the parts, cut out at b and b_1, balance the diagonal pressure in the main passage.

In order to equalize the wear of the cocks on all sides, Mr. Brerdel, of Freiberg, introduced the method of turning them round continuously in the same direction, instead of turning them backwards and forwards through only 90°. We shall see the application of this valve in a description of a water-pressure engine erected by M. Brendrel, in the sequel.

§ 184. *The Slide-piston Valve.*—The pistons are generally made of slips of leather, placed one above the other, and closely packed

Fig. 300.

Fig. 301.



together, as we have mentioned for the packing of the stuffing box in § 181. The engine at Huelgoat, was originally made with cylindrical slide valves of gun metal. These lasted, without repair, for seven years; but in 1839, the valves having worn loose, a depth of 5 inches, consisting of 24 discs, or rings of leather pressed together and accurately turned down, was substituted. Reichenbach made the cylinder valves of *tin*, and the engines in Bavaria, in most recent times, have had the valves made by a combination of leather and tin rings.

At the end of the stroke of the working piston, the valve piston *AK* (Fig. 301) rises, gradually shutting off the water from the cylinder, but in gradually checking the flow of water in the course *EC*, the piston is pressed on one side, and this gives rise to a very rapid wear. To prevent this, the end of the pipe *CD* communicating with the working cylinder is carried quite round the valve cylinder, so that it incloses it, and the water then presses equally on every side of the piston, as it moves up and down. The packing

suffers by this arrangement, as it has room to expand at this point, and has to be compressed as it passes into the cylinder above or below it. On this account the supply of water to the cylinder is carried through a series of openings, as shown in the horizontal section in Fig. 302. The objection to this arrangement is, that it increases the hydraulic resistances. The *form* of the valve piston *K* is of great importance. The communication between *C* and *E* must not be suddenly opened or shut, so that the column of water, in motion, may not be suddenly brought to rest; for this acts violently on the engine, on the same principle as is more fully developed in the so-called *hydraulic ram*. The gradual opening of the communication may be managed by giving the piston a particular form. We shall hereafter show how a slow motion of the valve piston is effected, and in the mean time point out, that, by giving a conical shape to the *head*, or that part of the piston which begins the closing of the *ports*, a ring-formed opening is made between *C* and *E*, which is gradually diminished as the piston ascends, until it is finally closed. Besides this arrangement, the top of the slide piston is perforated by slits that gradually diminish, but leave a narrow communication between *C* and *E*, even when the ring-formed opening above mentioned is quite closed, so that the passage is not perfectly closed until the slide-piston stroke is completed. This system of coning out the top, and perforating the upper part of the piston proper, is applied in the Clausthal water-pressure engine.

Fig. 302.

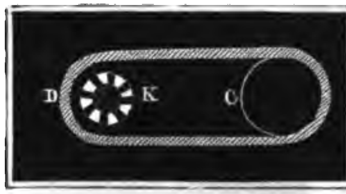
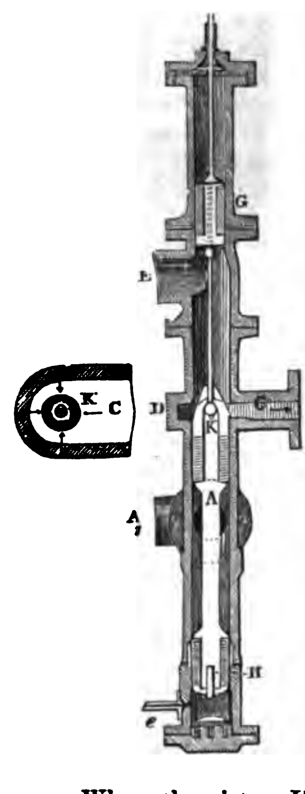


Fig. 303.

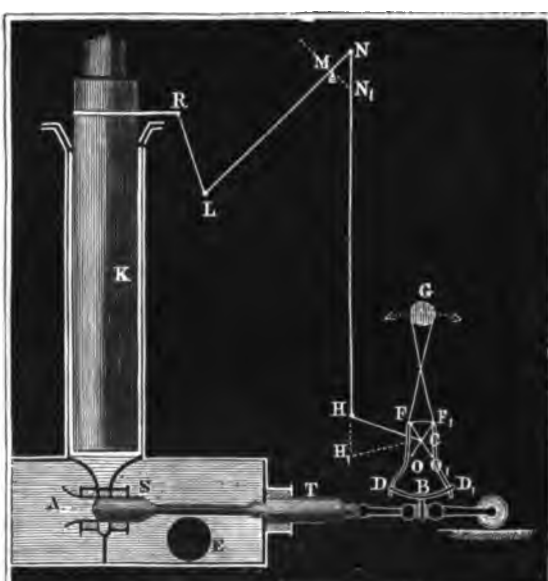


§ 185. *The Valve Gear.*—The gear for moving the valves of water-pressure engines is generally complicated, more so, for instance, than in the steam engine, because water is practically an incompressible fluid, exerting no pressure when cut off from the pressure column. When the piston *K*, Fig. 303, in ascending, cuts off the pressure column from the working cylinder *C*, then either the motion of the working piston ceases, or, in virtue of its *vis viva*, it moves away from the water in the

cylinder, as this has no expansive capability. But this formation of a vacuum under the piston must be carefully avoided, and, therefore, the valve piston should begin to rise, while the main piston stroke is still unfinished, and thus the *vis viva* of all the parts connected with it is gradually destroyed by the gradual cutting off of the pressure column. But although the stroke of the piston is completed as the slide valve closes the communication, the motion of the slides must not stop here. The water in the working cylinder must now be discharged. The valve must rise somewhat higher, in order to open the orifice of discharge. Hence it is not *possible* to work the valve gear *directly* from the moving parts of the engine, for then the motion of both would cease simultaneously. Intermediate *gear* must be introduced, by which the motion of the valve piston is continued after the working piston has come to rest. This gear may be worked either by weights, raised by the piston in its ascent, and let fall at a particular part of the course, or by springs, bent during the motion of the piston, and disengaged at the end of the stroke, or by a subsidiary engine regulated by the main engine, but whose working piston moves the valves of the main engine. The gear of water engines is, therefore, either *counter-balance gear*, *spring gear*, or *water-pressure gear*.

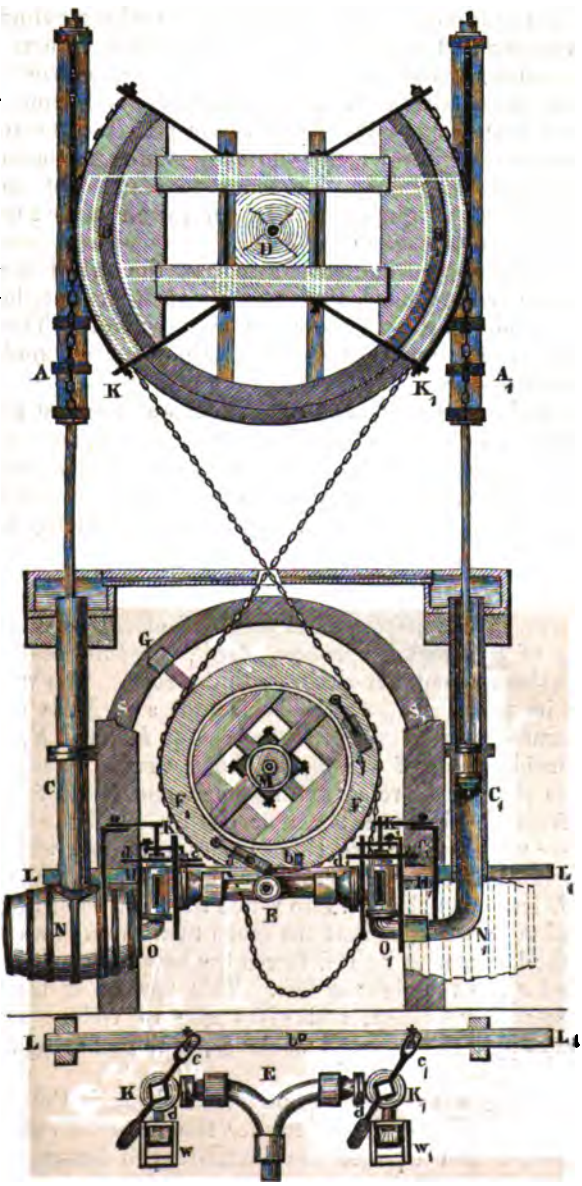
§ 186. *Counter-balance Gear.*—This gear was the first employed, and is now found only as the older water-pressure engines, under the name of *fall bob*, *valve hammer*, and other names. The principle of the different systems is always the same. They are essentially a heavy weight raised by the working piston, and suddenly let go to

Fig. 304.



work the cocks, or valves, by means of linked levers. We shall here describe only two of these arrangements. The small engine in the Pfingstwiese mine, near Ems, has gear connected with a pendulum or fall bob, moving two pistons *S* and *T*, lying horizontally under the working cylinder *K*, Fig. 304. The pendulum swings on an

Fig. 305.



axis C, and consists of a heavy bob G, and two fork-like springs FD and F_1D_1, carrying a cross head DBD_1, having a projecting piece B, in the centre, passing between two small rollers on the valve rod. The bob is raised so as to exceed the summit of its arc, by means of link work $CHNMLR$, connected with the *ram* of the engine at R. Motion is not communicated from the axis of the pendulum C, but by means of an arm CO, on a separate axis, and forming a single bent lever with CH, and which pushes out the springs FD and F_1D_1 alternately, so far that the *bob* G is brought beyond the position of stable equilibrium, and in its fall gives the valve rod the requisite extra push to right or left. At the commencement of the stroke of the working piston, the whole apparatus has necessarily a very slow motion. The coming into play of the arm CO, on the one or other spring, should only take place when the stroke is nearly completed, that, as the valve piston gradually advances, the retarded motion of the working piston may begin.

It is easy to perceive from our figure, how the pressure water is introduced into the cylinder, and discharged from it at the end of the stroke. When the piston S is in the orifice A, the pressure water from E enters by the opposite orifice into the cylinder; but if S be in the orifice next E, so that the orifice A is open to the cylinder, then the water that has raised the ram discharges into the waste-course at A.

Remark. This little engine has 60 feet fall, 4 feet stroke, 1½ foot diameter working cylinder, and made (in 1839) 1 stroke in 65 seconds.

§ 187. *The Valve Hammer.*—The arrangement of the valve hammer, is well illustrated by that on the water engine at Bleiberg, in Karinthia, and which is fully described in *Gerstner's* "Mechanics." Fig. 305 shows this arrangement in plan and elevation. A and A_1 are the rods of the working pistons, BDB_1 is a *balance beam* connected by chains and counter-chains with the rods. The valve hammer G, and its wheel FF_1, on the horizontal axis M, is connected with the balance beam by another set of chains FK and F_1K_1. An attentive consideration of the figure shows that the reciprocating motion of the piston rods raises the hammer, and lets it fall without hindrance from the balance beam or chains. On the fall of the valve-hammer wheel, there are two catches, a and a_1, which, when the hammer falls, catch upon a projection on the horizontal rod LL_1. This rod LL has two nobs $c\,c_1$, into which the handles or keys of the cocks, K and K_1 are set, so that the cocks turn through an angle of 90°, when the hammer in its fall forces the bolt b, by means of the catches a and a_1, to the right or left. This method of moving the cocks is necessarily sudden, and gives rise to violent shocks, so that it is only applicable to small machines, or those having moderate falls.

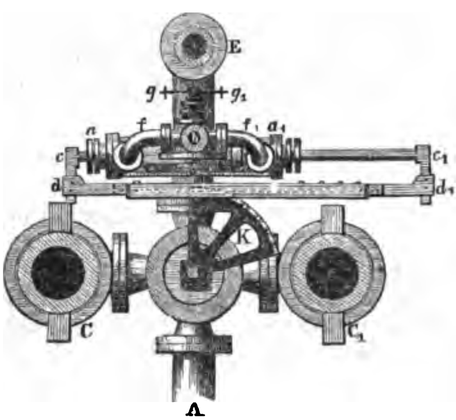
The cocks have a passage, or *are bored* through the axis, and through the side. Through the former the pressure water enters by knee pieces O and O_1 into the barrel-shaped bottom pieces N

and N_1 at the bottom of the working cylinders; and through the side passage, the pressure water is brought to the cock from the cylinder. In order that only as much water may be used as is necessary to fill the space passed through by the working piston, the discharge is made to take place *under* water into special reservoirs W and W_1.

Remark. The engine now described has a fall of 260 feet, stroke 6¼ feet, cylinder 7 inches diameter, 8 strokes per minute. It is in many respects an imperfect engine; but it is economically adapted to its position. We have not only to consider mechanical *perfection* in the construction of engines in general, but we have to weigh well the circumstances in which the engine is to work, the facilities for repair in the particular locality, and the relative supply and demand for the water power.

§ 188. *Auxiliary Water-Engine Valve Gear.*—No application of spring-valve gear has been made; but the method of using an auxiliary water engine is now come into very general use. The general arrangement of such an auxiliary engine gear is shown in Fig. 306, as applied to the great water-pressure engine in the *Leopold* shaft, near Chemnitz.

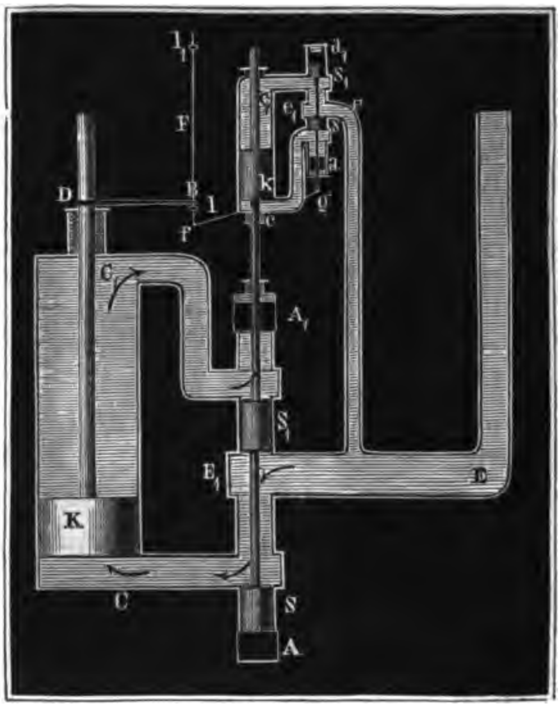
Fig. 306.



This engine has two cylinders, C and C_1; E is the pressure pipe, A the discharge pipe, H the main cock, K a quadrant key fastened on the cock. The auxiliary engine has a horizontal cylinder $a\ a_1$, with a piston b on the piston rod $c\ c_1$. The piston rod is connected with the valve rod $d\ d_1$ by cross pieces, so that the two united form a rectangular frame. The valve rod is connected with the quadrant by two chains, so that the reciprocating motion of the piston b communicates a rotary motion of 90° to the cock. The auxiliary engine is worked by means of the cock $h\ h_1$ lying horizontally, with two *bores*, or passages, as in the case of the main cock H. The little pipe e communicating with the pressure pipe E, takes the pressure water to the cock $h\ h_1$, from which it passes through the pipes $f\ f_1$ to one side or the other of the piston b, so that it is moved backwards and forwards, the water used in each alternate stroke being discharged by the other passage in the

cock, and thence by a pipe from h. The small cock $h\ h_1$ is turned by the double-handled key $g\ g_1$ connected by slender chains to a double-armed lever parallel to it, and which is on the same axis as the balance beam to which the piston rods of the two cylinders are

Fig. 307.



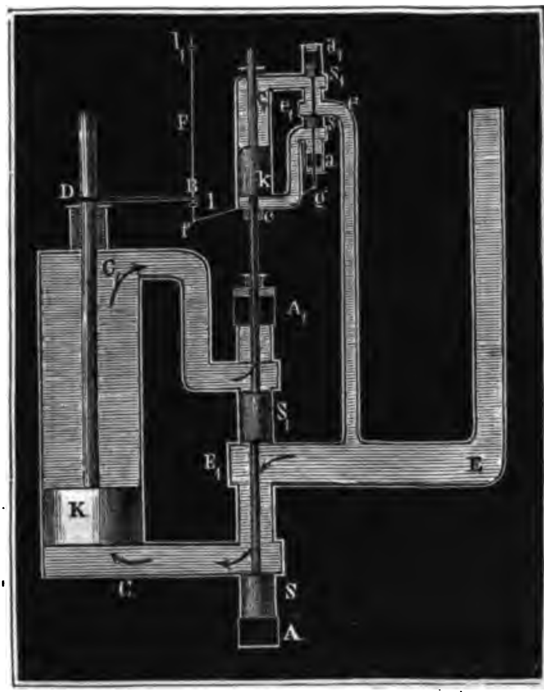
attached. The whole play of the valve gear is now evident. While the working piston rises and the other descends, the cock $h\ h_1$ is turned by the lever or key $g\ g_1$, thus the communication between the water and the cylinder $a\ a_1$ is opened or shut, and thus power is obtained for bringing the piston b, and the cock H into the opposite position, so that the first working cylinder is now shut off from the pressure pipe, and the second put in communication with it.

Remark. The engine in the Leopold shaft has 710 feet fall (Austrian measure), 8 feet stroke, 11 inch diameter of cylinder; each piston makes 3 strokes per minute.

§ 189. The working of the valves (Fig. 308) by means of an auxiliary engine, is well illustrated by that of the double-acting water-pressure engine at Ebensee, in Salzburg; the auxiliary engine being, in this case, an exact model of the working engine. CC_1 is the cylinder of the principal engine, and cc_1 that of the auxiliary. K is the piston of the one, and k that of the other cylinder. S and S_1 are the valve pistons of the working, and s and s_1 those of the

auxiliary engine. EE_1 (Fig. 308) is the main pressure pipe, and $e\ e_1$ the pipe communicating with the auxiliary engine. Lastly, A and A_1 are the orifices of discharge of the main, and $a\ a_1$ those of the auxiliary engine. Thus the one engine is an exact counterpart

Fig. 308.



of the other, the dimensions being, however, very different in the two. The valve gear of the auxiliary engine consists in the cantilever BD attached to the main piston rod at D—of the valve piston rod $g\ s$, connected by the link $f\ g$ to the rod $l\ l_1$, on which there are two studs placed, so that the lever DB catches upon them a little before the end of the up and down strokes, respectively, of the main piston, and thus the valve piston is moved. It is easy to trace how this motion admits the pressure water alternately above and below the piston K, so as to raise or depress the valve pistons $k\ S_1\ S$, giving the required alternation of admission of the pressure water above and below the main piston K.

Remark. The engine at Ebensee has a fall of only 36 feet, a stroke of 17 inches, and a cylinder of 9½ inches diameter. It makes 6 strokes per minute, and moves two double acting pumps.

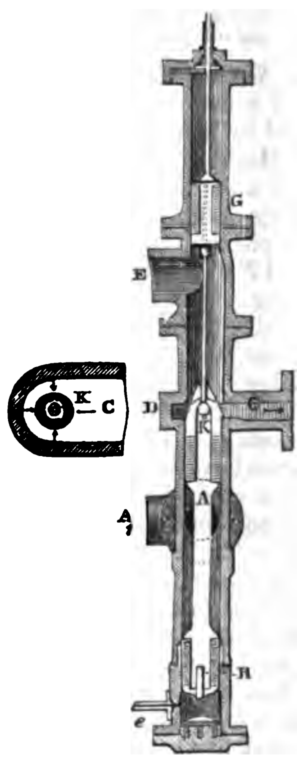
§ 190. *The Valve Cylinder.*—In the larger engines of recent date, the valve pistons of the main cylinder are inclosed in the same pipe, or cylinder, as the piston of the auxiliary engine; and in some engines the counter-pressure valve, or piston balancing the pressure

on the valve, is the working piston of the auxiliary engine, and thus great simplicity of construction is attained.

Fig. 309 shows a simple arrangement adopted in two engines in the Freiberg mining district. *S* is the main piston valve, and *G* the counter-pressure piston, *C* an intermediate pipe communicating with the main cylinder, *E* the entrance for the pressure water, and *A* the orifice of discharge for the water used, *e* is the communication with the valve of the auxiliary engine, which in this case is a cock. The piston *G* is larger than *S*, and, therefore, the valve

Fig. 309.

Fig. 310.



apparatus *S G* descends, when the pressure is admitted from above at *e*, and ascends when the pressure water is cut off at *e*, and the pressure acts underneath. For each stroke there is a consumption of a certain quantity of water for the valves, which is lost for useful effect. This amounts to the contents of the space passed through in the up or down stroke. In the construction, now under consideration, this is not so little as in some others, for the piston *G* must have, at least, one and a half times the area of the piston *S*, the sectional area of which is the same, or even greater than that of the pressure pipe.

The system of valves shown in Fig. 310, is that of the Clausthal engine, and here the waste of water is less than in the last mentioned system. For there are three pistons; namely: the main valve piston AK, the counter piston G, and the auxiliary piston H; the latter being somewhat less in area than the former. The water is brought into the valve cylinder by the pipe e, and the reverse motion of the piston is effected by a small cock through which the water enters before coming into e, and through which, also, when the revolution is completed, it is let off. The cock is moved by link work, by means of a *tap* on the main piston rod.

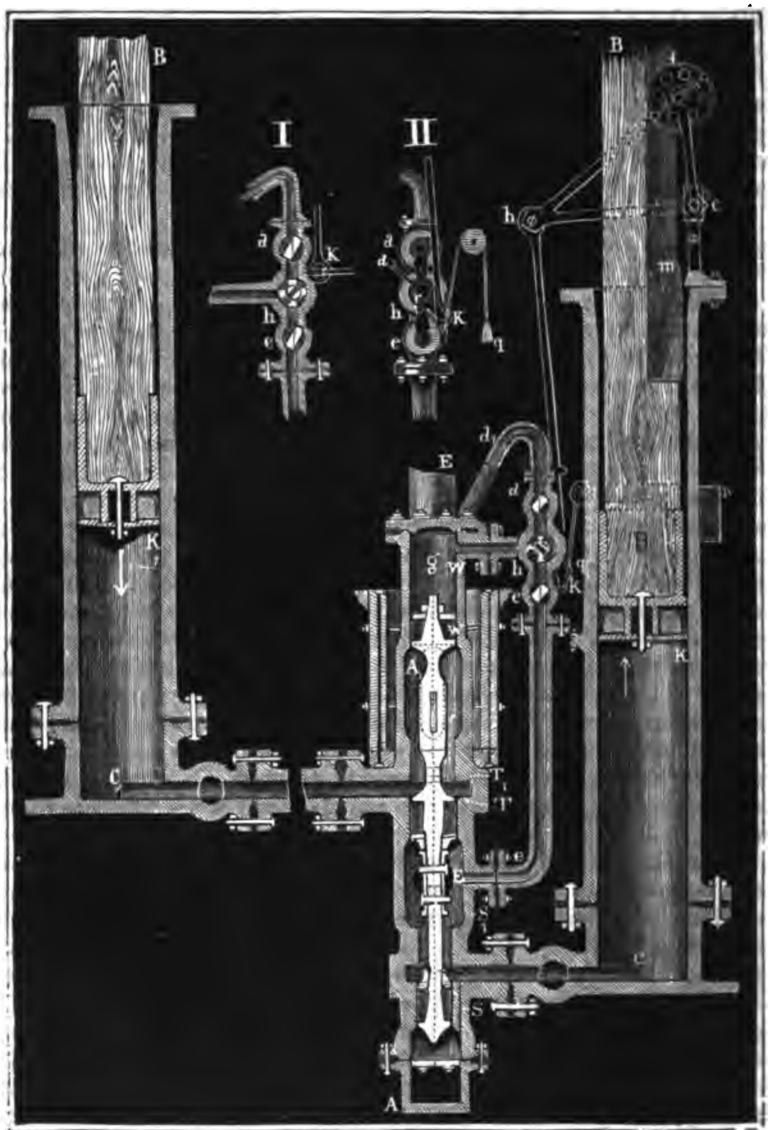
Remark. The engines at Clausthal have 612 feet fall, diameter of cylinder 16½ inches, stroke 6 feet, and make 4 strokes per minute.

§ 191. *Saxon Water-Pressure Engine.*—The arrangement and motions of a double cylinder water-pressure engine may be clearly understood by a study of a sectional view of the engine, erected in the Alte Mordgrube, near Freyberg, in Saxony, delineated in Fig. 311. CK and C_1K_1 are the two working cylinders, K and K_1 being the working pistons, S and T are the two valve pistons, W is the auxiliary piston, and S_1 T_1 and W_1 are the points in the valve cylinder ATW_1 at which the pistons are for the return stroke of the working pistons. E is the entrance of the pressure pipe E_1E into the valve cylinder, CS is the intermediate pipe communicating with the one, and C_1T the pipe communicating with the other working cylinder. A is the orifice of discharge of the one, and A_1 that of the other (this latter orifice is nearly covered by the piston rod in the drawing). The two piston rods BK and B_1K_1 are connected by a balance beam (not shown in the figure), so that as the one piston ascends the other descends. It is hence easy to perceive, that, for the lower position of the valve piston, here represented, the pressure water takes the course ES_1C, driving the piston K upwards, and that the piston K_1 is pushed downwards, the used water taking the course $C_1T_1A_1$ to the discharge orifice A_1.

The auxiliary valve consists of a four-way cock h (already described) shown at I. in the second position, and external elevation at II. This cock gives passage between the pipe e e_1 and the pressure pipe, and between g h and the valve cylinder.

It is evident that in the one position of h, the pressure water takes the course Ee_1 e h g W, and presses down the auxiliary piston W, whilst for the second position of h, the pressure water is shut off from W, and hence the ascent of the valve piston system STW, the return of the valve water through g h, and its discharge at a a_1, can take place. That the valve piston system may rise when the water is shut off from W, and may descend when it is let on, it is necessary that the piston T, pressed upwards by the pressure water, should have a greater sectional area than the piston S, which is pressed downwards by the pressure water; and also, the auxiliary piston must have sufficient area that the water pressure, or W and S together, may exceed the opposite pressure on T.

Fig. 311.



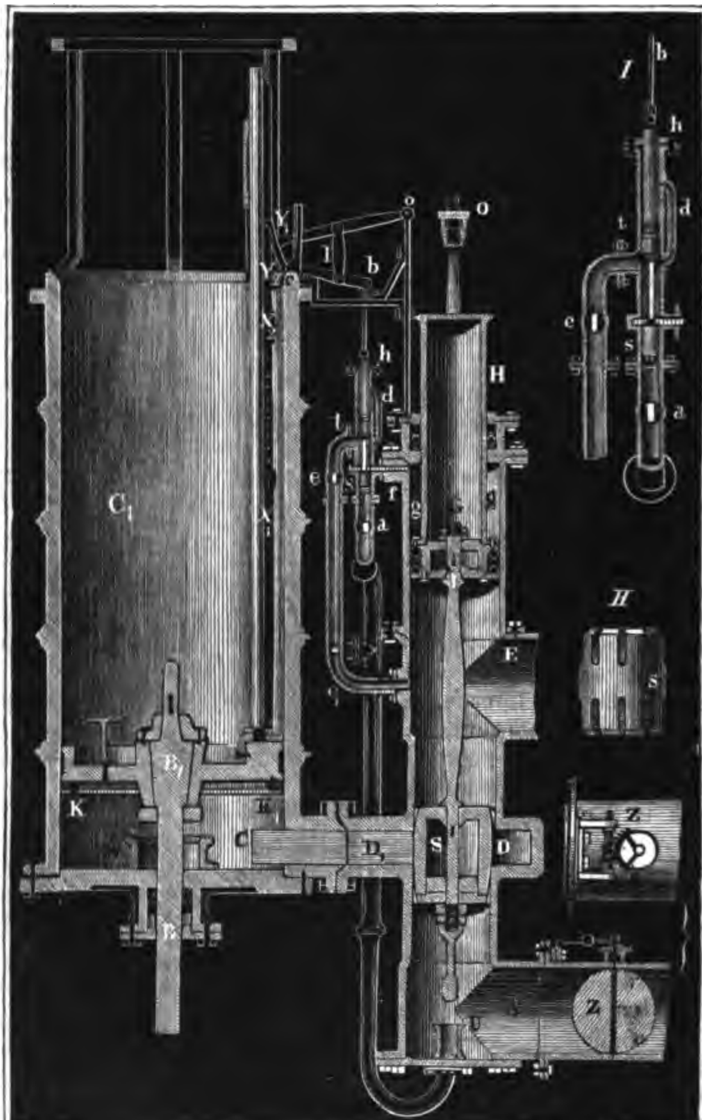
The valve gear of this machine is composed of a ratchet wheel r, a catch $r\,k$, a rod $k\,h$, and a bent lever $h\,c\,f$ with its friction wheel f, and the two wedge-formed pieces $m\,m_1$ set on each piston rod. The catch $r\,k$ is connected with the axis of the cock, and is held by a small balance weight q in its place on the ratchet wheel. When the piston K has reached nearly the end of its stroke, the wedge m (or m_1) passes under the friction wheel, and turns the lever $f\,c\,h$ to a certain extent, so that the rod $h\,k$ is drawn up, and the wheel and cock h are turned through a quadrant. As the working piston makes its return stroke, the lever falls back again, and the catch slides back over the next tooth of the ratchet, and is ready at about the end of this return stroke to push round the ratchet, &c.

Remark. The water-pressure engine in the Alte Mordgrube, has a fall of 356 feet, a stroke of 8 feet, 18 inches diameter of cylinder, and makes 4 double strokes per minute.

§ 192. *Huelgoat Water-pressure Engine.*—One of the largest and most perfect water-pressure engines hitherto erected, is that at Huelgoat, in Brittany. It is a single-cylinder, single-acting engine. Fig. 312 represents the essential parts of this engine, and its valve gear. CC_1 is the working cylinder, KK_1 the working piston, and BB_1 the piston rod working through a stuffing box at B. In the Saxon engine, the piston is packed by a single sheet of leather; but in this engine, the rim of the piston is packed, and there is also a sheet of leather, held in its place by a ring. The valve cylinder ASG is united to the working cylinder by the pipe DD_1, into which the pressure pipe opens at E, and the discharge pipe at A. To the valve piston S, a counter-balance piston T of greater diameter is connected by the rod ST. This system will, therefore, be forced upwards by the pressure water, if a third force be not brought into play. This third force is, however, produced by bringing the pressure water above T, through the pipe $e_1\,e\,f$, and in order to use only a small quantity of water for working this valve system, a hollow cylinder GH is placed on T, passing through a stuffing box at H, and, therefore, exposing only an annular area to the pressure of the water.

The alternate admission and exclusion of the pressure water of the hollow space $g\,g$, is effected by an auxiliary valve system, resembling the main valve system in every respect; consisting like it of a valve piston s, a counter-balance piston t, which is a solid piston passing through a stuffing box at h. For the position $s\,t\,h$, shown in our figure, the pressure water has free circulation through $c\,f$ to g; but if $s\,t\,h$ be raised, so as to bring s above f, this passage is stopped, and the valve water, in the hollow space $g\,g$, escapes through $a\,a_1$, when ST goes up. Lastly, to derive the motion of the auxiliary valve-piston system from the engine itself, there is let into the working piston KK_1 an upright rod with a feather edge attached to the side. This feather has a series of holes drilled in it, into which catches can be put as X_1, X_2, at the required distance apart. The link $b\,h$ is connected to two levers, centred at c and o, and connected

Fig. 312.



by the link *l*. The end of the one lever has an arc, on which there are two projections or catches Y_1, Y_2. As the up stroke of the piston comes to an end, the catch X_1 strikes on Y_1, and thus *s t h* is moved to its upper position, and at nearly the end of the down stroke of the piston, the catch X_2 strikes Y_2, and the valve system *s t h* descends to a lower position. It is now easy to perceive how the alternate positions of *ST*, necessary for the reciprocating motion of the piston, KK_1, are produced.

§ 193. The following are the details of the construction of Mr. Darlington's water-pressure engine. The first engine erected in England with cylinder or piston valves was that put up in the Alport mines, Derbyshire, in the year 1842. This was a single cylinder engine. Its success was complete, and others were erected on the same plan. But in 1845, a *combined cylinder engine* was designed, and erected by the same engineer, which is found practically to have several advantages for such large supplies of water as that consumed by the pumping engine, of which are subjoined accurate reductions of the working drawings.

Fig. 313.

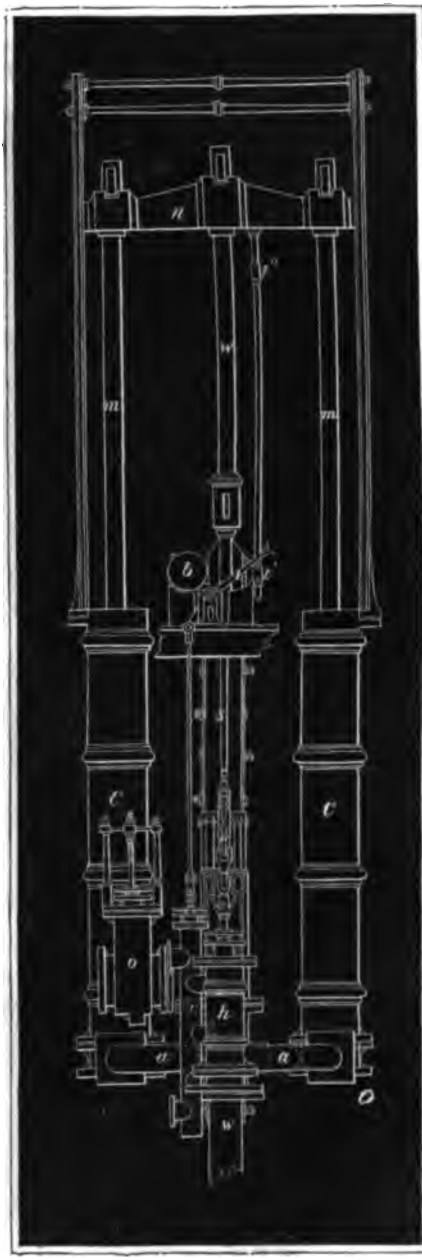
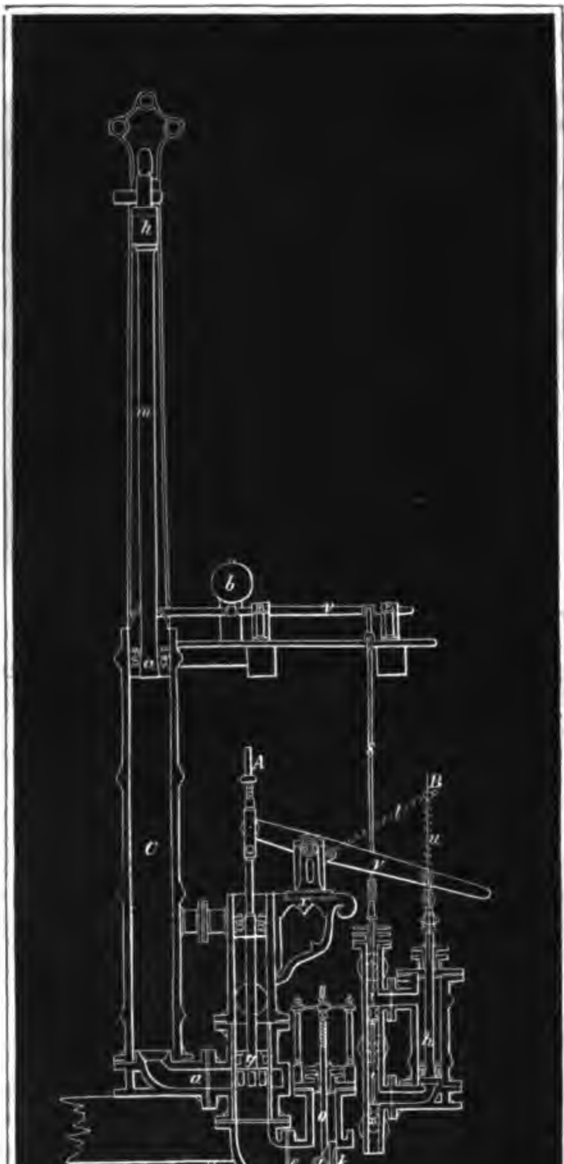


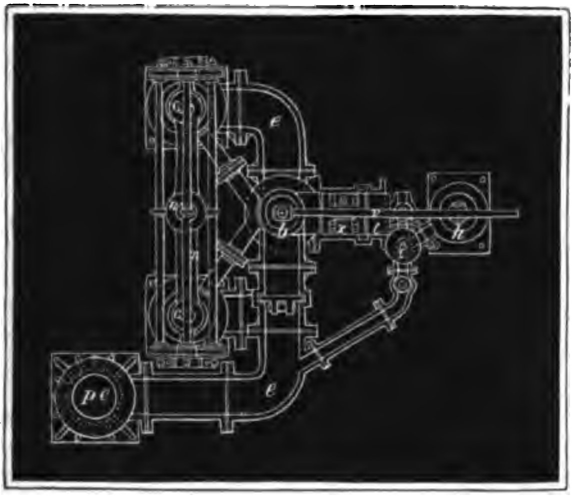
Fig. 313 is a front elevation of the combined cylinder engine. Fig. 314 is a sectional view, and Fig. 315 is a general plan. *PC*, is the bottom of the pressure column, 130 feet high, and 24 inches internal diameter, *CC* are the combined cylinders, each 24 inches diameter, open at top, with hemp-packed pistons *a* (Fig. 314), and piston rods

Fig. 314.



m, combined by a cross head *n*, working between guides in a strong frame. The admission throttle valve is a sluice valve, shown at *o*, Fig. 313, and between the letters *b* and *e* and Fig. 315. The main

Fig. 315.



or working valve, is a piston *g*, 18 inches in diameter, Fig. 314, with its counter or *equilibrium piston* above. The orifice for the admission of the pressure water is between the two pistons. The intermediate pipe *a* is a flat pipe, into which numerous apertures lead from the valve cylinder (seen immediately under *g*, Fig. 314). The valve piston is in the position for discharging the water from the cylinders through the pipe *e*, Fig. 314, by the sluice valve *k*.

The valve gear is worked by an auxiliary engine *h*, by means of the lever *v*. The auxiliary engine valves, are piston valves in the valve cylinder *i*, Figs. 314 and 315, communicating with the pressure pipes by a small pipe, provided with cocks, as shown in Fig. 315. The motion of the auxiliary engine valves is effected by a pair of tappets *t'*, *t''*, set on a vertical rod attached to the cross head *n*. These tappets move the fall bob *b*, by means of the canti-lever *t*, Fig. 313, the other end of the lever being linked to the rod *s*, Fig. 314, which again is linked to the auxiliary piston valve rod.

The play of the machine is now manifest. It is in every respect analogous to the Harz and Huelgoat engines, described above. The average speed of the engine is 140 feet per minute, or 7 double strokes per minute. This requires a velocity of something less than 2½ feet per second of the water in the pressure pipes; and as all the valve apertures are large, the hydraulic resistances must be very small. The engine is direct acting, drawing water from a depth of 135 feet, by means of the spear *w*, *w*, Figs. 313 and 314. The "box," or *bucket* of the pump, is 28 inches in diameter, so that the discharge is 266 gallons per stroke, or, when working full speed,

1862 gallons per minute. The mechanical effect due to the fall and quantity of water consumed is nearly 140 horse power. The mechanical effect involved in the discharge of the last-named quantity of water is nearly 74 horse power, so that, supposing the efficiency of the engine and pumps to be on a par with each other, the efficiency of the two being (§ 203), $\eta_1 = 71{,}15$, the efficiency of the engine alone $\eta = \frac{1 + \eta_1}{2} = \frac{1 + {,}71}{2} = {,}85$, or, in the language of Cornish engineers, 85 per cent. is the duty of the engine.

The cost of maintenance, grease, &c., of the engine, is only £40 per annum. In every particular, it redounds to the credit of Mr. Darlington's skill as an hydraulic engineer.

Balance.—For regulating the motion of water-pressure engines, several auxiliary arrangements are necessary, which we shall explain hereafter. The ascent and descent of the working piston is regulated by an arrangement called a *balance*, or *counter-balance*, which aids the motion of the piston in the one direction, and retards it in the other, so that the working of the machine goes on with a nearly uniform velocity. In the double-cylinder engine, the balance is effected by a simple beam, connecting the two cylinders. In the double-acting, single-cylinder engine, a fly wheel is necessary, and in the single-acting, single-cylinder engine, a counterbalance weight, either of a solid body, or of a column of water, an *hydraulic balance*, is employed. On the subject of "Regulators of Motion," we, for the present, make only a few general remarks. The *mechanical balance* consists of a beam with a weight at one end, and having the other end attached to the piston of the engine, so that the weight assists during the working stroke of the piston to counterbalance the piston and rods; and during the down stroke, or discharge of the used water, prevents the too rapid return of the piston and rods; the adjustment being such as to allow of the discharge stroke being made in about half the time that the working stroke occupies. The hydraulic balance consists of a second column of pipes, which ascends from the discharge orifice to such a height, that the water it contains counterbalances the extra weight of the piston and rods. The machine at Huelgoat, and also those at Clausthal, have hydraulic balances.

There is evidently neither loss nor gain of effect by the use of a counter-balance, save by the *prejudicial* resistances they give rise to in their motion. The balance beam has the advantage that its *balance weight* can be varied as required; and the hydraulic balance has the advantage of *simplicity*, when other circumstances do not interfere with its application.

§ 194. *Throttle Valves.*—The cocks or throttle valves of water-pressure engines are important organs, their function being to regulate the supply of water to, and its discharge from, the engine—that is *the speed* of the engine. These valves must have a prejudicial effect on the efficiency of the engine, and yet, they are a necessary evil. In order to regulate the ascent and descent of the working

piston and of the valve pistons, there are necessary, four cocks or valves—one in the pressure pipe, and one in the discharge pipe (as Z, Fig. 312); also a cock in the pipe leading the valve water above the auxiliary piston, and another similar in the pipe which discharges the water used in the valves, as e and a in Fig. 312.

To get the highest efficiency from a water-pressure engine, its work should be such as to render any contraction of the pressure pipes, by a throttle valve, unnecessary for its uniform motion. If, however, the useful effect of the engine is greater than is required by the work to be done, the excess must be taken away by checking the supply by means of the throttle valve, or by shortening the stroke of the engine.

If it be an object to save water, the latter means is the best when possible, because the efficiency of the machine is not thereby interfered with.

A change in the length of stroke of the piston is easily effected by altering the position of the catches on the rod X_1, X_2, Fig. 312. The nearer X_1 and X_2 are brought together, the earlier the reversing of the stroke ensues; and, therefore, the shorter is the stroke of the working piston.

§ 195. *Mechanical Effect of Water-pressure Engines.*—In computing the effect of water-pressure engines, we shall make use of the following symbols:—

F = the area of the working piston.
F_1 = the area of the pressure pipes.
d = the diameter of the working piston.
d_1 = the diameter of the pressure pipe.
d_2 = the diameter of the discharge pipe.
h = the fall from surface of reservoir to surface of water in discharge channel.
h_1 = the vertical distance from the surface of reservoir to the surface of piston at half stroke.
h_2 = the distance from surface of discharged water to the piston at half stroke.
s = the stroke of the piston.
l_1 = length of pressure pipe.
l_2 = length of discharge pipe.
v = mean velocity of piston.
v_1 = mean velocity of water in pressure pipe.
v_2 = mean velocity of water in discharge pipe.

We shall assume the engine to be single acting, making:

n = the number of strokes per minute.
Q = the quantity of water used per second.

The mean pressure of the water on the piston surface F is $P_1 = F h_1 \gamma$, and, therefore, the mechanical effect produced *per stroke*, prejudicial resistances neglected, is $P_1 s = F s h_1 \gamma$, and per minute $n P s = n F s h_1 \gamma$, and, therefore, the mean effect per second, is:

$$L_1 = \frac{n}{60} P s = \frac{n}{60} F s h_1 \gamma, \text{ or as } \frac{n F s}{60} = Q,\ L_1 = Q h_1 \gamma,$$

In the return of the piston, the mean effective resistance is: $P_2 = F h_2 \gamma$, and, therefore, the mechanical effect consumed is: $P_2 s = F h_2 s \gamma$, and hence the loss of effect per second is: $L_2 = Q h_2 \gamma$, and, therefore, the effect available

$$L = L_1 - L_2 = Q (h_1 - h_2) \gamma = Q h \gamma,$$

as in many other hydraulic recipient machines.

This formula is evidently not changed should the working piston not fill up the cylinder, *i. e.*, supposing a plunger is used, round which there is a free space, or supposing the piston does not descend to touch the bottom of the cylinder. Nor would the circumstance of the discharge taking place *below* the mean position of the piston —that is, of h_2 being negative, or $h = h_1 + h_2$ alter the formula. F is the area of a section of the piston at right angles to its axis, or $F = \frac{\pi d^2}{4}$, and, therefore, the form of the piston can have no effect.

§ 196. ***Friction of the Piston.***—Of the prejudicial resistances, the friction of the piston is a principal one. As there are no accurate experiments on this subject, we must content ourselves by estimating it from the pressure of the water, and a co-efficient of friction ascertained in the nearest possible analogous circumstances. If the packing be on the *hydrostatic* plan, the force with which each element e of the packing is pressed against the cylinder during the up stroke is $= e h_1 \gamma$, and during the down stroke it is $= f e h_2 \gamma$, and hence the friction $= f e h_1 \gamma$, and $f e h_2 \gamma$, respectively. The total friction will be the sum of the frictions of all the elements, or of the area of the whole packing. If the breadth of the packing be b, then $\pi d b$ is the area, and then the piston friction is $R_1 = f \pi d b h_1 \gamma$ for up stroke, and $R_2 = f \pi d b h_2 \gamma$ for down stroke.

It is convenient to express the various prejudicial resistances in terms of a column of water of the area of the piston, and whose height h_3 or h_4 is the head *lost* (in the present case) by the friction of the piston. Let us, therefore, put:
$R_1 = F h_3 \gamma$, and $R_2 = F h_4 \gamma$, or $F h_3 = f \pi d b h_1$, and $F h_4 = f \pi d b h_2$, or putting $\frac{\pi d^2}{4}$ for F, we have $\frac{d h_3}{4} = f b h_1$, and $\frac{d h_4}{4} = f b h_2$; and hence the loss of fall, corresponding to the friction of the piston $h_3 = 4 f \frac{b}{d} h_1$, and $h_4 = 4 f \frac{b}{d} h_2$.

If we deduct these heights, we get for the mean power during the up stroke:

$$P_1 = F (h_1 - h_3) \gamma = \left(1 - 4 f \frac{b}{d}\right) F h_1 \gamma,$$

and during the down stroke:

$$P_2 = F (h_2 + h_4) \gamma = \left(1 + 4 f \frac{b}{d}\right) F h_2 \gamma,$$

and hence the resultant mean effect:

$$L = \frac{n}{60}(P_1 - P_2)\,s = \frac{n}{60}\left((h_1 - h_2) - 4f\frac{b}{d}(h_1 + h_2)\right) F s \gamma$$

$$= \left(h - 4f\frac{b}{d}(h_1 + h_2)\right) Q \gamma.$$

If the rising pipe height $h_2 = 0$, or be very small, then we have more simply

$$L = \left(1 - 4f\frac{b}{d}\right) Q h \gamma.$$

We see from this that the loss of effect from friction of piston is so much the greater, the greater $\frac{h_1}{h}$ and $\frac{h_2}{h}$ are, that is, the greater the head, and the greater the counter-balance head.

To reduce this friction, the packing should not have unnecessary width. In existing machines $\frac{b}{d} = 0,1$ to $0,2$. The co-efficient of friction is to be taken as determined by Morin, $f = 0,25$. This being assumed, we see that $4f\frac{b}{d} = 0,1$ to $0,2$, or that the friction of the piston absorbs from 10 to 20 per cent. of the whole available power.

§ 197. *Hydraulic Prejudicial Resistances.*—Another source of loss of effect in water-pressure engines, is the friction of the water in the pressure and discharge pipes. According to the theory given in Vol. I. § 329, the pressure height or head corresponding to this loss, ζ being the co-efficient of friction, is

$h = \zeta \cdot \frac{l}{d} \cdot \frac{v^2}{2g}$. This applied to the pressure pipe, becomes

$h_5 = \zeta \cdot \frac{l_1}{d_1} \cdot \frac{v_1^2}{2g}$, and applied to the discharge pipe it is

$h_6 = \zeta \cdot \frac{l_2}{d_2} \cdot \frac{v_2^2}{2g}$. But the quantity of water, is

$$\frac{\pi d_1^2}{4} \cdot v_1 = \frac{\pi d_2^2}{4} \cdot v_2 = \frac{\pi d^2}{4} v, \text{ therefore,}$$

$d_1^2 v_1 = d_2^2 v_2 = d^2 v$, or $v_1 = \left(\frac{d}{d_1}\right)^2 v$, and $v_2 = \left(\frac{d}{d_2}\right)^2 v$, and, hence, we may put

$$h_5 = \zeta \cdot \frac{l_1 d^4}{d_1^5} \cdot \frac{v^2}{2g}, \text{ and } h_6 = \zeta \cdot \frac{l_2 d^4}{d_2^5} \cdot \frac{v^2}{2g},$$

and for velocities (v_1 and v_2) of from 5 to 10 feet,

$$\zeta = 0,021 \text{ to } 0,020.$$

In order to reduce these resistances, the pipes must be of as great diameter as possible, and the number of strokes as few as possible.

The motion of water in the pipes of a water-pressure engine is different from that in ordinary conduit pipes, inasmuch as in the former the velocity continually varies, whilst in the latter it is sensibly uniform.

Hence the *inertia of the water* plays a more conspicuous part in the one than in the other. In order to put a mass, M, into motion with a velocity v, there is required to be expended an amount of mechanical effect represented by $\frac{M v^2}{2}$; and hence to communicate to the column of water in the pressure pipes a velocity v_1, the weight being $F_1 l_1 \gamma$, there is required an amount of mechanical effect $= F_1 l_1 \gamma \cdot \frac{v_1^2}{2g}$. If the water column be cut off from the working cylinder only at the end of the stroke of the piston, this amount of effect would not be lost, for this column would restore, or give back the mechanical effect, during the gradual cessation of the piston's motion; but the cutting off of the water pressure from the working piston takes place, although near the end of the stroke, yet gradually and while the piston is in motion, so that the working piston and column of water come to rest at the same instant; and hence the valve piston causes a gradual absorption of all the *vis viva* of the water column during the first half of its ascent, inasmuch as it brings a gradually increasing resistance in the way, by gradually decreasing the passage, and hence we may assume that the mechanical effect due to inertia, $F_1 l_1 \gamma \cdot \frac{v_1^2}{2g}$ is *lost* at each stroke.

If we introduce $v_1 = \frac{d^2}{d_1^2} v$, and $F_1 = \frac{\pi d_1^2}{4}$, then we have for the above amount of mechanical effect $\frac{\pi d^2}{4} \cdot \frac{d^2 l_1}{d_1^2} \gamma \cdot \frac{v^2}{2g}$, and, hence, the mean *effort* during the whole stroke s,

$$= \frac{\pi d^2}{4} \cdot \frac{d^2 l_1}{d_1^2 s} \gamma \cdot \frac{v^2}{2g},$$

and the corresponding loss of fall or pressure head:

$$h_7 = \frac{d^2 l_1}{d_1^2 s} \cdot \frac{v^2}{2g}.$$

A loss, that would be expressed in a similar manner, takes place on the return stroke, during which the water is forced out of the cylinder with a velocity v_2, and the *vis viva* communicated to it at the commencement of the stroke is of course lost to the efficiency of the engine. The pressure head lost would be:

$$h_8 = \frac{d^2 l_2}{d_2^2 s} \cdot \frac{v^2}{2g}.$$

To keep these two losses by inertia as small as possible, it is requisite to have the pressure and discharge pipes of greatest diameter, and least length possible, to have a small velocity of the piston, and a long stroke.

Remark. To mitigate or to get rid of the prejudicial effect of shock, which the sudden cutting off of the water gives rise to, an air vessel has been introduced in many engines, at the lower end of the pressure pipes, and near the valves. This is a cylinder filled with compressed air, analogous to the air vessels on fire engines. The air in this case absorbs the excess of *vis viva* in the water, being compressed by it; and the air expanding again,

restores this *vis viva* at the commencement of the next stroke, the water being forced from the air vessel into the working cylinder, nearly as if under the original hydrostatic pressure. In the application of this arrangement to machines having very great falls, the air in the vessel has been found to mix with the water, so that it is gradually removed from it entirely. To prevent this, either a piston must be fitted into the air cylinder, or air must be continually supplied to it by a small air pump to make up the absorption of it by the water.

§ 198. *Changes in direction and in sectional areas* of the various pipes of a water-pressure engine are further causes of diminished efficiency. Although these losses may be calculated by the formulas given in the third and fourth parts of the sixth section of the first volume, it appears necessary that we should here bring together the formulas to be applied.

In the pressure and discharge pipes, there are bent knee pieces, the motion of the water through which involves a loss of head, which may be expressed, according to Vol. I. § 334, by the formula $h = \zeta_1 \frac{\beta}{\pi} \cdot \frac{v^2}{2g}$. Here β is the arc of curvature, generally $= \frac{\pi}{2}$, ζ_1 is a co-efficient depending on the ratio between the radius r of the sectional area of the pipe, and the radius of curvature of the axis of the pipe, and which may be calculated by the formula

$\zeta_1 = 0,131 + 1,847 \left(\frac{r}{a}\right)^{\frac{7}{2}}$, or may be taken from the tables given at the place cited. For a bend in the pressure pipe, the head due to the resistance is

$$h_9 = \zeta_1 \cdot \frac{\beta_1}{\pi} \cdot \frac{v_1^2}{2g} = \zeta_1 \frac{\beta_1}{\pi} \cdot \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g},$$

and for a bend in the discharge pipe:

$$h_{10} = \zeta_1 \cdot \frac{\beta_2}{\pi} \cdot \frac{v_2^2}{2g} = \zeta_1 \frac{\beta_2}{\pi} \cdot \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g}.$$

At the entrance of the water into the valve cylinder, as well as at its discharge from it, the water is suddenly turned aside at a right angle, exactly as in an elbow, or rectangular knee piece. There is, therefore, a loss of head in this case, which, according to Vol. I. § 333, may be put: $h = 0,984 \frac{v^2}{2g}$, or almost equal to $\frac{v^2}{2g}$. For uniformity's sake, we shall put this loss of head for the pressure pipe:

$$h_{11} = \zeta_1 \cdot \frac{v_1^2}{2g} = \zeta_2 \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g},$$

and for the discharge pipe:

$$h_{12} = \zeta_1 \cdot \frac{v_2^2}{2g} = \zeta_2 \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g}.$$

Sudden *changes in sectional area*, as, for example, at the entrance and discharge of the water into and from the working cylinder, give rise, in like manner, to a loss of pressure head. According to Vol. I. § 337, such a loss is determined by the formula, $h = \left(\frac{F}{F_1} - 1\right)^2 \frac{v^2}{2g}$. For the entrance of the water into the work-

ing cylinder, this formula applies directly, if F and F_1 be the areas of the cylinder and pressure pipes respectively. For the discharge on the other hand $\frac{F}{F_1} = \frac{1}{a}$, in which a is the co-efficient of contraction. If $a = 0,6$, then $\left(\frac{1}{a} - 1\right)^2 = \frac{4}{9}$; and hence the head due to the resistance to the entrance of the water into the cylinder:

$$h_{13} = \left(\frac{F}{F_1} - 1\right)^2 \cdot \frac{v^2}{2g},$$

and for the discharge:

$$h_{14} = \left(\frac{1}{a} - 1\right)^2 \cdot \frac{v_1^2}{2g} = \frac{4}{9} \cdot \left(\frac{F}{F_1}\right)^2 \cdot \frac{v^2}{2g}.$$

For simplicity's sake, however, we shall put

$$h_{13} = \zeta_3 \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g}, \text{ and } h_{14} = \zeta_4 \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g},$$

so that, when $F = \frac{\pi d^2}{4}$ is introduced, and $F_1 = \frac{\pi d_1^2}{4}$, then

$$\zeta_3 = \left(\frac{F}{F_1} - 1\right)^2 \left(\frac{d_1}{d}\right)^4 = \left[1 - \left(\frac{d_1}{d}\right)^2\right]^2, \text{ and}$$

$$\zeta_4 = \frac{4}{9} \left(\frac{F}{F_1}\right)^2 \left(\frac{d_2}{d}\right)^4 = \frac{4}{9}\left(\frac{d_2}{d_1}\right)^4.$$

To avoid loss of effect by sudden variations of velocity generally, the intermediate pipes, and parts of the valve cylinder through which the water passes, should have the same area as the pressure and discharge pipes, or, at all events, the intermediate passages should gradually widen out to the area of the main pipes.

There are further special losses of effect occasioned by the cocks and throttle valves. These are to be calculated by the formula $h = \zeta_s \cdot \frac{v^2}{2g}$ and the co-efficients ζ_s depend on the position or angle of the cocks, &c., and are to be taken from the tables, Vol. I. § 340. Hence for the ascent of the working piston:

$$h_{15} = \zeta_5 \cdot \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g}, \text{ and for the descent, } h_{16} = \zeta_6 \cdot \left(\frac{d}{d_2}\right)^4 \cdot \frac{v^2}{2g}.$$

By setting the regulating cock or valve, the co-efficient of resistance may be varied to any amount from 0 to ∞, or any excess of power may be absorbed, and the velocity of the piston checked at pleasure.

§ 199. *Formula for the Useful Effect.*—If in the mean time we leave the valves out of consideration, we can now construct a formula representing the useful effect of a water-pressure engine. The mean effort during an ascent of the piston is

$$P = [h_1 - (h_3 + h_5 + h_7 + h_9 + h_{11} + h_{13} + h_{15})]\, F\gamma,$$

and the resistance in the descent:

$$P_1 = (h_2 + h_4 + h_6 + h_8 + h_{10} + h_{12} + h_{14} + h_{16})\, F\gamma;$$

and hence the effect for a double stroke:

$$(P - P_1)\, s = [h_1 - (h_2 + h_3 + h_4 + \ldots + h_{16})]\, F s \gamma,$$

and the mechanical effect produced per second:

$$L = [h_1 - (h_2 + h_3 + h_4 + \ldots + h_{16})] \cdot \frac{n}{60} \cdot F s \gamma.$$

If, again, we put:

$$\zeta \frac{l_1 d^4}{d_1^5} + \frac{d^2 l_1}{d_1^3 s} + \zeta_1 \frac{\beta_1}{\kappa}\left(\frac{d}{d_1}\right)^4 + \zeta_2\left(\frac{d}{d_1}\right)^4 + \zeta_3\left(\frac{d}{d_1}\right)^4 + \zeta_s\left(\frac{d}{d_1}\right)^4, \text{ or}$$

$$\left(\zeta \frac{l_1}{d_1} + \frac{d_1^2 l_1}{d^2 s} + \zeta_1 \cdot \frac{\beta_1}{\kappa} + \zeta_2 + \zeta_3 + \zeta_s\right)\left(\frac{d}{d_1}\right)^4, = \kappa_1\left(\frac{d}{d_1}\right)^4, \text{ and}$$

$$\zeta \frac{l_2 d^4}{d_2^5} + \frac{d^2 l_2}{d_2^3 s} + \zeta_1 \cdot \frac{\beta_2}{\kappa}\left(\frac{d}{d_2}\right)^4 + \zeta_2\left(\frac{d}{d_2}\right)^4 + \zeta_4 \cdot \left(\frac{d}{d_2}\right)^4 + \zeta_s\left(\frac{d}{d_2}\right)^4, \text{ or}$$

$$\left(\zeta \frac{l_2}{d_2} + \frac{d_2^2 l_2}{d^2 s} + \zeta_1 \frac{\beta_2}{\kappa} + \zeta_2 + \zeta_4 + \zeta_s\right)\left(\frac{d}{d_2}\right)^4 = \kappa_2\left(\frac{d}{d_2}\right)^4,$$

then we may express the useful effect very simply and comprehensively, by

$$L = \left[h - \left(4 f \frac{b}{d}(h_1 + h_2) + \left[\kappa_1\left(\frac{d}{d_1}\right)^4 + \kappa_2\left(\frac{d}{d_2}\right)^4\right] \cdot \frac{v^2}{2g}\right)\right] \cdot \frac{n}{60} F s \gamma.$$

On account of the greater length of the pressure pipes, κ_1 is considerably more than κ_2; and, therefore, the time for the up stroke t_1 is usually allowed to be longer than that for the down stroke t_2.

If we make the ratio $\frac{t_1}{t_2} = \nu = \frac{3}{1}$, then

$$t_1 = \frac{\nu}{\nu + 1} \cdot \frac{60''}{n}, \text{ and } t_2 = \frac{1}{\nu + 1} \cdot \frac{60''}{n};$$

and if we retain v as the value of the mean velocity of a double stroke $\frac{2s}{t_1 + t_2} = \frac{2ns}{60''}$, then the mean velocity during the up stroke

$$= \frac{s}{t_1} = \frac{\nu + 1}{\nu} \cdot \frac{ns}{60} = \frac{\nu + 1}{\nu} \cdot \frac{v}{2},$$

and that during the down stroke

$$= \frac{s}{t_2} = (\nu + 1) \cdot \frac{ns}{60''} = (\nu + 1) \cdot \frac{v}{2},$$

and, hence, the useful effect may be expressed more generally:

$$L = \left[h - \left(4 f \frac{b}{d}(h_1 + h_2) + \left[\kappa_1\left(\frac{\nu + 1}{2\nu}\right)^2\left(\frac{d}{d_1}\right)^4 \right.\right.\right.$$
$$\left.\left.\left. + \kappa_2\left(\frac{\nu + 1}{2}\right)^2\left(\frac{d}{d_2}\right)^4\right]\frac{v^2}{2g}\right)\right] \cdot \frac{n}{60} F s \gamma,$$

or, introducing $\frac{n}{60} \cdot F s = Q$,

$$L = \left[h - \left(4 f \frac{b}{d}(h_1 + h_2) + \left[\kappa_1\left(\frac{1}{\nu}\right)^2\left(\frac{d}{d_1}\right)^4 \right.\right.\right.$$
$$\left.\left.\left. + \kappa_2\left(\frac{d}{d_2}\right)^4\right]\left(\frac{\nu + 1}{2}\right)^2 \cdot \frac{v^2}{2g}\right)\right] Q \gamma,$$

or, introducing $v = \frac{Q}{F} = \frac{4 Q}{\kappa d_2}$,

$$L = \left(h - \left[4 f \frac{b}{d}(h_1 + h_2) + \left(\frac{\kappa_1}{\nu^3 d_1^4} + \frac{\kappa_1}{d_2^4}\right)\left(\frac{\nu + 1}{2}\right)^2 \cdot \frac{1}{2g} \cdot \left(\frac{4\,Q}{\pi}\right)^2\right]\right) Q \gamma.$$

In the double-acting, water-pressure engine, the mechanical effect produced is of course doubled.

This formula shows, very clearly, that the useful effect of a water-pressure engine is greater, the greater d, d_1 and d_2 are, or the wider the cylinder and pipes. It is also demonstrable, by aid of the higher calculus, that for a given number of strokes the useful effect is a maximum, or the prejudicial resistances are a minimum, when $\frac{\kappa_1}{\nu^3 d_1^4} = \frac{\kappa^2}{d_2^4}$, that is, when $\nu = \sqrt[3]{\frac{\kappa_1 d_2^4}{\kappa_2 d_1^4}}$. If, for example, $d_2 = d_1$, and $\kappa_1 = 8\,\kappa_2$, then $\nu = \sqrt[3]{8} = 2$, or the time for the up stroke would be double that for the down stroke. By applying a balance beam, attached to the working piston rod, this ratio ν, between the time for the up and down stroke, may be adjusted by the counter-balance weight applied. Any regulation by means of the throttle valve, or cocks, on the pressure or discharge pipes, can only be effected at the cost of useful effect, as by these a loss of power measured by ζ_s is occasioned, and which increases in proportion as the passages are contracted.

If the mechanical effect required be less than the best effect of the engine, the excess must be destroyed or checked by the throttle valves.

Example. It is required to make the calculations necessary for establishing a single-acting, single cylinder, water-pressure engine for a fall $h = 350$ feet, and a quantity of water $Q = 1$ cubic foot per second.

Suppose v the mean velocity of the up and down stroke $= 1$ foot, then for its area, we have $F = \frac{2\,Q}{v} = \frac{2\,.\,1}{1} = 2$ square feet; and if we arrange that the water shall move through the pressure and discharge pipes with a velocity $v_1 = v_2 = 5$ feet, then for the section of these pipes, we have $F_1 = \frac{2\,Q}{v_1} = \frac{2}{5} = 0{,}4$ square feet. Hence the diameter of the working piston, $d = \sqrt{\frac{4\,F}{\pi}} = \sqrt{\frac{8}{\pi}} = 1{,}5958$ feet; and that of the pressure and discharge pipes, $d_1 = d_2 = \sqrt{\frac{4\,F_1}{\pi}} = \sqrt{\frac{1{,}6}{\pi}} = 0{,}71364$ feet. For simplicity and certainty, we shall assume $d = 20$ inches, and $d_1 = d_2 = 7$ inches.

If, for counter-balancing the rods, &c., we carry up the discharge pipe 50 feet above the mean height of the piston, or make $h_2 = 50$ feet, then $h_1 = h + h_2 = 400$ feet. We shall assume further, that the total length of pressure pipe $l_1 = 450$ feet, and that of the discharge pipe $l_2 = 66$ feet. For a diameter of 20 inches,

$$F = \frac{\pi d^2}{4} = \frac{\pi}{4} \cdot \frac{25}{9} = 2{,}182 \text{ square feet} \therefore v = \frac{2\,Q}{F} = \frac{2}{2{,}182} = 0{,}9166 \text{ feet.}$$

Suppose we have 4 strokes per minute, then the length of stroke

$$s = \frac{60\,v}{2\,n} = \frac{60\,.\,0{,}9166}{8} = 6{,}8745 \text{ feet.}$$

If, again, we suppose the width of the packing of the piston $b = \frac{1}{8} d = 2\frac{1}{2}$ inches, we get as the pressure height absorbed by the friction of the piston:

$$4\,f \frac{b}{d}\,(h_1 + h_2) = 4\,.\,0{,}25\,.\,\tfrac{1}{8}\,(400 + 50) = \frac{450}{8} = 56{,}25 \text{ feet,}$$

or there remains, after deducting the piston friction, the head 350 — 56,25 = 293,75 feet. To calculate the hydraulic resistances, we must, in the first place, determine x_1 and x_2. That for the pressure pipe,

$$x_1 = \zeta \frac{l_1}{d_1} + \frac{d_1^2 l_1}{d^2 s} + \zeta_1 \frac{\beta_1}{\pi} + \zeta_2 + \zeta_3 + \zeta_5,$$

and that for the discharge pipe:

$$x_2 = \zeta \frac{l_2}{d_2} + \frac{d_2^2 l_2}{d^2 s} + \zeta_1 \frac{\beta_2}{\pi} + \zeta_2 + \zeta_4 + \zeta_5,$$

and in these expressions:

$$\zeta = 0{,}021,\ \frac{l_1}{d_1} = \frac{450}{\frac{3}{4}} = 600, \text{ and } \frac{l_2}{d_2} = \frac{66}{\frac{3}{4}} = 88;$$

therefore, $\zeta \frac{l_1}{d_1} = 0{,}021 \,.\, 600 = 12{,}6$, and $\zeta \frac{l_2}{d_2} = 0{,}021 \,.\, 88 = 1{,}85$. Again,

$$\frac{d_1^2 l_1}{d^2 s} = \left(\frac{9}{20}\right)^2 \cdot \frac{450}{6{,}87} = 13{,}26, \text{ and } \frac{d_2^2 l_2}{d^2 s} = \left(\frac{9}{20}\right)^2 \frac{66}{6{,}87} = 1{,}94.$$

If we further assume, that the bends in the pipes have radii of curvature $a = 4r$, or if $\frac{r}{a} = \frac{1}{4}$, we have as the co-efficient of resistance in bends:

$\zeta_1 = 0{,}131 + 1{,}847 \left(\frac{1}{4}\right)^{\frac{7}{2}} = 0{,}145$, and if the aggregate angle of deflexion by curves in the pressure and discharge pipes $= 270°$, or if:

$$\frac{\beta_1}{\pi} = \frac{\beta_2}{\pi} = \frac{270°}{180°} = \frac{3}{2} \text{ then, } \zeta_1 \frac{\beta_1}{\pi} = \zeta_1 \frac{\beta_2}{\pi} = 0{,}145 \,.\, \tfrac{3}{2} = 0{,}22.$$

If, further, the water, before and after its work is done in the cylinder, makes two rectangular deviations in its progress through the valve cylinder, we have, in the formulas for x_1 and x_2, $\zeta_2 = 2 \,.\, 1 = 2$; and if the valve cylinder is of the same diameter as the pressure and connecting pipes, the co-efficient of resistance for the up stroke

$\zeta_3 = \left[1 - \left(\frac{d_1}{d}\right)^2\right]^2 = (1 - 0{,}2025)^2 = 0{,}64$, whilst for the down stroke $\zeta_4 = \frac{4}{9} = 0{,}44$.

If the throttle and other passage valves be fully open, then $\zeta_5 = 0$, and, therefore, we have $x_1 = 12{,}60 + 13{,}26 + 0{,}22 + 2{,}00 + 0{,}64 = 28{,}72$, and

$$x_2 = 1{,}85 + 1{,}94 + 0{,}22 + 2{,}00 + 0{,}44 = 6{,}45$$

Lastly, we have the best ratio of the times for the up stroke and down stroke:

$$\nu = \sqrt[3]{\frac{x_1}{x_2}} = \sqrt[3]{\frac{28{,}72}{6{,}45}} = 1{,}646, \text{ or nearly 5 to 3.}$$

By introducing these values, we get the height of column remaining:

$$h - \left[4 f \frac{b}{d} (h_1 + h_2) + \left(\frac{x_1}{\nu^2 d_1^4} + \frac{x_2}{d_2^4}\right) \left(\frac{\nu + 1}{2}\right)^2 \cdot \frac{1}{2g} \left(\frac{4 Q}{\pi}\right)^2\right]$$

$$= h - \left[4 f \frac{b}{d} (h_1 + h_2) + \left(\frac{x_1}{\nu_2} + x_2\right) \left(\frac{\nu + 1}{2}\right)^2 \cdot \frac{1}{2g} \cdot \left(\frac{4 Q}{\pi d_1^2}\right)^2\right]$$

$$= 293{,}75 - \left(\frac{28{,}72 \,.\, 9}{25} + 6{,}45)\right) \left(\frac{4}{3}\right)^2 . 0{,}0155 . \left(\frac{4 \,.\, 16}{9 \,.\, \pi}\right)^2$$

$$= 293{,}75 - 16{,}79.\ 0{,}0155 \,.\, \frac{16 \,.\, 4096}{729 \,.\, \pi^2} = 293{,}75 - 2{,}37 = 291{,}38 \text{ feet.}$$

From this we get the efficiency of this engine, neglecting the mechanical effect required for working the valves, $\eta = \frac{291{,}38}{350} = 0{,}832$, and the useful effect:

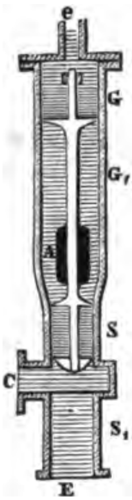
$L = 291{,}38 \,.\, 1 \,.\, 62{,}5 = 18211$ feet lbs., or 3,1 horse power, nearly.

§ 200. *Adjustment of the Valves.*—The arrangement and proper adjustment of the valves is a most important part of the water-pressure engine. As in all the engines we have described piston valves are used, we shall, in what follows, confine ourselves to the consideration of these arrangements.

We shall first consider the system having *two* pistons, as used in some of the Saxon engines, and represented in Fig. 316.

If we assume that the valve piston S is pressed upwards with a mean pressure h_1, and downwards with a pressure h_2; and if the height of the counter piston G above $S = e$, and, therefore, the height of the hydrostatic column under $G = h_2 - e$, and that above G according as the water is let on or shut off, $h_1 - e$, or $h_2 - e$. If further, $d_1 =$ the diameter of S, and $d_2 =$ that of G, and we shall assume that the packing of the two pistons consists of leather discs pressed together, and that they are about the same height or thickness. If, now, this piston valve system be *up*, as shown in Fig. 316, the letting on of the pressure water above G would occasion a descent of the valves, and, therefore, the difference of the water pressure on S and G, in combination with the weight R of the system, must be sufficient to overcome the friction of the piston S and G. The pressure downwards on

Fig. 316.



$$G = \frac{\pi d_2^2}{4} (h_1 - e) \gamma,$$

and the counter pressure under

$$G = \frac{\pi d_2^2}{4} (h_2 - e) \gamma.$$

The downward pressure on $S = \frac{\pi d_1^2}{4} h_2 \gamma$, and the counter pressure under $S = \frac{\pi d_1^2}{4} h_1 \gamma$, and, hence, the power to push the system down:

$$P = \frac{\pi d_2^2}{4} (h_1 - e - h_2 + e) \gamma + \frac{\pi d_1^2}{4} (h_2 - h_1) \gamma + R$$

$$= \frac{\pi}{4} (d_2^2 - d_1^2)(h_1 - h_2) \gamma + R,$$

or the fall $h_1 - h_2$ being represented by h:

$$P = \frac{\pi}{4} (d_2^2 - d_1^2) h \gamma + R.$$

The friction of the pistons, even though they be not on the hydrostatic principle, is proportional to the circumference of the piston, and to the difference of pressure on the two sides of the piston, and may be represented by $F = \phi \pi d h \gamma$. Hence, in the case in question $P = \phi \pi (d_1 (h_1 - h_2) + d_2 [h_1 - e - (h_2 - e)]) \gamma = \phi \pi (d_1 + d_2) h \gamma$. Hence we have the following formula:

$$\frac{\pi}{4} (d_2^2 - d_1^2) h \gamma + R = \phi \pi (d_1 + d_2) h \gamma,$$

or, simplified:

$$1.) \; d_2^2 - d_1^2 + \frac{4 R}{\pi h \gamma} = 4 \phi (d_1 + d_2).$$

If, on the other hand, the valve has to rise from its lowest position after the water has been cut off, then the excess, or the differ-

ence of the water pressure on S alone, must overcome the weight of the valve system, and the friction of the pistons; because then the pressures on both sides of G cease, we must, therefore, have

$$\frac{\pi}{4} d_1^2 (h_1 - h_2) \gamma = R + \phi \pi (d_1 + d_2) h \gamma,$$

or, more simply:

$$2.)\ d_1^2 - \frac{4 R}{\pi h \gamma} = 4 \phi (d_1 + d_2).$$

These formulas will serve for calculating the diameters d_1 and d_2 of the two pistons. Neglecting R, which, in considerable falls, is almost always of small amount:

$$d_2^2 - d_1^2 = 4 \phi (d_1 + d_2), \text{ and } d_1^2 = 4 \phi (d_1 + d_2), \text{ therefore,}$$
$$d_2^2 - d_1^2 = d_1^2, \text{ or } d_2^2 = 2 d_1^2,$$

and, hence, the diameter of the counter piston:

$$d_2 = d_1 \sqrt{2} = 1{,}414\, d_1,$$

or about $\frac{7}{5}$ of the diameter of the piston valve, which is determined by the first equation:

$$d_2^2 - d_1^2 = 4 \phi (d_1 + d_2), \text{ or } d_1 - d = 4 \phi,$$

if we substitute in it: $d_1 \sqrt{2}$ for d_2.

We then have:

$$d_1 = \frac{4 \phi}{\sqrt{2} - 1} = (\sqrt{2} + 1) \,.\, 4 \phi = 2{,}414 \,.\, 4 \phi, \text{ and } d_2 = 3{,}414 \,.\, 4 \phi.$$

Taking the weight of the pistons into account, we have, with sufficient accuracy,

$$d_2 = \sqrt{2 d_1^2 - \frac{8 R}{\pi h \gamma}} = d_1 \sqrt{2} - \frac{4 R}{\pi h \gamma d_1 \sqrt{2}}$$
$$= d_1 \sqrt{2} - \frac{(\sqrt{2} - 1) R}{\phi \pi h \gamma \sqrt{2}},$$

and from this, we have by the equation 1:

$$d_2 - d_1 = 4 \phi - \frac{4 R}{\pi h \gamma (d_1 + d_2)}, \text{ i. e.}$$
$$(\sqrt{2} - 1) d_1 = 4 \phi + \frac{(\sqrt{2} - 1)}{\phi \pi h \gamma \sqrt{2}} R - \frac{(\sqrt{2} - 1) R}{\phi \pi h \gamma (1 + \sqrt{2})}, \text{ i. e.}$$
$$d_1 = (\sqrt{2} + 1) 4 \phi + \frac{(2 - \sqrt{2}) R}{2 \phi \pi h \gamma}, \text{ and}$$
$$d_2 = (\sqrt{2} + 2) 4 \phi + \frac{(3 \sqrt{2} - 4) R}{2 \phi \pi h \gamma}.$$

For the sake of certainty in the working, both diameters are made somewhat greater, and the excess of power is absorbed by setting the regulating cocks, already mentioned, so as to exactly adjust the area of passage. Judging from the best existing engines, we may take $4 \phi = 0{,}1$, or $\phi = \frac{1}{40}$. In order that, in the passage of the pressure water through the valve cylinder, there may be the least possible hydraulic resistance, it is usually made of equal area, at that part, with the area of the pressure and intermediate pipes; and supposing the formulas give a diameter d_1, which is less than

that of the pressure pipes, we may consider that there exists an excess of power, which must be adjusted by the regulating cocks.

Example. It is required to determine the proportions of a two-piston valve system for a water-pressure engine of 400 feet fall. Suppose the weight of the pistons and rod, &c. $= 150$ lbs. Leaving this weight out of the calculation, the diameter $d_1 = 2{,}414 \,.\, 4\,\phi = 2{,}414 \,.\, 0{,}1 = 0{,}2414$ feet $= 2{,}897$ inches, and $d_2 = 3{,}414 \,.\, 0{,}1 = 0{,}3414 = 4{,}097$ inches. Taking the weight of pistons, &c., into account $d_1 = 0{,}2414 + \frac{0{,}586 \,.\, 150}{0{,}05 \,.\, 400 \,.\, 62{,}5\,\pi} = 0{,}2414 + \frac{0{,}586}{8{,}33 \,.\, \pi} = 0{,}2414 + 0{,}0223 = 0{,}2637$ ft. $= 3{,}164$ inches, and $d_2 = 0{,}3414 + \frac{0{,}243 \,.\, 150}{0{,}05 \,.\, 400 \,.\, 62{,}5\,\pi} = 0{,}3414 + 0{,}0092 = 0{,}3516$ feet $= 4{,}219$ inches. It will be sufficient in this case, if we take $d_1 = 3\frac{1}{2}$, and $d_2 = 5$ inches. For so small a counter-balance to piston valve, only a small supply of water is necessary; but the resistance in the passage through the valve cylinder would be great. If, on this account, we put $d_1 = 6$ inches, then we should have to make d_2 at least $= d_1\sqrt{2} = 8{,}484$ inches, that is from $8\frac{3}{4}$ to 9 inches, the excess of power being absorbed by adjusting the cocks.

§ 201. In the three-piston valve system, the mode of calculation is very similar to that gone through above. The advantage of this system is, that we may make one of the pistons, the valve piston proper, for example, of the same diameter as the pressure pipes. The calculations for the valves in the engine represented in Fig. 311, may be made as follows: Putting $d_1 =$ the diameter of the lower piston, or first valve piston, and d_2 that of the second, and d_3 that of the upper or counter piston; then, for the descent, we have

$$1.)\ d_1^2 - d_2^2 + d_3^2 + \frac{4\,R}{\pi\,h\,\gamma} = 4\,\phi\,(d_1 + d_2 + d_3),$$

and for the ascent:

$$2.)\ d_2^2 - d_1^2 - \frac{4\,R}{\pi\,h\,\gamma} = 4\,\phi\,(d_1 + d_2 + d_3).$$

From d_1 we can, by means of these formulas, determine d_2 and d_3, making d_3, however, somewhat greater than the calculation gives for insuring certainty of action. If we put the value thus found into the formula

$$2\,(d_1^2 - d_2^2) + d_3^2 + \frac{8\,R}{\pi\,h\,\gamma} = 0,$$

we get as the diameter of the third piston:

$$d_3 = \sqrt{2\,(d_2^2 - d_1^2) - \frac{8\,R}{\pi\,h\,\gamma}},$$

which, for the reasons already given, should be made something more than the absolute result of calculation.

For the valve system of the engine in Fig. 312, we have the following formulas. Let $h_1 =$ the mean height of the pressure column, and $h_2 =$ the mean height of counter-balance column; d_1 the diameter of the valve piston, d_2 that of the counter piston, and d_3 that of the projection forming a third piston. The power in the descent, is then

$$\frac{\pi}{4}\,[d_1^2\,(h_1 - h_2) + (d_2^2 - d_3^2)\,h_1 - d_2^2\,h_1]\,\gamma + R,$$

and that in the ascent:

$$\frac{\pi}{4}[d_2^2 h_1 - (d_2^2 - d_3^2) h_2 - d_1^2 (h_1 - h_2)] \gamma - R;$$

therefore:

$$1.)\ d_1^2 - \frac{h_1}{h} d_3^2 + \frac{4R}{\pi h \gamma} = 4 \phi (d_1 + d_2 + d_3), \text{ and}$$

$$2.)\ d_2^2 - d_1^2 + \frac{h_2}{h} d_3^2 - \frac{4R}{\pi h \gamma} = 4 \phi (d_1 + d_2 + d_3).$$

If d_1 be given, we can then calculate d_2 and d_3, but we must keep d_2 somewhat above, and d_3 somewhat below the result of the formula. The formulas

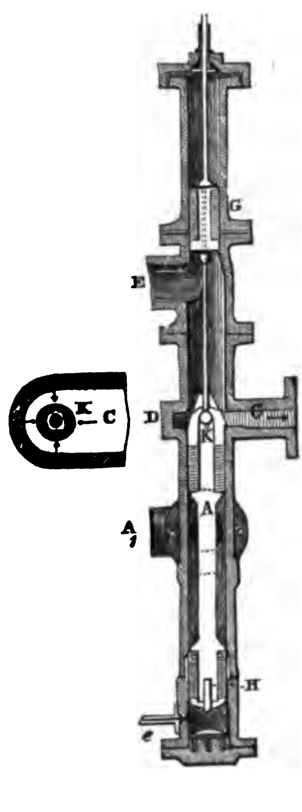
$$1.\ d_2^2 - d_3^2 = 8 \phi (d_1 + d_2 + d_3), \text{ and}$$

$$2.\ d_2^2 + \left(\frac{h_1 + h_2}{h}\right) d_3^2 = 2 d_1^2 + \frac{8R}{\pi h \gamma},$$

are of rather simpler application.

For the valve system shown in Fig. 317, already mentioned as that of the Clausthal engines, we have, when d_1 = the diameter of valve piston, d_2 the diameter of upper or counter piston, and d_3 that of the lower or auxiliary piston, the power for descent:

Fig. 317.



$$\frac{\pi}{4}[d_1^2(h_1-h_2)-d_2^2 h_1)]\gamma + R,$$

The power of ascent:

$$\frac{\pi}{4}[d_3^2(h_1-h_2)-d_1^2(h_1-h_2)+d_2^2 h_1]\gamma - R; \text{ therefore,}$$

$$1.)\ d_1^2-\frac{h_1}{h}d_2^2+\frac{4R}{\pi h\gamma}=4\phi(d_1+d_2+d_3), \text{ and}$$

$$2.)\ d_3^2-d_1^2+\frac{h_1}{h}d_2^2-\frac{4R}{\pi h\gamma}=4\phi(d_1+d_2+d_3).$$

Example. Supposing, as in the last-mentioned engine, $h_1 = 688$ feet, and $h_2 = 76$ feet, $R = 170$ lbs., and $d_1 = \frac{1}{2}$ foot, we get the diameters of the other pistons as follows: $d_3^2 = 8\phi(d_1+d_2+d_3)$, and also $= 2d_1^2 - \frac{2h_1}{h}d_2^2 + \frac{8R}{\pi h\gamma}$, or, in numbers: $d_3^2 = 0{,}2(0{,}5+d_2+d_3)$, and $= 0{,}5 - 2{,}248\, d_2^2 + 0{,}0107$. If, now, we assume $d_2 = 0{,}3$ feet, we have by one formula $d_3^2 = 0{,}5107 - 0{,}2023 = 0{,}3084$, that is $d_3 = 0{,}555$; and by the second formula, $d_3^2 = 0{,}2 \,.\, 1{,}355 = 0{,}2710$, *i. e.* $d_3 = 0{,}5205$. But if we put $d_2 = 0.33$, then $d_3^2 = 0{,}5107 - 0{,}2448 = 0{,}2659$, or $d_3 = 0{,}516$, and, again, $d_3^2 = 0{,}2 \,.\, 1{,}346 = 0{,}2692$, or $d_3 = 0{,}519$. Hence $d_2 = 0{,}33 \,.\, 12 = 3{,}96$, or about 4 inches, and $d_3 = 0{,}52 \,.\, 12 = 6{,}24$, or $6\frac{1}{4}$ inches. Jordan, the engineer, who erected these machines, has made $d_2 = 4$ inches, 1,6 lines, and $d_3 = 5$ inches, $9\frac{3}{4}$ lines, from which we deduce that 4ϕ is somewhat less than 0,1 in this case.

Remark. To calculate more accurately, the diameter of the valve rod would have to be taken into account.

§ 202. *Water for the Valves.*—The quantity of water required, for the motion of the valves, gives rise to the loss of a certain amount of mechanical effect, or to a diminution of the engine's efficiency, because it is abstracted from the water working the engine. It should, therefore, be rendered as little as possible, that is d_3, the diameter of the counter piston, and its *stroke* should be as small as possible. The stroke depends on the depth of the valve piston, or on the diameter of the intermediate pipe. The intermediate pipe is, therefore, made rectangular: of the width of the working cylinder, and low in proportion. As it is made of the same area as the pressure pipes, we have $ad = \frac{\pi d_1^2}{4}$, and, therefore, the height, or least dimension of the intermediate pipe $a = \frac{\pi d_1^2}{4d}$.

That the valve piston may cut off the water exactly at the end of the stroke, it is made three times the height of the pipe, or its depth is $a_1 = 3a$; and, hence, the stroke of the valve piston proper $s_1 = a_1 + a = 3a + a = 4a$, and the quantity of water expended for each stroke is $= \frac{\pi d_3^2}{4}s_1 = \pi a d_3^2$.

If the engine makes n strokes per minute, the quantity of water expended by the valves per second.

$$Q_1 = \frac{ns_1}{60}\cdot\frac{\pi d_3^2}{4} = \frac{na}{60}\pi d_3^2,$$

and, hence, the loss of effect corresponding:

$$L_1 = \frac{n s_1}{60} \cdot \frac{\pi d_3^2}{4} \cdot h \gamma,$$

or the loss is the less the longer the stroke of the engine.

As to the valve gear, the power required to work it is so small that it may be left out of consideration. The study of the arrangement of the mechanism arises under another section of our work.

Example. If in the water-pressure engine, the subject of the example calculated, there be applied a valve piston of 9 inches diameter, and, therefore, a counter piston $= 9\sqrt{2} = 13$ inches diameter. If, further, the intermediate pipe have a height $a = \frac{\pi d_1^2}{4 d} = \frac{9^2 \pi}{4 \cdot 20} = \frac{81 \pi}{80} = 3{,}18$ inches, then the valve piston must have a height $a_1 = 3\,a = 9{,}54$ inches, and its stroke $s_1 = a_1 + a = 12{,}72$ inches $= 1{,}06$ feet; and therefore the quantity of water expended each stroke $= \frac{\pi}{4}\left(\frac{13}{12}\right)^2 \cdot 1{,}06 = 0{,}977$ cubic feet; and hence the loss of effect per second:

$L_1 = \frac{n}{60} \cdot 0{,}977 \cdot h\gamma = \frac{4}{60} \cdot 0{,}977 \cdot 350 \cdot 62 \cdot 5 = 1424.^{3}$ feet lbs., or nearly 3 *horse power.*

It would certainly be better in this case to make the piston valves less in diameter, and have a lower intermediate pipe; for although this would increase the hydraulic resistances, still it would not involve so great a loss as the waste of water we have calculated implies.

§ 203. *Experimental Results.*—There are not many good experiments on the effect of water-pressure engines. These engines are usually employed as pumping engines in mines, and the experiments that have been made involve the whole machinery, as well as the engines themselves, in the results as to the efficiency. But it is very easy to get an approximate determination of this efficiency, if we assume that the efficiency of water-pressure engines and pumps are in certain proportions to each other. This assumption we may make with perfect propriety, as the engine and machine are very analogous in their construction and movements. We shall not give any advantage to the water-pressure engine, nor be far from the truth, if we suppose the loss of effect of the whole apparatus to be one-half due to the water-pressure engine. The calculation then becomes very simple.

The effect at disposition is $\frac{n}{60}(Fs + F_1 s_1) h \gamma$, in which F_1 is the section, and s_1 the stroke of the auxiliary piston. The effect produced, however, is $\frac{ns}{60} F_2 h_2 \gamma$; if $F_2 =$ the section of the pump piston, and h_2 the height, the water is raised by the pump. The loss of effect is, therefore,

$$= \frac{n}{60}(Fs + F_1 s_1) h \gamma - \frac{ns}{60} F_2 h_2 \gamma,$$

the half of which is:

$$= \frac{n\gamma}{120}\left[(Fs + F_1 s_1) h - F_2 s h_2\right],$$

and hence the efficiency of the water-pressure engine:

$$\eta = 1 - \tfrac{1}{2}\frac{(Fs + F_1 s_1) h - F_2 s h_2}{(Fs + F_1 s_1) h} = \tfrac{1}{2} + \frac{F_2 s h_2}{2(Fs + F_1 s_1) h} = \tfrac{1}{2}(1 + \eta_1),$$

if η_1 be the efficiency of the combined engine and pumps. In this mode of calculation, it is assumed that there are no losses of water, and when the machinery is in good order, this loss is so small that it may be neglected. Jordan found for the Clausthal engines, that the loss of water in the water-pressure engines is only ½ per cent., and in the pumps 2½ per cent. The experiments are made by opening the regulating apparatus in pressure and discharge pipes, and then raising the height of the *pump* column, or increasing the work to be done till the required number of strokes is performed *uniformly*.

By experiments on this principle, Jordan found that one of the Clausthal engines gave, when making 4 strokes per minute, $\eta_1 = 0{,}6568$, and making 3 strokes $\eta_1 = 0{,}7055$, and, therefore, in the first case, $\eta = \frac{1{,}6568}{2} = 0{,}8284$, and in the second:

$\eta = \frac{1{,}7055}{2} = 0{,}8527$, and hence as a mean $\eta = 0{,}84$. When the greatest effect of a water-pressure engine cannot be determined by the method of heightening the pump column till a uniform motion is established, it may be done perhaps by diminishing the water-pressure column. This, however, can only be done when the excess of power of the engine is small, that is, when the part of the water-column to be taken off is small. The water may be kept at a certain level in the pressure pipes, ascertained by a float, and in this way the efficiency for a *certain head* be determined. The engine in Alte Mordgrube, near Freyberg, was experimented on in this way, and it was found that for 3 strokes per minute $\eta_1 = 0{,}684$, and hence the efficiency of the water-pressure engine alone is to be estimated as $\eta = \frac{1{,}684}{2} = 0{,}842$.

The most of the results reported in reference to the effect of water-pressure engines are too uncertain to be worthy of much confidence, having been deduced from experiments in which essential circumstances were not noted. If we take ζ as the co-efficient of resistance corresponding to a certain position of the regulating valves or cocks, as given in the table, Vol. I. § 340, the fall y, lost by this contraction, may be estimated by the formula:

$$y = \zeta \cdot \frac{v_1^2}{2g} = \zeta \cdot \left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g},$$

and we can, therefore, estimate the efficiency by the formula:

$$\eta = \frac{1}{2}\left(1 + \frac{F_2 s h_2}{Fs\left[h - \zeta\left(\frac{d}{d_1}\right)^4 \cdot \frac{v^2}{2g}\right] + F_1 s_1 h}\right).$$

Example. A pressure engine consumes 10 cubic feet of water per second, besides 0,4 cubic feet for the valves. The fall = 300 feet, the mean velocity of the water in the pressure pipes = 6 feet per second, and the circular throttle valves in the main pipe stands at 60°. Suppose, that by this engine, there is raised at each stroke 3,5 cubic feet 420 feet high, at what is the efficiency of the engine to be estimated? According to Vol. I. § 340, for the position of the valve 60°,

$$\zeta = 118, \therefore \zeta \cdot \frac{v_1^2}{2g} = 118 \cdot 0{,}0155 \cdot 6^2 = 62{,}2 \text{ feet; and hence,}$$

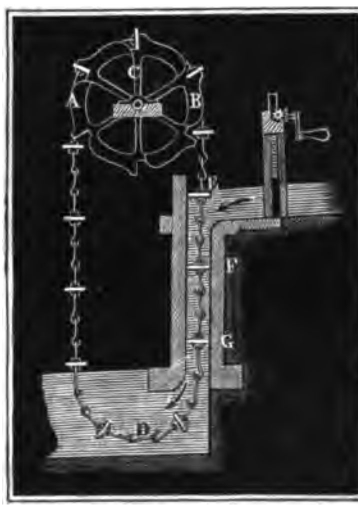
$$\eta = \tfrac{1}{2}\left(1 + \frac{3{,}5 \cdot 420}{10\,(300 - 62{,}2) + 0{,}4 \cdot 300}\right) = \tfrac{1}{2}\left(1 + \frac{3{,}5 \cdot 42}{237{,}8 + 12}\right) = \tfrac{1}{2} \cdot 1{,}588 = 0{,}794.$$

§ 204. *Chain Wheels.*—There are other water-power machines, neither wheels nor pressure engines, but which are to be met with from time to time. We may mention the following:—

The chain of buckets (Fr. *roue à piston;* Ger. *Kolbenrad*) has recently been revived as a machine recipient of water power by Lamolières (*see* "Technologiste," Sept. 1845).

The principal parts of this machine are *ACB*, Fig. 318, over which passes a chain *ADB*, forming the axis connecting a series of pistons (called buckets or saucers), *E*, *F*, *G*, &c., and a pipe *EG*, through which the chain passes in such manner, that the pistons nearly fill the section of the pipe. The water flowing in at *E*, descends in the pipe *EG*, carrying the buckets along with it, thus setting the whole chain in motion, and turning the sprocket wheel *ACB* round with it. Lamolière's piston wheel, consists of two chains having from 10 to 15 buckets with leather packing. The buckets have an elliptical form, the major axis being 8 times the minor axis. The sprocket wheel consists of two discs with six cuts to receive the buckets.

Fig. 318.



For a fall of two metres (6′ — 8″), the surface of buckets being 0,25 square feet, the quantity of water 31 litres (6,82 gallons) per second, the number of revolutions 36 to 39, it is said that an efficiency = 0,71 to 0,72 was obtained.

Remark. This machine is the chain pump used in the English navy, converted into a recipient of power. For a description of the chain pumps, *see* Nicholson's "Operative Mech.," p. 268.

The chain of buckets (Fr. *noria*, *chapelet*, *pater-noster;* Ger. *Eimerkette*) is a similar apparatus. The chain in this machine has a series of buckets attached, Fig. 319, of such form that no pipe is required. The water enters at *A*, fills the buckets successively, and sets the whole series in motion, so that the sprocket wheel *C* is made to revolve. This wheel should give a very high efficiency, seeing that the whole fall may be made use of; but from the great number of parts of which it is composed, their liability to wear, and other sources of loss of effect, it is practically a very inefficient machine.

Fig. 319.

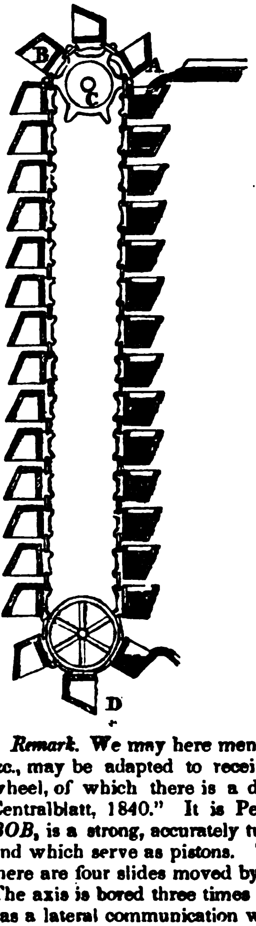
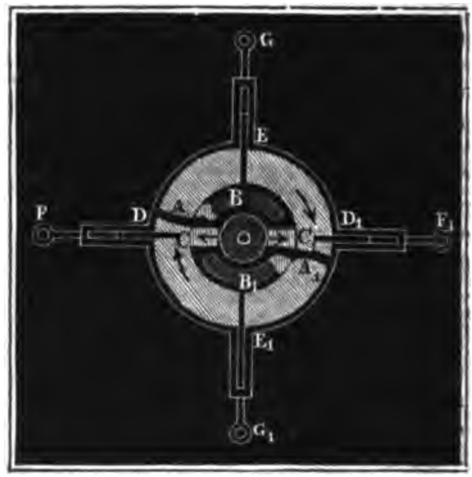


Fig. 320.



Remark. We may here mention, that the so-called rotary pump, rotary steam engines, &c., may be adapted to receive water power. Fig. 320, represents a water-pressure wheel, of which there is a detailed description and theory given in the "Polytechn. Centralblatt, 1840." It is Pecqueur's rotary steam engine adapted to water power. BOB_1 is a strong, accurately turned axis, A and A_1 being two wings connected with it, and which serve as pistons. These pistons are enclosed in a cover DED_1E_1, in which there are four slides moved by the engine itself, and performing the functions of valves. The axis is bored three times in the direction of its length, and each of the hollow spaces has a lateral communication within the cover. The pressure water flows through the inner bore O, enters through the side openings C and C_1, into the, in other respects, isolated space between the axis and the cover; presses against the pistons A and A_1, and in that way sets the axis in rotation. That the rotation may not be interrupted by the slides, they must always recede before the piston comes up to them; and on the other hand, that no water pressure may act on the opposite side of the piston, the slides must fall back instantly on the piston passing them, so that the spaces ABE and $A_1B_1E_1$, are shut off, and communicate only with the passages B and B_1, through which the water is discharged when it has done its work.

Mr. Armstrong, of Newcastle, constructed a water-pressure wheel of about 5 H. P., in 1841, a description of which will be found in the "Mechanic's Magazine," vol. xxxii.

Literature.—We shall conclude by some account of the literature and statistics of water-pressure engines. Belidor, in the "Architecture Hydraulique," describes a water-pressure engine with a horizontal working cylinder; and mentions, also, that, in 1731, MM. Denisard and De la Duaille had constructed a water-pressure engine. But this machine had only 9 feet fall, and raised about $\frac{1}{16}$ part of the weight of the power water to a height of 32 feet. It appears pretty certain, however, that the water-pressure engine was employed for raising water from mines, first by Winterschmidt, and soon afterwards by Höll. The details of this historical fact are to be found in Busse's "Betrachtung der Winterschmidt und Höll'schen Wassersäulenmaschine, &c., Freiberg, 1804." A drawing and description of Winterschmidt's engine is given in Calvör's "Historisch chronolog.

Nachricht, &c., des Maschinenwesens, &c., auf dem Oberharze, Braunschweig, 1763." Höll's engine is described in Delius's "Introduction to Mining," originally published at Vienna, 1773, and in the description of the engines erected at Schemnitz, by Poda, published at Prague, 1771.

Smeaton mentions the water-pressure engine in 1765, as an old invention, improved by Mr. Westgarth, of Coalcleugh, in the county of Northumberland, at which time several had been erected, in different mines, on Mr. Westgarth's plan. *See* Smeaton's "Report," vol. ii. p. 96.

Trevethick, the celebrated Cornish engineer, also invented or reproduced the water-pressure engine; and erected one, still at work in the Druid copper mine, near Truro, about the year 1793. *See* Nicholson's "Operative Mech."

The water-pressure engine is now in use in nearly every mining district in the world. The Bavarian engineer, Reichenbach, greatly improved and has made a most extensive application of this power for raising the brine to the boiling establishments in the Salzburg district. These engines have never been accurately described, but notices of them will be found in Langsdorf's "Maschinenkunde," in Hachette's "Traité élémentaire des Machines," and in Flachat's "Traité élémentaire de Mécanique." The engines erected by Brendel, in Saxony, are described in Gerstner's "Mechanik," where also the engines in Karinthia and at Bleiberg are described in detail. The water-pressure engines in the Schemnitz district are described by Schitko, in his "Beiträgen zur Bergbaukunde." Jordan has given a very detailed account of the engines at Clausthal, in Karsten's "Archiv für Mineralogie," &c., b. x., published as a separate work by Reimer, of Berlin. Junker has described his engines, at Huelgoat, in the "Annales des Mines," vol. viii. 1835, and the description is published as a separate work, by Bachelier.

No description of the engines erected by Mr. Deans, of Hexham, has been published. They are, however, simple and efficient. The engine erected by him at Wanlockhead, in Scotland, in 1830 or 31, having the fall-bob for working the valves, is one of the largest, and considered very efficient.

But the water-pressure engine erected in 1842, at the Alport Mines, near Bakewell, in Derbyshire, and several others on nearly the same model, are perhaps the most perfect of this description of engine hitherto made. These engines have been constructed from the designs of Mr. Darlington, engineer of the Alport Mines, under Mr. Taylor, by the Butterly Iron Company. There is a beautiful model of the first erected at Alport, in the Museum of Economic Geology, but no description of it has yet been published. Its arrangement—the construction of its parts—the valves, and their gear—are each of them admirable and peculiar to this engine, though, in its general features, it resembles the engines of Brendel, and Junker, and Jordan, which have been described.—Tr.

CHAPTER VII.

ON WINDMILLS.

§ 205. *Windmills.*—The atmospheric currents caused by a local expansion of the air by the sun's heat, are a source of mechanical effect, as is the expansive force of air heated artificially.

The machines, recipients of this wind power, are windmills (Fr. *roues à vent;* Ger. *Windräder*). They serve to convert a portion of the *vis viva* of the mass of air in motion into useful effect. As the direction of the wind is more or less horizontal, windmills or sail wheels usually have the axis nearly horizontal, that is, they are themselves nearly vertical.

Horizontal windmills, having concave buckets or sails, have been erected. The force of the wind against a hollow surface is greater

than against a plane or a convex surface, and hence such a wheel revolves under very light winds, but not advantageously for the production of mechanical effect.

Remark. For some account of Beatson's horizontal windmills, *see* Nicholson's "Practical Mechanic," and Gregore's "Mechanic," vol. ii.

§ 206. The advantage of *sail* wheels over any construction of *bucket* wheel is, that for the same weight, or in the same conditions generally, they produce a greater effect than these latter. We shall, therefore, in what follows, confine ourselves to the consideration of sail wheels, of which the general arrangement is as follows: First, there is the axle of wood, or better, of iron. This shaft, or axle, is inclined at an angle of from 5 to 15 degrees to the horizon, in order that the wheels may hang free from the structure on which they are placed, and also because the wind is supposed to blow at an inclination amounting to that number of degrees. This axle has a *head*, a *neck*, a *spur wheel*, and a *pivot*. At the head are the *arms*—the neck is the journal, or principal point of support on which it revolves. The spur wheel transmits the motion to the work to be done, and the pivot, at the low end of the axis, takes up a certain amount of the weight and counter-pressure of the machine. The loss of effect arising from the friction of the axle on the points of support is considerable, on account of the great weight and strain upon them, as also on account of the velocity with which it generally revolves, and hence every means must be taken to reduce it. On this account, iron shafts and bearings are to be preferred to wooden ones, as they may be made of much less diameter. The diameter of a wooden neck being $1\frac{1}{2}$ to 2 feet; that of an iron one substituted, need not be more than 6 to 9 inches. The friction of wooden axles is also in itself greater than that of iron.

§ 207. *Windmill Sails.*—A windmill sail consists of the arm or *whip*, of the *cross bars*, and of the *clothing*. The whips are radial arms of any required length, up to 40 or even 50 feet, usually about 30 feet. The number of arms is generally four, less frequently 5 or 6. For 30 feet in length, these arms are made 1 foot thick by 9 inches broad at the shaft, and 6 inches by $4\frac{1}{2}$ inches at the outer end. The mode of setting them in, or fastening them to the shaft, is various. When the axle is of wood, the arms are put through two holes, morticed at right angles to each other, thus getting 4 arms. The arms are sometimes made fast by screws to the shaft head, like the arms of a water wheel, and we refer to our description of water wheels for hints applicable to this subject. The bars are wooden cross arms, passing through the whip, which is morticed through at intervals of from 15 to 18 inches for the purpose, at right angles to the leading side of the whip. According as the sail is to have a rectangular or a trapezoidal form, the bars are all of the same length, or they increase in length from the shaft outwards. The first bar is placed at $\frac{1}{7}$ of the length of the whip from the shaft, and its length is = to this $\frac{1}{7}$ to $\frac{1}{6}$ of the length of the whip. The outermost bar is

made from $\frac{1}{4}$ to $\frac{2}{5}$ of the length of the whip. The whips are not generally made the *centre* line of the sails, but they divide them so that the part next the wind equals from $\frac{1}{5}$ to $\frac{1}{4}$ of the entire width of the sail. Therefore, the bars project much less from the one side than from the other. The narrower side is usually covered by the so-called *windboard*, and, on the wide side, the winddoor or a sail-cloth clothing is used.

The sails are made plane, or surfaces of double curvature, *i. e.*, warped, or concave. The slightly hollow surfaces of double curvature give the greatest effect, as we shall learn in the sequel. For plane sails, the bars have all the same inclination of from 12 to 18 degrees to the plane of rotation. In the double-curvature sails, the first bars are set at 24°, and the outer bars at 6° from the plane of rotation, and the inclinations of the intermediate bars form a transition between these two angles. To give the sails concavity, the whips must be curved, as also the bars. Although, according to the theory of the wind's impulse, this form gives an increased effect, the difficulty of execution renders it nearly inapplicable. The ends of the bars are connected or strung together by *uplongs*, and sometimes there are 8 of these uplongs on the driving and 2 on the leading side of the sail, to strengthen the lattice on which the sailcloth lies, on a series of frames of not more than 2 square feet each.

§ 208. *Postmills.*—As the direction of the wind is variable, and the axis has to be in that direction, the support of the wheel must have a motion on a vertical axis.

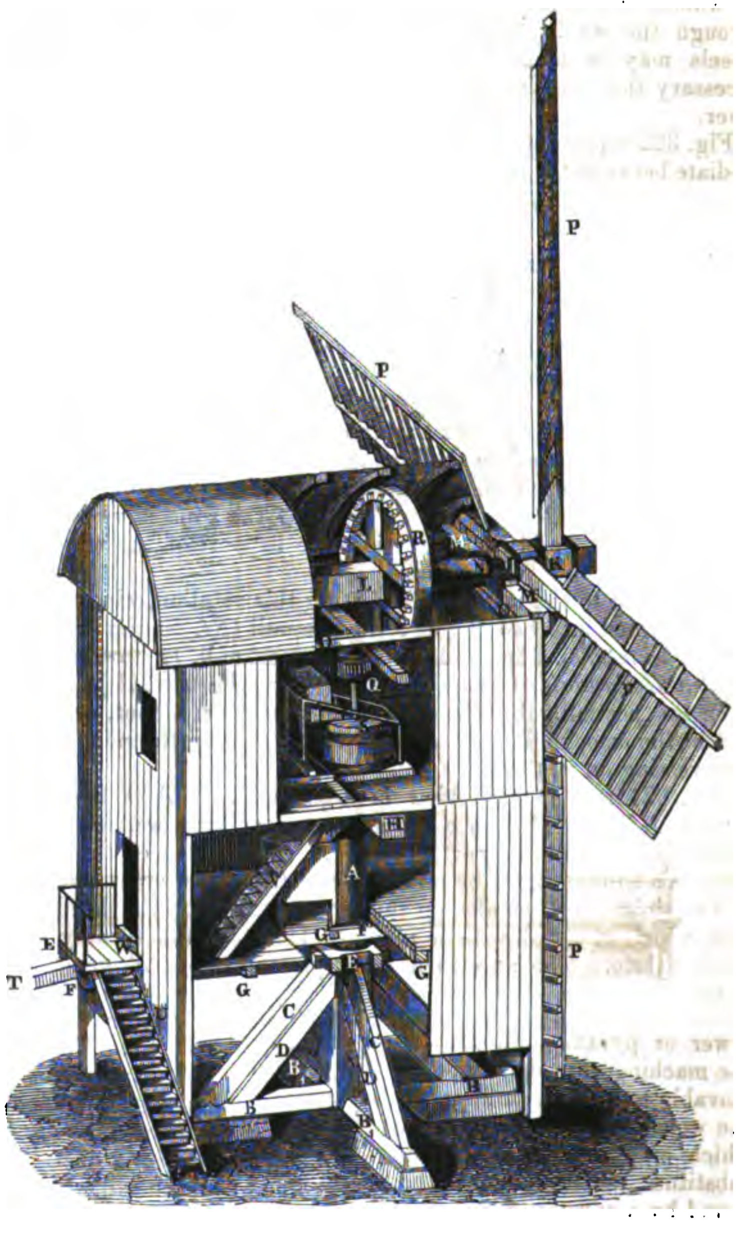
According to the manner of effecting this rotation, windmills may be subdivided into two classes: the postmill (Fr. *moulin ordinaire;* Ger. *Bockmühle*), Fig. 321, and the smockmill, or towermill (Fr. *moulin Hollandais;* Ger. *Holländische*, or *Thurmmühle*).

In the postmill, the whole structure turns on a foot, or centre; and in the smockmill, only the cap, with the gudgeon and pivot bearings resting on it, turns.

Fig. 321 is a general view of a postmill. *AA* is the post or centre, *BB* and B_1B_1 are cross bearers or sleepers, framed with struts *C* and *D*, to support the post. On the top of the framing there is a saddle *E*. The mill house rests on two cross beams *FF*, and on joists *GG*, as also on the cross beam *H* on the head of the post, which is fitted with a pivot to facilitate the turning of the whole fabric. The axis turns in a plumber-block *N*, generally of metal, sometimes of stone (basalt), lying on the beam *MM*, supported on the framing *OO*. *KP*, *KP*, &c., are the arms, passing through the shaft and carrying 4 plane sails *PP*, &c. The figure represents a grindingmill, and, hence, the wheel transmitting the power, *R*, works into a pinion *Q*, driving the upper millstone *S*. In order to turn the whole house, a long lever, strongly connected with the beams *EF*, projects 20 to 30 feet from the back of it. This lever is loaded, to counterbalance the weight of the sail wheel, &c. When the mill is set in the right direction, the lever *FT* (cut off in

the figure), is anchored and held fast, and generally there is a small movable capstan for getting power to turn the mill house, &c.

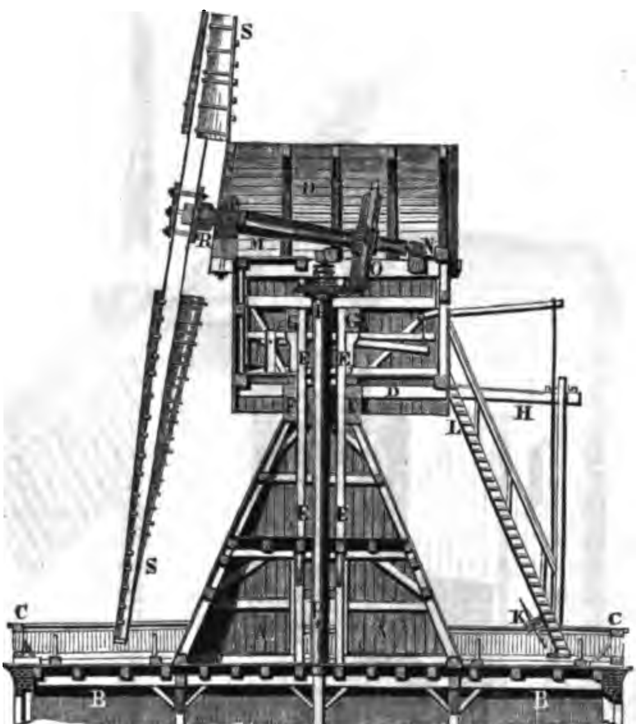
Fig. 321.



§ 209. *Smockmills.*—Smockmills are made in two different ways. Either the movable cap encloses the windshaft alone, or a greater part of the mill house, from the windshaft downwards, turns on a vertical axis. The motion of the sail wheel is transmitted by a pair of wheels to the *king post*, that is, a strong vertical axis going through the whole height of the mill house. In order that the wheels may be in gear in every position of the windshaft, it is necessary that the axis of the one shaft should intersect that of the other.

Fig. 322 represents the latter arrangement, which is, in fact, intermediate between the postmill and the smockmill. *AA* is a stationary

Fig. 322.

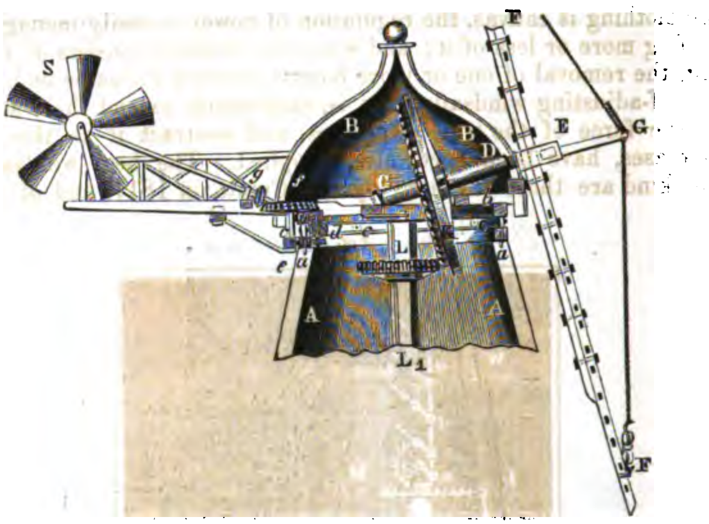


tower or pyramid, raised above which is the building containing the machinery, in driving which the power is consumed. *DD* is the movable top of the mill, supported by the wooden ring *FF*, and by the wooden ring *GG*, by means of the uprights *EE* and E_1E_1, and which only admits of rotation round these, which are, in fact, the substitute for the post in the postmill. The mill wheel is drawn round by a capstan *R*, attached to the lever *H*, framed by the stairs to the movable part of the structure. The windshaft is of cast

iron, and rests at *M* and *N* in plumber blocks, lined with brasses. *O* and *P* are iron-toothed wheels, for transmitting the motion of the wind shaft to the upright, or *king post PP*. The windsails *RS*, *RS*... are warped surfaces: the arms are fastened by screws into the cast iron socket piece *R*, attached by wedges to the head of the windshaft.

The upper part of a smockmill, properly so called, is shown in Fig. 323. *AA* is the upper part of the tower, or mill house, built

Fig. 323.



of wood, or of masonry. *BB* is the movable cap, *CDE* is the wind shaft, *EE* the arms of the sails, strengthened by the ties *GF*, *GF*, supported by a king post *EG*. *K* and *L* are bevelled gear for transmitting the power from the windshaft to the vertical shaft.

The sails are set to the wind, sometimes by means of a lever, as described in reference to the last-mentioned construction of cap, but sometimes by means of a large wind vane, the plane of which is in that of the axis of the wind shaft, but more generally by means of a small windmill *S*. That the cap may revolve easily, it is placed on rollers *c*, *c*, *c*... connected together in a frame, and running between two rings, one of which is laid on the summit of the tower, the other is attached to the under side of the cap. To prevent the cap from being raised up and displaced, there is an internal ring *d*, which has likewise friction rollers running on the internal surface of *a a*. When this method of adjustment is used, the outer surface of *a a* is toothed, and a small pinion *e* working into it, is moved by the auxiliary windmill, by means of the bevelled gear *f* and *g*, Fig. 323, and thus the whole cap is made to revolve, until the auxiliary wheel, and therefore the axis of the windshaft is in the direction of the wind.

§ 210. *Regulation of the Power.*—As the wind varies in intensity as well as in direction, when the work to be done is a constant resistance, unless some means of regulating the power be applied, the motion of the machinery would not be uniform. One means of absorbing any excess of power, is a friction strap, applied to the outside of the wheel on the windshaft. Another means is, to vary the extent of sail, or the quantity of clothing exposed. When the sails are quite spread out, the maximum power depends on the intensity of the wind, and if this intensity be constant, the power may be varied by taking in more or less of the clothing of the sail. When the clothing is canvas, the regulation of power is easily managed by reefing more or less of it; and when the clothing consists of boarding, the removal of one or more boards answers the same end.

Self-adjusting windsails, that is, sails which extend their surface as the force of the wind decreases, and contract it as this force increases, have been successfully applied. The best windsails of this kind are those invented by Mr. Cubitt, in 1817, and of which

Fig. 324.

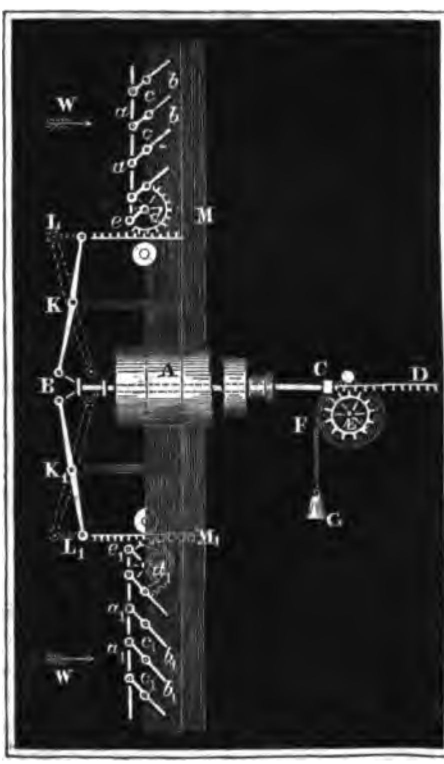


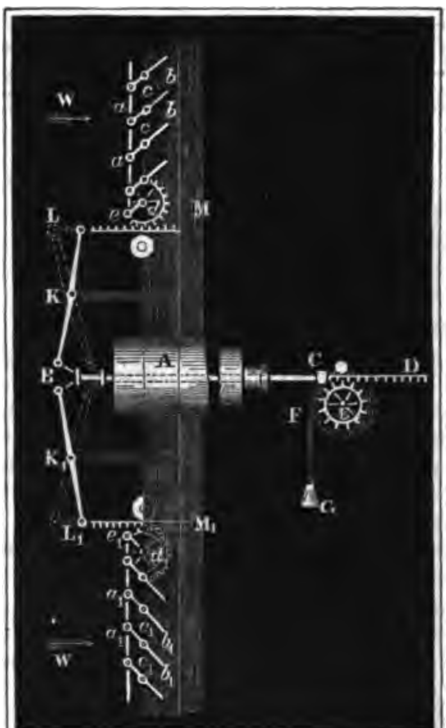
Fig. 324 represents the section of a part. *A* is a hollow windshaft, *BC* a rod passing through it, *CD* a ratchet fastened to *BC*, so that

it does not turn with it, but serves to move it in the direction of the axis.

The ratchet works into a toothed wheel E, on the same axis as the pulley F, round which there passes a string with a weight G. The sail clothing consists of a series of boards, or sheet-iron doors bc, b_1c_1, &c., movable by the arms ac, a_1c_1, &c., round the axis c, c_1, &c. These arms are connected by the rods ae, a_1e_1, &c., and by the levers or cranks, de, d_1e_1, with toothed wheels d, d_1, so that, by the turning of the latter, the opening and closing, or, in general, the *adjustment* of the flaps or doors is possible.

There are besides, levers BL, BL_1, Fig. 325, revolving on centres

Fig. 325.



K and K_1, and attached at one end to the rod BC, and at the other to the ratchets LM and L_1M_1, working into the small wheels d and d_1. The drawing explains how the wind, coming in the direction W, works backwards on the counter-balance weight G, which is adjusted so that the surface exposed shall be that required to do the work regularly, always supposing that, for the maximum surface that can be exposed, there is wind sufficient.

Remark. Mr. Bywater invented a mode of furling and unfurling the clothing when it consists of sailcloth. There are two rollers moved by toothed wheels, and the action of

these is to cover more or less of the sail frame, according to the force of the wind. This plan is described in detail in Barlow's "Treatise on the Manufactures and Machinery, &c. &c."

§ 211. *Direction of the Wind.*—The direction of the wind may be any of the 32 points of the compass, but the indications are generally noted as one of the 8 following: N., N.E., E., S.E., S., S.W., W., N.W., *i. e.*, north, north-east, east, south-east, south, south-west, west, north-west; or naming them according to the direction from which they blow. In the course of the year, the direction of the wind is more or less frequently from each of all these directions; some winds blowing more frequently than others. From Kämtz's "Meteorology," we extract the following table of the winds that blow during 1000 days, in different countries.

Country.	N.	N.E.	E.	S.E.	S.	S.W.	W.	N.W.
Germany . . .	84	98	119	87	97	185	198	131
England . . .	82	111	90	81	111	225	171	120
France . . .	126	140	84	76	117	192	155	110

We see from this that, in the three countries named, the south-west wind predominates; the passage of the wind from one direction to another is usually in the course from S., S.W., W., &c., and seldom in the opposite course of S., S.E., E., &c. That is, the latter course is generally only taken through a small angle, and then retraced.

The wind *vane*, or fane (Fr. *girouette, flouette;* Ger. *Wind-* or *Wetterfahne*), gives the direction of the wind. To give it facility of movement, the friction on its pivot or collar must be as small as possible, and hence the blade or plane of the vane has to be balanced by a counter-weight to bring the centre of gravity line to pass through the axis of rotation. (Whether the form resulting from this combination gave rise to the term *weathercock* (Fr. *coq à vent;* Ger. *Wetterhahn*), or whether "a king-fisher hanging by the bill, converting the breast to that point of the horizon from whence the wind doth blow, be the introducing of *weathercocks*," we cannot pretend to say.)

§ 212. *Intensity of the Wind.*—The miller is, however, dependent on the intensity of the wind, and not on its direction; for on the former the mechanical effect to be obtained from given wind-sails depends.

Accordingly, the velocity of the wind is

Scarcely sensible for	1½	feet per second.
Very gentle wind for	3	" "
Gentle breeze for	6	" "
Brisk breeze for	18	" "
Good breeze for windmills	22	" "
Brisk gale for	30	" "
High wind for	45	" "
Very high wind for	60	" "
Storm for	70 to 90	" "
Hurricane	100 or more.	

A breeze of 10 feet per second is not in general sufficient to drive a loaded windsail, and if the velocity rises above 35 feet per second, the intensity becomes too much for the strength of the arms, unless the clothing be very close reefed, and stormy weather is dangerous even to "bare poles."

Windgauges, or *anemometers*, are used for ascertaining the velocity of the wind. Many anemometers have been proposed and adopted, but few of them are sufficiently convenient or trustworthy in their indications. The anemometers have great resemblance to the hydrometers described in Vol. I. § 376. The velocity of a current of air may be measured by noting the rate of progress of a body floating in it, as a feather, smoke, soap bubbles, small air balloons, &c. This means will not suffice in the case of high velocities, for the eddies, that invariably accompany wind, disturb the progress of such bodies.

Anemometers may be divided into three classes. Either the velocity of the wind is deduced from that of a wheel moved by it, or it is measured by the height of a column of fluid, counterbalancing the force of the wind, or the pressure on a given surface is determined. We shall give a succinct account of each of these methods.

Remark. There is a very complete treatise on Anemometers, in the "Allgemeinen Maschinenencyclopädie, by Hülsse." In the transactions of the British Association for 1846, there is a report, by Mr. J. Phillips, on Anemometers, in which the essential points to be aimed at in these instruments, and the merits of those of Whewell, Osler, and Lind, respectively, are discussed. The chapters on Wind, in Kämtz's "Meteorology," and in Gehler's "Wörterbuch," are standards of reference on this subject.

§ 213. *Anemometers.*—Woltmann's wheel may be used for ascertaining the velocity of the wind as conveniently as it is for ascertaining the velocity of currents of water. When its axis of rotation is set in the direction of the wind, which is insured by means of a vane set on the same vertical axis with it, the number of revolutions made in a given time may be observed, and from this the velocity may be deduced, as explained, Vol. I. § 378, by the formula $v = v_0 + \alpha u$, in which v_0 is the velocity of the wind, for which the wheel begins to stop, and α is the ratio $\frac{v - v_0}{u}$. If the impulse of the wind were of the same nature as that of water, and if they both increased exactly as the squares of the relative velocities, then $\alpha = \frac{v - v_0}{u}$ would answer for wind and water, but as this is only *nearly* true, we can only expect that the co-efficient α is nearly the same for wind and water. As to the initial velocity v_0, this is, in the case of wind, about $\sqrt{800} = 28,3$ times greater than for water, as the density of water is about 800 times greater than that of air; and thence a column of air 800 times as high as that of water, or the impact of a stream of $\sqrt{800} = 28,3$ times the velocity of the water. This high value of the constant v_0, makes it necessary to construct the anemometer sails with great lightness, and to have the axis of hard steel running on agate or other hard bearings; as, for instance,

in Combe's anemometer. The constants v_0 and u are generally determined by moving the instrument through air at rest; but this method is objectionable, because the impact of a fluid in motion is not the same as the resistance of a fluid at rest (Vol. I. § 391).

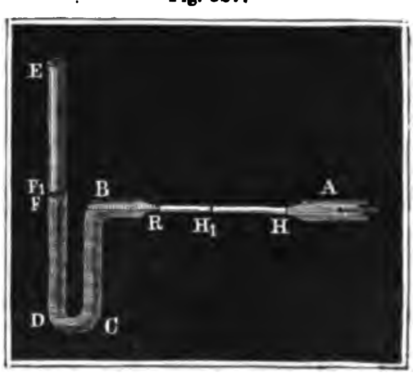
It is better, on every account, to deduce the constants from experiments on currents of air, deducing the low velocities by direct observations on light, floating bodies. By placing the instrument in the main pipe of a blowing engine, the observations for calculating the constants might be made. The calculation of constants from a series of experiments for which v and u are known, should be done as shown in Vol. I. § 379.

§ 214. Pitot's tube may also be very conveniently applied as an anemometer. This is Lind's anemometer, and its arrangement is shown in Fig. 326. AB and BE are two upright glass tubes $\frac{5}{12}$ of an inch in diameter, and filled with water, and BCD is a narrow, bent, connecting piece between the two, of only $\frac{1}{20}$ inch diameter. FG is a scale, by which to read off the height of the water. When the mouth A is turned to the wind, its force presses down the column in AB, and raises that in DE, and hence the difference of level between the two surfaces may be read on the scale h. From this, the velocity of the wind may be calculated by the formula $v = v_0 + a\sqrt{h}$; v_0 and a being co-efficients deduced from experiments for each instrument.

Fig. 326.

The use of this instrument is very limited, as pressures which move the water to a difference of level of $\frac{1}{20}$ of an inch can scarcely be noted accurately, but may be estimated to $\frac{1}{40}$ or $\frac{1}{50}$. This gives 5 to 7 miles per hour as the limit of wind velocity really measurable. To obviate these disadvantages, and render the instrument useful for moderate velocities, Robison and Wollaston introduced the following improvements.

Fig. 327.



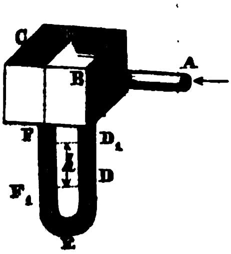
In Robison's anemometer there is a narrow, horizontal pipe HR, Fig. 327, between the mouthpiece A, and the upright pipe BC; and there is poured as much water into the instrument, before using it, as brings the surface F to the level of HR, and filling the small tube to H. When the mouth A is turned to the wind, the water is driven back in the narrow tube, and a column FF_1,

counterbalancing the force of the wind, rises in the tube DE, but which is measurable by the length of tube HH_1, in which the water has been driven back. If d and d_1 be the diameters, and h and h_1 the height of the columns FF_1 and HH_1, then

$\frac{\pi d^2}{4} h = \frac{\pi d_1^2}{4} h_1$, and $\therefore h = \left(\frac{d}{d_1}\right)^2 h_1$, or $h_1 = \left(\frac{d}{d_1}\right)^2 h$ or h_1 is always greater in the ratio $\left(\frac{d}{d_1}\right)^2$ than h, and can, therefore, be read with much greater accuracy than h. If $\frac{d}{d_1} = 5$, then the indications in HH_1 are 25 times greater than in FF_1.

Again, the differential anemometer of Wollaston, shown in Fig. 328, may be used for ascertaining the velocity of the wind. This instrument consists of two vessels B and C, and of a bent pipe DEF, which unites the two vessels by their bottoms. The one vessel is shut at top, and has a side orifice A, which is turned to the wind. The instrument is filled with *water and oil*. The former fills the two legs to about $\frac{1}{2}$, and the oil fills them up, and occupies part of each of the vessels. The force of the wind raises the water in one leg higher than it stands in the other, and the amount of this force is measured by the difference of pressure between the water column FF_1, and the oil column DD_1. If h = the height of each of these columns, and ϵ = the specific gravity of the oil, we then have in the last formula $h(1-\epsilon)$, instead of h, and, therefore, $v = v_0 + \alpha \sqrt{(1-\epsilon)h}$. If, for example, the oil be linseed, $\epsilon = 0{,}94$, $v = v_0 + \alpha \sqrt{(1-0{,}94)h} = v_0 + \alpha \sqrt{0{,}06 \, . \, h} = v_0 + 0{,}245 \, \alpha \sqrt{h}$, or h is $\frac{100}{6} = 16\frac{2}{3}$ times as high as in the case of the tubes being simply filled with water. By mixing, or combining the water with alcohol, the density of the water may be brought even nearer to that of the oil; and, therefore, $1-\epsilon$ becomes still less, or the difference of level to be read; and, therefore, the accuracy of the reading is increased.

Fig. 328.



§ 215. Anemometers, analogous to the *stream quadrant* (Vol. I. § 381), have also been proposed and applied on the same principle, there being substituted a thin plate for the spherical ball used in gauging water streams. But a very thin metallic sphere is certainly preferable to a thin plate, for then the force of the wind remains the same for all inclinations of the rod to which it is attached, whereas it changes with the angle of inclination of the thin plate; whilst, when a sphere is used, the formula $v = \psi \sqrt{tang. \beta}$ (in which β is the deviation of the rod from the vertical), is sufficient. The application of a thin plate involves a complicated expression for the calculation of the velocity.

Lastly, the velocity of the wind may be ascertained by the force with which it acts directly on a plane surface opposed to it at right angles, and for this the instruments used are more or less similar to the hydrometer, described in Vol. I. § 382. If the law of the impact of wind were accurately known, the velocity of the wind might be determined without further research by these means. But this is not the case, and the formulas given in Vol. I. § 390, and the co-efficient given in § 392, lead to only approximate results. Retaining these, however, for the present, or putting the impulse of the wind

$$P = \zeta \cdot \frac{v^2}{2g} F\gamma, = 186 \cdot \frac{v^2}{2g} F\gamma,$$

or, rendered in English measures, as

$$\frac{1}{2g} = 0{,}0155,\ P = 0{,}02883\ v^2 F\gamma,$$

and if the density of the air $\gamma = 0{,}07974$ lb. per cubic foot, then $P = 0{,}002299$ or $0{,}0023\ v^2 F$, and ∵ for two square feet of surface:

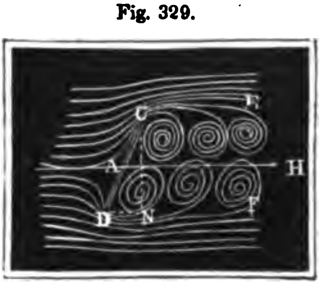
$$P = 0{,}0023\ v^2, \therefore v = \sqrt{\frac{P}{0{,}0023}} = 20{,}85 \sqrt{P} \text{ feet.}$$

For velocities $v =$	10	15	20	25	30	35	40	45	50 feet.
The impulsive force of the wind on 1 square ft. =	0,2455	0,5524	0,982	1,534	2,209	3,007	3,928	4,971	6,1375 lb.

Admitting the above premises, the force of the wind on any surface at right angles to its direction may be easily calculated.

§ 216. *Force of Wind.*—We shall now study more closely the effect of the impulse of wind on the sails of windmills. Let us, for this purpose, conceive the whole sail surface divided into an infinite number of normal planes on the axis of the sail or arm, and suppose *CD*, Fig. 329, to be such an elementary plane. Owing to the considerable extent, and particularly owing to the great length of a sail, we may assume that all the wind of the column pressing on the surface *CD*, coming in the direction *AH*, will be turned off by the impact in directions parallel to *CD*, and, therefore, we may make use of the formulas in Vol. I. § 388. If $c =$ the velocity of the sail, Q the quantity of wind striking on *CD per second*, $\gamma =$ the density of the wind, and $\alpha =$ the angle *CAH*, which the direction of the wind makes with *CD*, then, on the assumption

Fig. 329.



that the plane CD moves away in the direction of the wind, the normal impulse of the wind on CD, is $N = \frac{c - v}{g} \sin. \alpha . Q \gamma$.

Putting the section $CN = G$, then the quantity of wind Q coming into action, is not $G c$, but $G (c - v)$, as the sail, moving with the velocity v, leaves a space $G v$ behind it, which takes up a proportion of the quantity of wind $G c$ following it, equal to $G v$, without undergoing any change of direction. Hence, the normal impulse may be put

$$N = \frac{c - v}{g} \sin. \alpha . (c - v) G \gamma = \frac{(c - v)^2}{g} \sin. \alpha\, G \gamma,$$

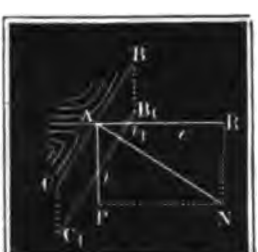
or, if F = the area of the element CD, and we substitute $F \sin. \alpha$ for G, then $N = \frac{(c - v)^2}{g} \sin. \alpha^2 F \gamma$.

Besides this impulse on the face of CD, there is a counter action on the back; inasmuch as one part of wind, passing in the directions CE and DF, at the outside of the plane, takes an eddying motion to fill up the space behind, and consequently loses pressure corresponding to the relative velocity $(c - v) \sin. \alpha$, and represented by $\frac{(c - v)^2}{2g} \sin. \alpha^2 . F \gamma$. If we combine the two effects, we get the normal impulse of the wind on the element F of the sail:

$$N = \frac{(c - v)^2}{g} \sin. \alpha^2 F \gamma + \frac{(c - v)^2}{2g} \sin. \alpha^2 F \gamma = 3 . \frac{(c - v)^2}{2g} \sin. \alpha^2 F \gamma.$$

§ 217. *Best Angle of Impulse.*—In the application of this formula to windmills, we have to bear in mind that the windsail BC, Fig. 330, does not move away in the direction AR of the wind, but in a direction AP at right angles to it, and hence, in the formula

Fig. 330.



$N = 3 . \frac{(c - v)^2}{2g} \sin. \alpha^2 . F \gamma$, we have to substitute, for v, the velocity $Av_1 = v_1$, with which the windsail moves in reference to the direction of the wind. If v = the actual velocity of rotation $\overline{A v}$ (Fig. 330), then $Av_1 = v_1 = v \text{ cotg. } \overline{Av_1 v} = v \text{ cotg. } \alpha$; and, therefore, for the case in question:

$$N = 3 . \frac{(c - v \text{ cotg. } \alpha)^2}{2g} . \sin. \alpha^2 F \gamma, \text{ or} = 3 \frac{(c \sin. \alpha - v \cos. \alpha)^2}{2g} F \gamma.$$

This normal impulse is to be decomposed into two others, P and R, one acting in the direction of rotation, the other in the direction of the axis of the element of the sail; then

$$P = N \cos. \alpha = 3 \frac{(c \sin. \alpha - v \cos. \alpha)^2}{2g} \cos. \alpha . F \gamma, \text{ and}$$

$$R = N \sin. \alpha = 3 \frac{(c \sin. \alpha - v \cos. \alpha)^2}{2g} \sin. \alpha . F \gamma.$$

By multiplying by the velocity of rotation v, we get, from the formula for P, the mechanical effect of the windsail.

$$L = Pv = 3\frac{(c\ sin.\ \alpha - v\ cos.\ \alpha)^2}{2g}\ v\ cos.\ \alpha\ .\ F\gamma.$$

The parallel or axial force R, gives no mechanical effect, but on the contrary increases the pressure on the pivot or footstep at the lower end of the windshaft, and so gives rise to a *loss* of effect.

The last formula indicates, and it is self-evident, that the effect increases with the velocity c, and with the area F; but it is not so evident from it, how the angle of impulse a, affects the mechanical effect produced. That L may not be $= 0$, $c\ sin.\ \alpha$ must be $> v\ cos.\ \alpha$; that is, $tang.\ \alpha > \frac{v}{c}$, and $cos.\ \alpha > 0$, and, therefore, $\alpha < 90^\circ$. There must, therefore, be a value of α between the limits $tang.\ \alpha > \frac{v}{c}$, and $\alpha < 90^\circ$, corresponding to a maximum value of L. To find this value, let us instead of α put $\alpha \pm x$, x being a very small angle. Then we have $sin.\ (\alpha \pm x) = sin.\ \alpha\ cos.\ x \pm cos.\ \alpha\ sin.\ x$, or putting $cos.\ x = 1$, and $sin.\ x$ being put $= x$,

$sin.\ (\alpha \pm x) = sin.\ \alpha \pm x\ cos.\ \alpha$, further:

$cos.\ (\alpha \pm x) = cos.\ \alpha\ cos.\ x \mp sin.\ \alpha\ sin.\ x = cos.\ \alpha \mp x\ sin.\ \alpha$, and these values give us as the effect:

$$L = \frac{3c^2v}{2g}F\gamma\left(sin.\ \alpha - \frac{v}{c}cos.\ \alpha\right)^2 cos.\ \alpha,$$

$$L_1 = \frac{3c^2v}{2g}F\gamma([sin.\ \alpha \pm x\ cos.\ \alpha - \frac{v}{c}(cos.\ \alpha \mp x\ sin.\ \alpha)]^2(cos.\ \alpha \mp x\ sin.\ \alpha))$$

$$= \frac{3c^2v}{2g}F\gamma[sin.\ \alpha - \frac{v}{c}cos.\ \alpha \pm (cos.\ \alpha + \frac{v}{c}sin.\ \alpha)x]^2(cos.\ \alpha \mp x\ sin.\ \alpha)$$

$$= \frac{3c^2v}{2g}F\gamma(sin.\ \alpha - \frac{v}{c}cos.\ \alpha)^2 cos.\ \alpha$$

$$+ [2(sin.\ \alpha - \frac{v}{c}cos.\ \alpha)(cos.\ \alpha + \frac{v}{c}sin.\ \alpha)cos.\ \alpha - (sin.\ \alpha - \frac{v}{c}cos.\ \alpha)^2$$

$$sin.\ \alpha]x +), \&c., \&c.$$

$$= L \pm \frac{3c^2v}{2g}F\gamma([2(sin.\ \alpha - \frac{v}{c}cos.\ \alpha)(cos.\ \alpha + \frac{v}{c}sin.\ \alpha)cos.\ \alpha$$

$$- (sin.\ \alpha - \frac{v}{c}cos.\ \alpha)^2 sin.\ \alpha]x + \&c.)$$

In order that α may give the maximum value, L_1 must be less than L, α being increased or diminished by x, that is, x being positive or negative. But the last formula gives in the one case $L_1 > L$, and in the other $< L$, so long as the second member $\pm \frac{3c\,v}{2g}F\gamma[\ldots]x$ is a real quantity. Therefore, for obtaining the maximum value, it is necessary that this second member should be 0, or, that

$$2\left(\sin. a - \frac{v}{c}\cos. a\right)\left(\cos. a + \frac{v}{c}\sin. a\right)\cos. a - \left(\sin. a - \frac{v}{c}\cos. a\right)^2 \sin. a = 0,$$

$$\text{or } 2\left(\cos. a + \frac{v}{c}\sin. a\right)\cos. a = \left(\sin. a - \frac{v}{c}\cos. a\right)\sin. a,$$

$$\text{or } \sin. a^2 - \frac{3v}{c}\sin. a \cos. a = 2\cos. a^2.$$

Dividing by $\cos. a^2$, and putting $\frac{\sin. a}{\cos. a} = tang.\ a$, we have

$$tang.\ a^2 - \frac{3v}{c}\,tang.\ a = 2,$$

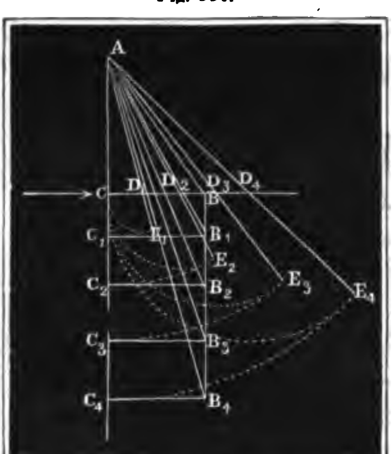
from which we deduce as the angle for the maximum effect;

$$tang.\ a = \frac{3v}{2c} + \sqrt{\left(\frac{3v}{2c}\right)^2 + 2}.$$

As, in the windsails, the outer elements of the sail have a greater velocity than those nearer the axis of rotation, it follows that the outer part of the sail should be set at a greater angle of impulse than the inner, in order to insure a maximum effect. Hence the sails must not be plane surfaces, but "*surfaces gauches*," or surfaces of double curvature, warped so that the outer part deviates less from the plane of the axis of rotation, than the inner part.

Remark. The most advantageous angles of impulse of a sail, may also be ascertained by the following construction. Fig. 331, take $CB = 1$, set off $CA = \sqrt{2}$ at right angles to it, *i. e.*, the diagonal of a square on CB. and draw AB. The $tang.\ ABC = \sqrt{2}$, and, therefore, $\angle ABC = 54°, 44', 8''$, which is the angle of impulse close up to the axis of rotation. If in $y = \frac{3\omega x}{2c}$, we make c the velocity of the wind, and ω the angular velocity, and for x successively, the distance of the sail bars from the windshaft axis, and set off these values of y from C on CB as CD_1, CD_2, CD_3, &c. Further draw the hypothenuses AD_1, AD_2, AD_3, &c., and prolong them, so that $D_1E_1 = CD_1$, $D_2E_2 = CD_2$, $D_3E_3 = CD_3$, &c. Lastly, lay off AE_1, AE_2, AE_3, in the direction AC as AC_1, AC_2, AC_3, &c., raise at C_1, C_2, C_3, &c., the perpendiculars C_1B_1, C_2B_2, C_3B_3, &c., $= CB = 1$, and draw AB_1, AB_2, AB_3, &c., then AB_1C_1, AB_2C_2, AB_3C_3, &c., are the angles required; for

Fig. 331.



$$tang.\ AB_1C_1 = \frac{AC_1}{B_1B_1} = \frac{AE_1}{1} = D_1E_1 + AD_1 = y_1 + \sqrt{y_1^2 + 2},$$

$$tang.\ AB_2C_2 = \frac{AC_2}{A_2C_2} = \frac{AE_2}{1} = D_2E_2 + AD_2 = y_2 + \sqrt{y_2^2 + 2}, \text{ \&c.}$$

§ 218. *The Mechanical Effect.*—The formula for the best angle of the sails may be used conversely to determine the best velocity of rotation for a given angle a. For this

$$tang. \alpha^2 - \frac{3v}{c} tang. \alpha = 2,$$

and, therefore, very simply,

$$v = \left(\frac{tang. \alpha^2 - 2}{tang. \alpha}\right) \cdot \frac{c}{3} = (tang. \alpha - 2\, cotang. \alpha)\frac{c}{3}.$$

If we put this value in the formula for the mechanical effect, we have

$$L = \frac{3\, c^2}{2g} F\gamma \cdot \frac{tang. \alpha^2 - 2}{tang. \alpha} \cdot \frac{c}{3} \cdot \left(sin. \alpha - \frac{tang. \alpha^2 - 2}{3\, tang. \alpha} cos. \alpha\right)^2 cos. \alpha$$

$$= \tfrac{4}{9} \cdot \frac{c^3}{2g} F\gamma \cdot \frac{(tang. \alpha^2 - 2)\, cos. \alpha^2}{sin. \alpha^3} = \tfrac{4}{9} \cdot \frac{c^3}{2g} F\gamma \cdot \frac{(3\, sin. \alpha^2 - 2)}{sin. \alpha^3}.$$

The theoretical effect of a windsail, may hence be calculated for any given velocity of wind, and of rotation. From a given number of revolutions per minute, we have the angular velocity $\omega = \frac{\pi u}{30} = 0{,}1047 \,.\, u$. If the whole length of whip be divided into 7 equal parts, and if, as usual, the sail begins at the lowest point of division, so that its total length $= \frac{6}{7} l$, we can very easily, by means of the formula

$$tang. \alpha = \frac{3\, v}{2\, c} + \sqrt{\left(\frac{3\, v}{2\, c}\right)^2 + 2},$$

calculate the best angle of sail $\alpha_0, \alpha_1, \alpha_2$, &c., or for each of the points of division of the whip, by substituting successively

$$v_0 = \omega \cdot \frac{l}{7},\ v_1 = \omega \cdot \frac{2\, l}{7},\ v_2 = \omega \cdot \frac{3\, l}{7} \ldots \text{ to } v_6 = \omega \cdot \frac{7\, l}{7} \text{ or } \omega\, l.$$

If, further, $b_0, b_1, b_2 \ldots b_6$ be the width of sail to be put on each of these points, we can calculate, by aid of Simpson's rule, from

$$\left(\frac{3\, sin. \alpha_0^2 - 2}{sin. \alpha_0^3}\right) b_0,\ \left(\frac{3\, sin. \alpha_1^2 - 2}{sin. \alpha_1^3}\right) b_1,\ \left(\frac{3\, sin. \alpha_2^2 - 2}{sin. \alpha_2^3}\right) b_2, \text{ \&c., a}$$

mean value k, and, hence, we arrive at the whole effect of the sail $L = \frac{4}{9} k\gamma \cdot \frac{6}{7} l \cdot \frac{c^3}{2g}$, or, more generally, l_1 being the length of *sail*, properly so called, $L = \frac{4}{9} \gamma\, k\, l_1 \frac{c^3}{2g}$.

If the sail were a plane surface, that is, if α were constant throughout its whole extent, then, by means of

$$v_0 = \frac{\omega l}{7},\ v_1 = \omega \cdot \frac{2\, l}{7}, \text{ \&c.,}$$

we should first calculate the corresponding values:

$$\left(sin. \alpha - \frac{v_0}{c} cos. \alpha\right)^2 \frac{v_0}{c} cos. \alpha \,.\, b_0,\ \left(sin. \alpha - \frac{v_1}{c} cos. \alpha\right)^2 \frac{v_1}{c} cos. \alpha \,.\, b_1, \text{ \&c.,}$$

and then from these, by Simpson's rule, deduce the mean value k_1, and introduce this into the formula for the mechanical effect developed, $L = 3\,\gamma\, k_1 \cdot l_1 \cdot \frac{c^3}{2g}$.

If n be the number of sails, we have of course to multiply the

last found value by this number, to get the whole mechanical effect developed by the windsail wheel, or $L = 3\, \eta\, \gamma\, k_1\, l_1 \frac{c^3}{2g}$.

Example 1. What angle of impulse is required for a windsail wheel, the velocity of the wind being 20 feet, the number of sails 4, each being 24 feet in length, and 6 to 9 feet in width? Number of revolutions 16 per minute. What will be the theoretical effect of this windmill?

In the first place, the angular velocity $\omega = 0{,}1047 \,.\, 16 = 1{,}6755$ feet, and if the distance of the first sail bar be 4 feet from the axis of the shaft, or the total length of whip $= 24 + 4 = 28$ feet, then for the

Distances:	4	8	12	16	20	24	28 Feet.
The velocities:	6,702	13,404	20,106	26,808	33,510	40,212	46,914 ft.
The tangents of the angles of impulse:	2,004	2,740	3,575	4,469	5,397	6,347	7,311
The angles:	63°,29′	69°,57′	74°,22′	77°,23′	79°,30′	81°,3′	82°,13′
The values of $\frac{3\, sin.\, a^2 - 2}{sin.\, a^2}$:	0,5612	0,7810	0,8759	0,9220	0,9472	0,9622	0,9716
The width of sails:	6,0	6,5	7,0	7,5	8,0	8,5	9,0 feet
The product of the two last:	3,367	5,076	6,131	6,915	7,578	8,179	8,744

And from the last product the mean value:

$$k = \frac{3{,}367 + 8{,}744 + 4\,.\,(5{,}076 + 6{,}915 + 8{,}179) + 2\,.\,(6{,}131 + 7{,}578)}{18}$$

$$= \frac{12{,}111 + 80{,}680 + 27{,}418}{18} = \frac{120{,}209}{18} = 6{,}679, \text{ and if we put:}$$

$$\gamma = 0{,}07974 \text{ lbs. } \tfrac{4}{9}\; l = 24, \text{ and } \frac{c^3}{2g} = 0{,}0155 \times 20^3 = 124,$$

then the effect of this wheel:

$L = 4 \,.\, \frac{4}{9} \,.\, 6{,}679 \,.\, 0{,}07974 \,.\, 24{,}124 = 11{,}874 \,.\, 1{,}91 \,.\, 124 = 2798$ feet lbs. $= 5$ horse power.

Example 2. What effect may be expected from a windmill wheel. having four plane sails, and the angle of impulse 75°, the other dimensions and proportions being the same as those of the wheel in the last example? In this case

The velocities of ratio $\frac{v}{c}$:	0,3351	0,6702	1,0053	1,3404	1,6755	2,0106	2,3457
The differences $sin.\, a - \frac{v}{c}\, cos.\, a$:	0,8792	0,7925	0,7057	0,6190	0,5323	0,4456	0,3588
The width b:	6,0	6,5	7,0	7,5	8,0	8,5	9,0 feet
The products $(sin.\, a - \frac{v}{c}\, cos.\, a)^2 \times \frac{v}{c}\, cos.\, a\; b$:	0,4023	0,7081	0,9071	0,9967	0,9830	0,8783	0,7034

From the latter products we deduce, by Simpson's rule, the mean value

$k_1 = \frac{1}{18}\,[0{,}4023 + 0{,}7034 + 4\,(0{,}7081 + 0{,}9969 + 0{,}8783) + 2\,(0{,}9071 + 09830)]$

$= \frac{1}{18}\,(1{,}1057 + 10{,}3324 + 3{,}7802) = \frac{15{,}2183}{18} = 0{,}8455$, and from this we have the effect required $L = 4 \,.\, 3 \,.\, 0{,}8455 \,.\, 0{,}7974 \,.\, 24 \,.\, 124 = 2390 = 4{,}34$ horse power, instead of 5 horse power, found when the sails are *warped.*

§ 219. ***Loss by Friction.***—A considerable part of the mechanical effect developed by the wind on the sails, is consumed by the friction of the windshaft at the neck, especially if the diameter of this

be great, as is not unfrequently the case. We may assume that the whole weight of the sail wheel bears on the neck, and thus leave out of consideration the pressure on the lower or back bearing. Although we shall thus find an excess of friction, yet this is compensated by leaving out of consideration the friction arising on the back pivot from the force of the wind in the axial direction. As the back pivot is much less in diameter than the neck or front gudgeon, this simplification of the problem may be the more readily admitted. This being assumed, we have from the weight G of the whole wheel, $F = f\,G =$ the friction, and if $r =$ the radius of the neck, and $\omega\, r$ the angular velocity, the mechanical effect consumed:

$$F \omega r = f\, G \omega r = 0{,}1047 \,.\, u f\, G\, r = f\, G \frac{r}{l}\, v,$$

if v be the velocity at the periphery of the sail wheel.

This being allowed, the useful effect of a windmill with plane sails:

$$L = 3\, n\, \gamma\, k_1\, l_1 \,.\, \frac{c^3}{2g} - f\, G \frac{r}{l}\, v,$$

and that of one with warped sails:

$$L = \frac{4}{3}\, n\, \gamma\, k\, l_1 \,.\, \frac{c^3}{2g} - f\, G \frac{r}{l}\, v.$$

From the formula:

$$L = \frac{3\,(c \sin. \alpha - v \cos. \alpha)^2}{2g}\, v \cos. \alpha \,.\, F \gamma,$$

for the theoretical effect of an element of a sail, we may deduce the influence of the velocity of the sail on the mechanical effect, and we find that for $v \cos. \alpha = \frac{c \sin. \alpha}{3}$ (compare Vol. II. § 118), that is, for $v = \frac{c\, tang. \alpha}{3}$, the effect is a maximum. If we introduce this value into the above formula, we get

$$L = 3 \,.\, \tfrac{4}{27} \,.\, \frac{c^3 \sin. \alpha^3}{2g} F \gamma,$$

and from this we deduce that the effect will be greatest when the angle $\alpha = 90°$, or $v = \infty$. These conditions cannot be fulfilled; because, even for moderately great velocities, the prejudicial resistances, and more particularly the friction at the neck, consume so much mechanical effect, that the useful effect remaining is very small. The velocity of rotation should be great to insure a good efficiency, but it must in each case be made a special subject of calculation, as to what number of revolutions will give the maximum effect. This can only be done by calculating the effect for a series of velocities of rotation, and from these choosing the greatest, or deducing it by interpolation.

Example. Supposing the windshaft, sails, &c., of the mill in the last example weighs 7500 lbs., that the radius of the neck or gudgeon $r = \frac{1}{3}$ foot, that the co-efficient of friction $f = 0{,}1$, then the mechanical effect lost by friction at the neck $= 0{,}1 \,.\, 7500 \,.\, \omega r = 419$ feet lbs. There remains, therefore, in the wheel with *warped* sails $2798 - 419 = 2590$ feet lbs., or about 86 per cent. of the theoretical effect. When the shaft is of wood, the

neck is double the above diameter, and, hence, the loss of effect by friction is double, or the efficiency is only 0,70.

§ 220. *Experiments.*—Experiments or observations on windmills, of accuracy sufficient to test our theory, are not extant. There is no lack of general statements of the results of the effects of different windmills, but these are not of a nature to serve for judging of the *efficiency* of the machines referred to, inasmuch as the velocity of the wind has been either altogether undetermined, or ascertained by instruments not sufficiently trustworthy. The experiments of Coulomb and Smeaton are still the most complete, there being, in fact, none of recent date. Coulomb made his experiments on one of the many windmills in the neighborhood of Lille; and from the circumstance of the work done, being the pressing of oil by means of stampers, a kind of work, the mechanical effect consumed in which is easily calculated, deductions from these experiments may be very safely made. The four sails of this mill were *warped* in the Dutch style, with the angle of impulse from $68\frac{3}{4}°$ to $81\frac{1}{4}°$, and each of them contained about 20 square metres, or 215 square feet. The experiments were made when the velocity of the wind was from 7 to 30 feet per second, the velocity at the periphery being from 23 to 70 feet, and the results correspond, according to Coriolis (*see* "Calcul de l'effet des Machines), with those of the theory above given. It is, besides, easy to perceive that, for the better construction, when *warped* sails are used, the mean value of $\frac{3\ sin.\ a^2 - 2}{sin.\ a^3}$, cannot vary very much from that which is deduced by calculation in the first example § 218, viz. $= 0,880$. If, now, we introduce this into the general formula, we obtain the following very simple expression for the effect of a windmill:

$$L = \tfrac{4}{9} \cdot 0,88 \cdot 0,0781 \cdot n\,F\frac{c^3}{2g} = 0,000473\ n\,F\,c^3 \text{ ft. lbs.}$$

The mean of Coulomb's observations, gives

$$L = 0,026\ n\,F\,c^3 \text{ kilogrammetres, or}$$
$$L = 0,000511\ n\,F\,c^3 \text{ ft. lbs.}$$

or a near approximation to the theoretical determination. We may with safety assume

$$L = 0,00048\ n\,F\,c^3 \text{ ft. lbs.}$$

This formula only gives satisfactory results, however, when the velocity at the extremity of the sails is about $2\frac{1}{2}$ times that of the wind, as indicated by theory to be the best velocity.

Example. Suppose a windmill of 4 horse power, when the velocity of the wind is 16 feet per second is required. What sail surface must it have? According to the last formula, $n\,F = \frac{4\ .\ 510}{0,00048\ .\ 16^3} = \frac{4249320}{4096} = 1030$ square feet; that is, for 5 sails each, 206 square feet. If l_1 the length $= 5$ times the mean breadth b, then $5\ b^2 = 206$ $\therefore b = \sqrt{41} = 6\frac{1}{3}$ feet, and the length $l_1 = 31\frac{2}{3}$ feet.

§ 221. *Smeaton's Maxims.*—The great English civil engineer, John Smeaton, instituted a very complete inquiry into the power of wind, and made a series of experiments, the results of which are given in the following table:—

TABLE,

Exhibiting the Results of Nineteen Sets of Experiments on Windmill Sails, of various Structures, Positions, and Extents of Surface,

The description of sails made use of.	Number.	Angle at the extremities.	Greatest angle.	Turns of the Sails unloaded.	Turns of the Sails at the Maximum.	Load at the Maximum.	Greatest Load.	Product.	Extent of Surface.	Ratio of greatest Velocity to the Velocity at a Maximum.	Ratio of greatest Load to the Load at a maximum.	Ratio of Surface to the Product.
		degrees	degrees			lb.	lb.		sq. in.			
Plane sails, at an angle of 55° . .	1	35	35	66	42	7.56	12.59	318	404	10.7	10:6	10:7.9
Plane sails, weathered according to the common practice . .	2	12	12	—	70	6.3	7.56	441	404	—	10:8.3	10:10.1
	3	15	15	105	69	6.72	8.12	464	404	10:6.6	19:8.3	10:10.15
	4	18	18	96	66	7.0	9.81	462	404	10:7	10:7.1	10:10.15
Weathered according to M'Laurin's Theory	5	9	26½	—	66	7.0	—	462	404	—	—	10:11.4
	6	12	29½	—	70½	7.35	—	518	404	—	—	10:12.8
	7	15	32½	—	63½	8.3	—	527	404	—	—	10:13.
Sails weathered in the Dutch manner, tried in various positions.	8	0	15	120	93	4.75	5.31	442	404	10:7.7	10:8.8	10:11.
	9	3	18	120	79	7.0	8.12	553	404	10:6.6	10:8.6	10:13.7
	10	5	20	—	78	7.5	8.12	585	404	—	10:9.2	10:14.5
	11	7½	29½	123	77	8.3	9.81	639	404	10:6.8	10:8.5	10:15.8
	12	10	25	108	73	8.69	10.37	634	404	10:6.8	10:8.4	10:15.7
	13	12	27	100	66	8.41	10.94	580	404	10:6.6	10:7.7	10:14.4
Sails weathered in the Dutch manner, but enlarged towards the extremities	14	7½	22½	123	75	10.65	12.59	799	505	10:6.1	10:8.5	10:15.8
	15	10	25	117	74	11.08	13.69	820	505	10:6.3	10:8.1	10:16.2
	16	12	27	114	66	12.09	14.23	799	505	10:5.8	10:8.4	10:15.8
	17	15	30	96	63	12.09	14,78	762	505	10:6.6	10:8.2	10:15.1
Eight sails, being sectors of ellipses, in their best positions .	18	12	22	105	64½	16.42	27.87	1059	854	10:6.1	10:5.9	10:12.4
	19	12	22	96	64½	18.06	—	1165	1146	10:5.9	—	10:10.1

The experimental wheel had whips 21 inches long, the sails being 18 inches long, and 5,6 inches broad. This wheel was not moved by the impulse of wind, but was moved round in air at rest, whence it was the resistance of the air, and not its impulse, which was observed—a circumstance taking considerably from the value of the experiments. The motion of the sails against the wind, was given by means of an upright shaft, from which projected an arm $5\frac{1}{4}$ feet long, at the end of which was a seat for the model mill wheel. This upright shaft was set in motion by the observer having a cord wound round it like the peg of a top. To measure the *resistances of the air*, supposed here to be identical with the impulse of wind of the same velocity, there was a scale with weights, attached by a fine cord to the shaft of the wind wheel, and this was wound up by the power communicated to the sails. The results of these experiments correspond well *qualitatively* with our theory. They show to demonstration that the warped sail gives the best effect, and that the angles of impulse deduced by theory are actually the best. In the example to § 218, we found the angles for 7 bars, starting from next the axle, to be: 63° 29′; 69° 57′; 74° 22′; 77° 23′; 79° 30′; 81° 8′, and 82° 18′, and Smeaton found the following 6 angles to be the best, or at least very good, 72°; 71°; 72°; 74°; $77\frac{1}{2}$°; 83°; or very little different from the theory.

Smeaton remarks, too, that a deviation of 2 degrees in the angle of impulse, has no sensible influence on the mechanical effect produced by the wheel.

Smeaton draws the following *maxims* from his experiments, made at velocities varying from $4\frac{1}{2}$ to $8\frac{3}{4}$ feet per second.

1. The velocity of the windmill sails, whether unloaded or loaded, so as to produce a maximum, is nearly as the velocity of the wind, their shape and motion being the same.

2. The load at the maximum is nearly, but somewhat less than, as the square of the velocity of the wind, the shape and position of the sails being the same.

3. The effects of the same sails at a maximum are nearly, but somewhat less than, as the cubes of the velocity of the wind.

4. The load of the same sails at the maximum is nearly as the squares, and their effects as the cubes of their number of turns in a given time.

5. When the sails are loaded so as to produce a maximum at a given velocity, and the velocity of the wind increases the load containing the same: first, the increase of effect, when the increase of the velocity of the wind is smaller, will be nearly as the squares of those velocities; secondly, when the velocity of the wind is double, the effects will be nearly as 10 to $27\frac{1}{2}$; but, thirdly, when the velocities compared are more than double of that where the given load produces a maximum, the effects increase nearly in a simple ratio of the velocity of the wind.

6. If sails are of a similar figure and position, the number of

turns in a given time will be reciprocally as the radius or length of the sail.

7. The load at a maximum that sails of a similar figure and position will overcome, at a given distance from the centre of motion, will be as the cube of the radius.

8. The effect of sails of similar figure and position are as the square of the radius.

9. The velocity of the extremity of Dutch sails, as well as of the enlarged sails, in all their usual positions when unloaded, or even loaded to a maximum, is considerably quicker than the velocity of the wind.

According to these experiments, the effect of the wind on windmill sails is greater than theory indicates, or than Coulomb's experiments gave.

Literature. The most complete exposition of the theory of windmills is given in Weisbach's "Bergmaschinen Mechanik," vol. ii., and in Coriolis's "Traité du Calcul à l'effet des Machines." Smeaton's experiments are recorded in the "Philosophical Transactions," 1759 to 1776. They were collected into a separate volume, and published under the title "An experimental Enquiry concerning the natural powers of Water and Wind to turn Mills and other Machines depending on a circular motion." These papers were translated into French by Girard, in 1827. There are extracts from them in Barlow's "Treatise on the Manufactures," &c. In Nicholson's "Operative Mechanic," Brewster's, Ferguson's, &c., &c. Coulomb's experiments are given in his oft-quoted work "Théorie des Machines simples."

Mariotte wrote upon the impulse of wind, in his "Hydrostatics." He makes the impulse $P = 1{,}73 \frac{c^2}{2g} F \gamma$.

Borda, in the "Mémoires de l'Académie de Paris," 1763, has a paper; Rouse, Hutton, Woltmann, have all handled this subject. The two latter authors find P much smaller than Mariotte did, because they measured the *resistance*, not the *impulse* of the wind. The co-efficient $\zeta = \frac{4}{3}$, as found by Woltmann, is too small, or $P = \frac{4}{3} \frac{c^2}{2g} F \gamma$ is certainly too little, for he did not obtain the *constants* for his windsail wheel by direct experiment (*see* "Theorie und Gebrauch des Hydrometrischen Flügels," Hamburg, 1790). Hutton deduces from his experiments, that it is more accurate to consider the impulse and resistance of the air as increasing as $F^{0,1}$ (*see* "Philosophical and Mathematical Dictionary," vol. ii.). If we assume $\zeta = 1{,}86$ for a small surface of 1 square foot, then, for a sail of 200 square feet surface, we should have $\zeta = 200^{0,1} . 1{,}86 = 1{,}7 . 1{,}86 = 3{,}162$, which agrees well with the theoretical determination, and with what we have said above, where $\zeta = 3$ and $P = 3 . \frac{c^2}{2g} F \gamma$. In Poncelet's "Introduction à la Mécanique industrielle," there is an admirable collection and discussion of the experiments on impulse and resistance of wind.

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THE END.

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